

ENCE 353 Solutions to Midterm 1

Question 1 (15 points): Bending Moments and Qualitative Displacements in a Multi-span Beam Structure.

Consider the multi-span connected beam/column structure shown in Figure 1.

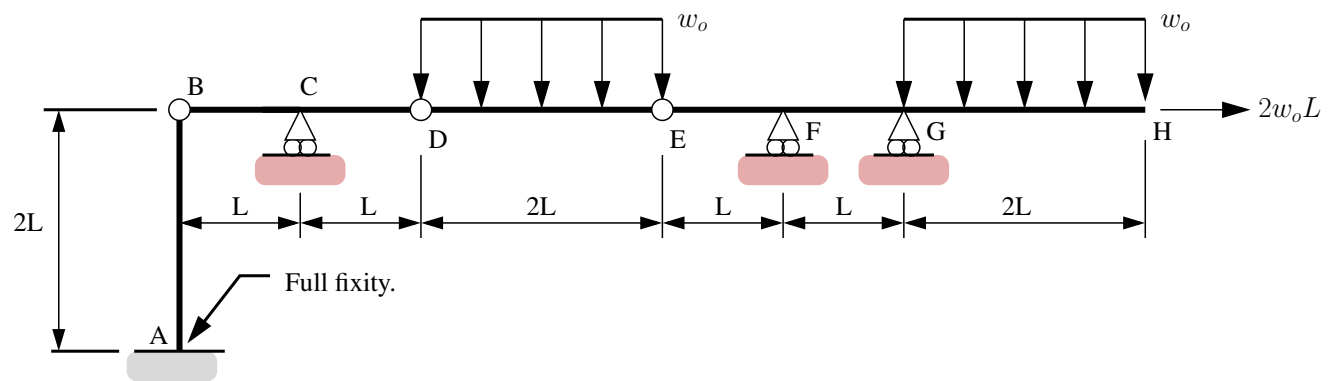


Figure 1. Front elevation view of multi-span connected beam/column structure.

The cantilever is fully-fixed to the foundation at Point A. Points B, D and E are hinges. The beam segments D-E and G-H carry a uniformly distributed load w_o (force/per unit length). Finally, a horizontal point load $2w_oL$ is applied at point H.

Part [1a]: (3 pts). Compute the vertical reaction forces at nodes C, F and G, and shear, moment, and axial reaction forces at node A.

Sol'n: First, we observe that the structure is statically determinate. Support reactions $r = 12$, $n = 4$ substructures, hence:

$$\hat{i} = r - 3n = 0 \quad \longrightarrow \quad \text{statically determinate.} \quad (1)$$

Next, we can pull the structure apart and systematically examine equilibrium of the individual segments.

Figure 2 summarizes reaction forces on the individual segments. Notice that the shear and axial forces in B-C are transferred into column A-B as axial and shear forces, respectively.

Part [1b]: (4 pts). Draw and label diagrams showing how the **bending moment** and **axial force** vary along the beam, nodes B through H. Clearly indicate on your bending moment diagram, regions that are in

