Department of Civil and Environmental Engineering, Fall Semester, 2026

ENCE 353 Midterm 2, Open Notes and Open Book

Name: _____

E-mail (print neatly!): _____

Exam Format and Grading. This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are three questions. Partial credit will be given for partially correct answers, so please show all your working.

Please see the **class web page for instructions on how to submit your exam paper**.

Question	Points	Score
1	15	
2	10	
3	15	
Total	40	

Question 1: 15 points

Moment-Area Method. Figure 1 is a front elevation view of a cantilevered beam carrying a single point load P. EI is constant along the beam structure A-B-C-D.

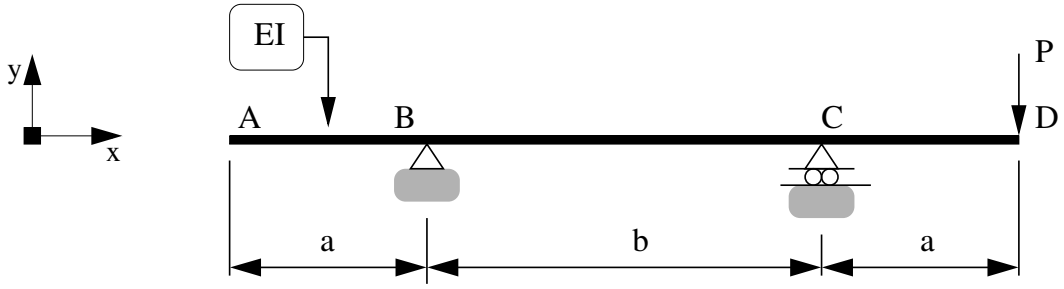


Figure 1: Cantilevered beam carrying a single applied load P.

[1a] (2 pts) Draw and label the M/EI diagram.

[1b] (2 pts) Draw and label the **moment area analysis diagram** (i.e., with rotations, tangents, displacements, etc) for this problem.

[1c] (3 pts) Use the method of **moment area** to show that the anticlockwise rotation of the beam at B is:

$$\theta_B = \frac{Pab}{6EI} \tag{1}$$

and the clockwise rotation of the beam at C is:

$$\theta_C = \frac{Pab}{3EI}. \tag{2}$$

Show all of your working.

[1d] (3 pts) Use the method of **moment area** to show that the vertical deflection of the beam at points A and D (measured downwards) is:

$$y_A = \frac{Pa^2 b}{6EI} \tag{3}$$

and

$$y_D = \frac{Pa^2}{3EI}\,(a+b). \tag{4}$$

Show all of your working.

[1e] (5 pts) Using parts [1a] - [1d] as a starting point, show – do not simply assume – that the maximum upwards deflection of the beam occurs at a distance $b/\sqrt{3}$ from B, and that its value is:

$$y_{\text{maximum upward}} = \frac{Pab^2}{9\sqrt{3}EI}. \tag{5}$$

Show all of your working.

Question 2: 10 points

Derive Elastic Curve for Beam Deflection. Figure 2 is a front elevation view of a cantilevered beam carrying a single point load P. EI is constant along the beam structure A-B-C-D.

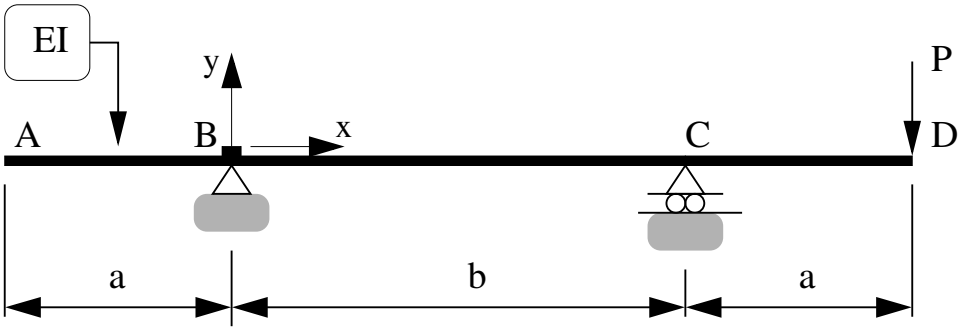


Figure 2: Cantilevered beam carrying a single applied load P.

[2a] (5 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{-M(x)}{EI}\right], \tag{6}$$

(notice the minus sign on M(x)) and appropriate boundary conditions, show that that vertical beam deflection along the beam segment B-C is:

$$y(x) = \frac{Pa}{6EIb}\left(b^2 - x^2\right)x. \tag{7}$$

[2b] (5 pts) Hence (i.e., using your results from part [2a] as a starting point), show that the maximum upwards deflection of the beam occurs at a distance $b/\sqrt{3}$ from B, and that its value is:

$$y_{\text{maximum upward}} = \frac{Pab^2}{9\sqrt{3}EI}. \tag{8}$$

Show all of your working.

Question 3: 15 points

Compute Cable Profile and Tension in a Suspension Bridge. Figure 3 is a front elevation view of a cable span in a suspension bridge.

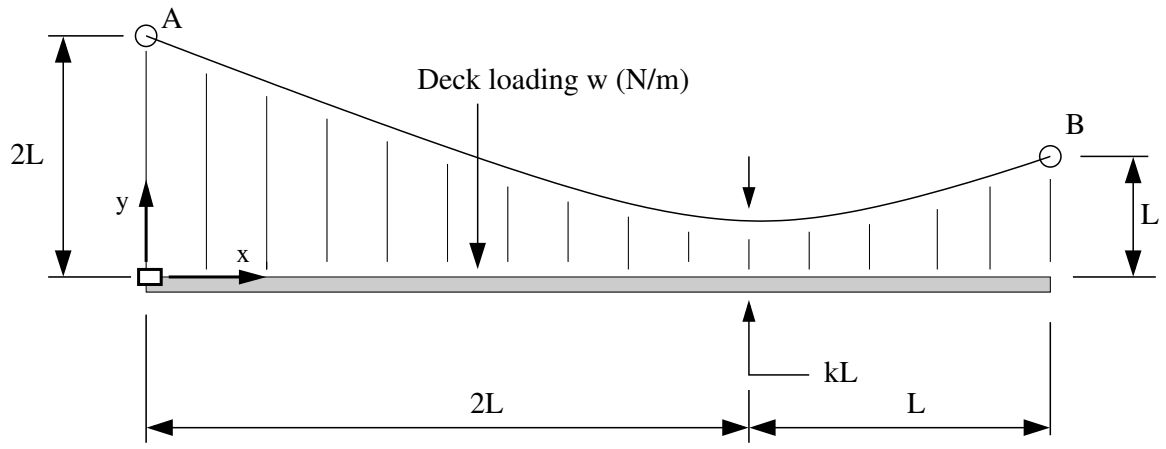


Figure 3: Elevation view of a single cable span in a suspension bridge (not to scale).

The span has length 3L and carries a uniform load w (N/m). The lowest point of the cable profile is kL above the bridge deck.

[3a] (4 pts) If the lowest point in the cable profile is at distance 2L from the left-hand side, show that:

$$k = \left[\frac{2}{3}\right]. \tag{9}$$

[3b] (4 pts) Hence, show that the horizontal component of cable force is:

$$H = \left[\frac{3}{2}\right] wL. \tag{10}$$

[3c] (4 pts) Compute the horizontal and vertical components of cable force at supports A and B.

[3d] (3 pts) Draw and label a sketch showing how the tension force in the cable varies along its length. Clearly indicate where the maximum and minimum values of cable tension will occur.