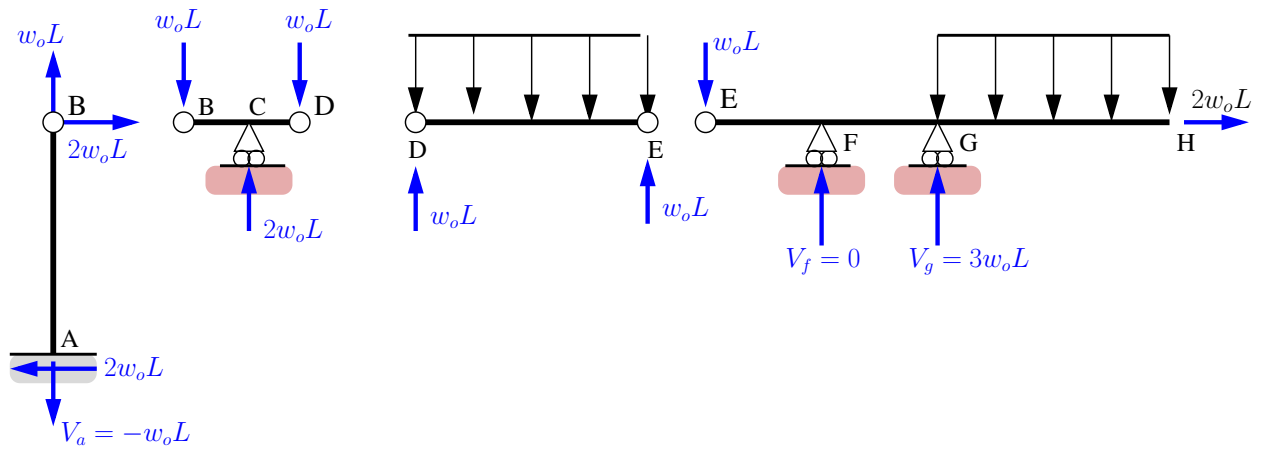
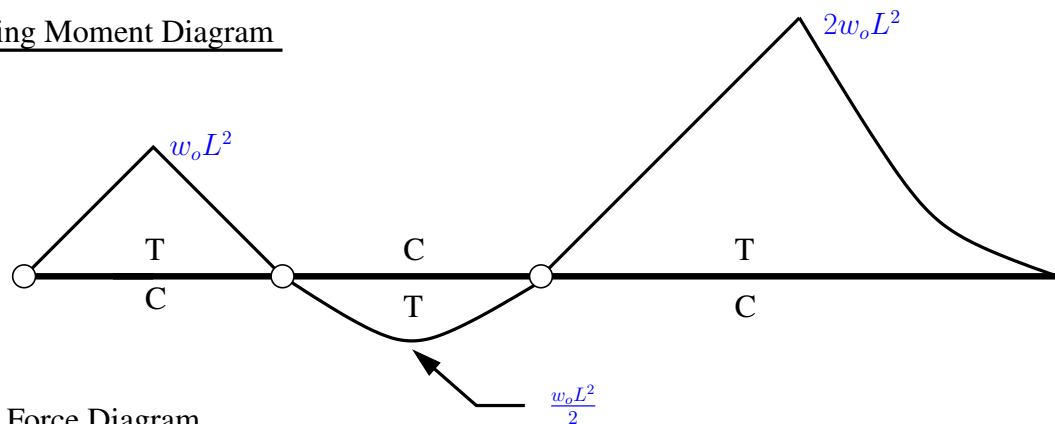


Figure 2. Reaction forces in individual beam/column elements.

tension/compression.

Sol'n: Summary of bending moments and axial forces in B-H:

Bending Moment Diagram



Axial Force Diagram

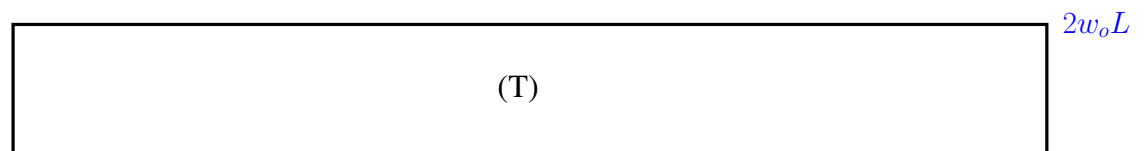


Figure 3. Bending moments and axial force in beam elements B-C-D-E-F-G-H.

Part [1c]: (4 pts). Draw and label a diagram showing how the **bending moment** and **axial force** in column element A-B. Clearly indicate on your bending moment diagram, regions that are in tension/compression.

Sol'n: Summary of bending moments and axial forces in A-B:

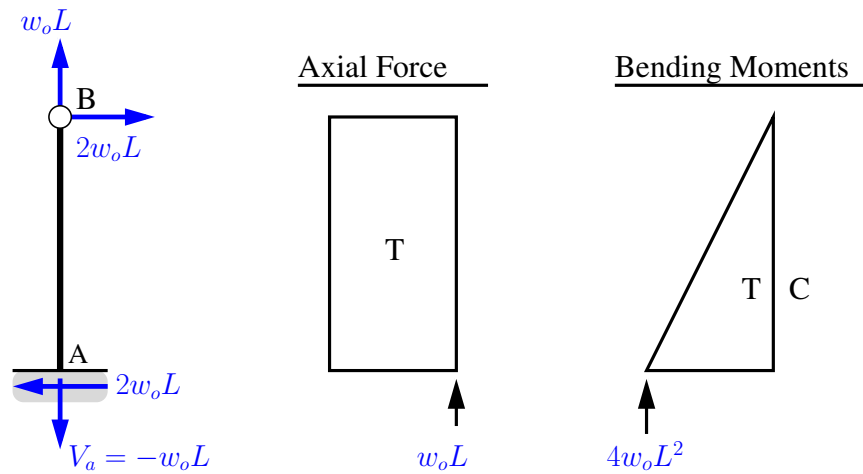


Figure 4. Bending moments and axial force in beam element A-B.

Now suppose that the flexural stiffness of the beam and column elements is EI throughout.

Part [1d]: (4 pts). With your solutions to Parts 1b and 1c in mind, create a sketch of the beam and column displacements, highlighting in an **exaggerated qualitative way**, movement of the nodes and curvature in the beam elements and column.

Sol'n: Original structure and qualitative sketch of displacements.

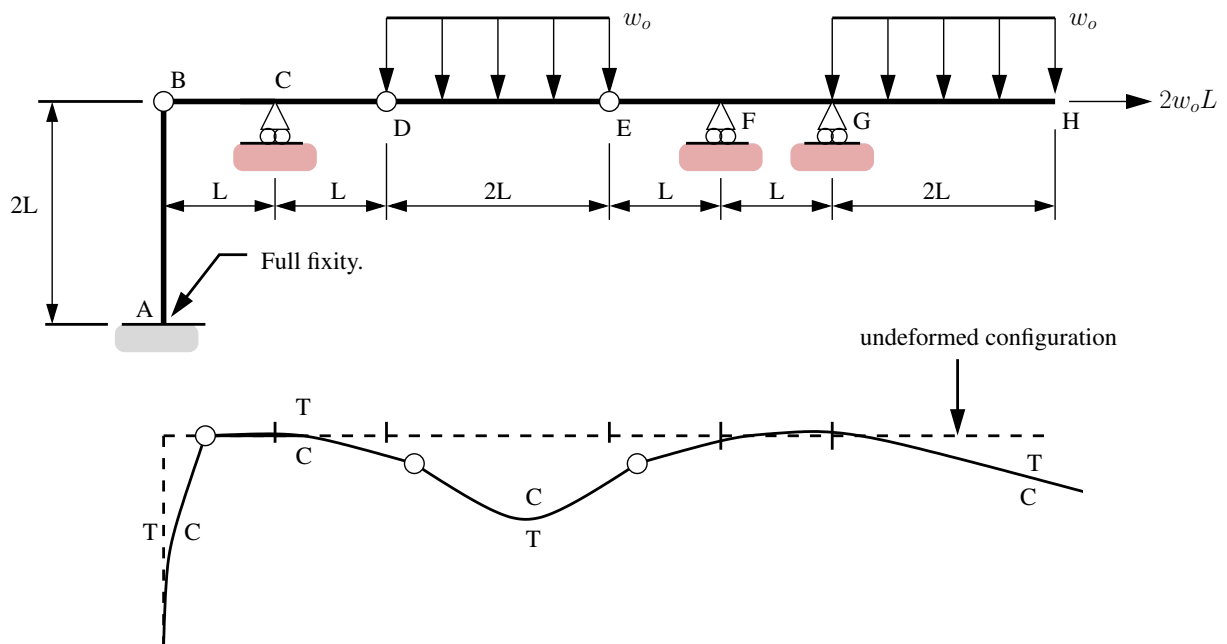


Figure 5. Qualitative sketch of displacements.

Question 2 (17 points): Analysis of a Zigzag Truss Structure.

Consider the zigzag truss structure shown in Figure 6.

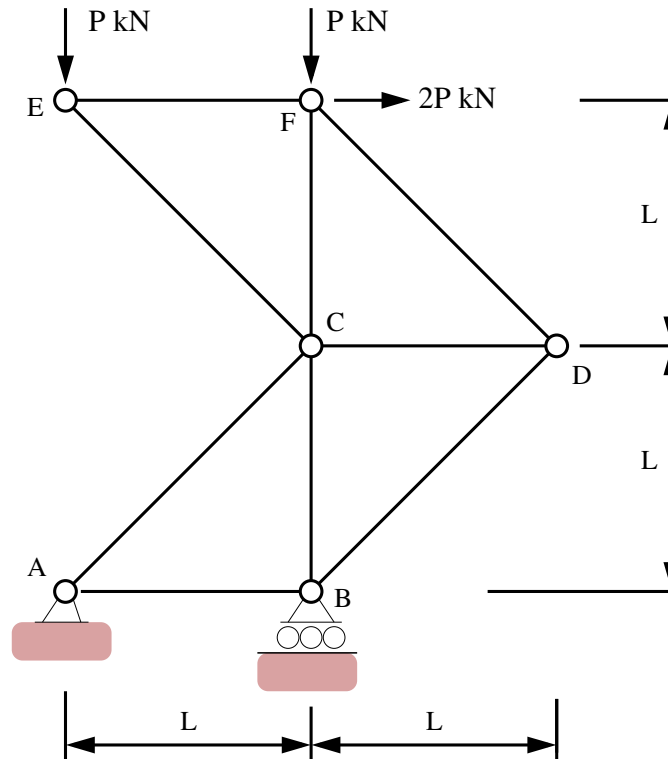


Figure 6. Elevation view of a crooked truss structure.

Horizontal and vertical loads P (kN) and $2P$ (kN) are applied at nodes E and F as shown in the figure.

Part [2a] (3 pts). Compute the degree on statical indeterminacy for this system.

Sol'n: $m = 9$, $r = 3$, $j = 6$. Hence,

$$m + r - 2j = 9 + 3 - 2(6) = 0, \quad \longrightarrow \quad \text{statically determinate.} \quad (2)$$

Part [2b] (3 pts). Compute the **total support reactions** at A and B.

Sol'n: Taking moments about A,

$$\sum M_A = 0, \quad PL + 2P(2L) - V_B L = 0, \quad \longrightarrow \quad V_B = 5P. \quad (3)$$

Sum forces in the vertical direction:

$$\sum V = 0, \quad V_A + V_B - 2P = 0, \longrightarrow V_A = -3P. \quad (4)$$

Sum forces in the horizontal direction:

$$\sum H = 0, \quad H_A + 2P = 0, \longrightarrow H_A = -2P. \quad (5)$$

Total support reaction at A = $[V_A^2 + H_A^2]^{1/2} = \sqrt{13}$ kN. Total support reaction at B = 5 kN.

Part [2c] (10 pts). Using the method of joints (or otherwise) compute the distribution of tension and compression forces throughout the structure. Show all of your working.

Joint A:

$$\sum V = 0, \quad V_A + \frac{F_{ac}}{\sqrt{2}} = 0, \longrightarrow F_{ac} = 3\sqrt{2}P \text{ (T)}. \quad (6)$$

$$\sum H = 0, \quad H_A + \frac{F_{ac}}{\sqrt{2}} + F_{ab} = 0, \longrightarrow F_{ab} = -P \text{ (C)}. \quad (7)$$

Joint E:

$$\sum V = 0, \quad -\frac{F_{ce}}{\sqrt{2}} - P = 0, \longrightarrow F_{ce} = -\sqrt{2}P \text{ (C)}. \quad (8)$$

$$\sum H = 0, \quad \frac{F_{ce}}{\sqrt{2}} + F_{ef} = 0, \longrightarrow F_{ef} = P \text{ (T)}. \quad (9)$$

Joint B:

$$\sum H = 0, \quad \frac{F_{bd}}{\sqrt{2}} - F_{ab} = 0, \longrightarrow F_{bd} = -\sqrt{2}P \text{ (C)}. \quad (10)$$

$$\sum V = 0, \quad \frac{F_{bd}}{\sqrt{2}} + F_{bc} + V_B = 0, \longrightarrow F_{bc} = -4P \text{ (C)}. \quad (11)$$

Joint D:

$$\sum V = 0, \quad \frac{F_{bd}}{\sqrt{2}} - \frac{F_{df}}{\sqrt{2}} = 0, \rightarrow F_{df} = -\sqrt{2}P \text{ (C)}. \quad (12)$$

$$\sum H = 0, \quad F_{cd} + \frac{F_{bd}}{\sqrt{2}} + \frac{F_{df}}{\sqrt{2}} = 0, \rightarrow F_{cd} = 2P \text{ (T)}. \quad (13)$$

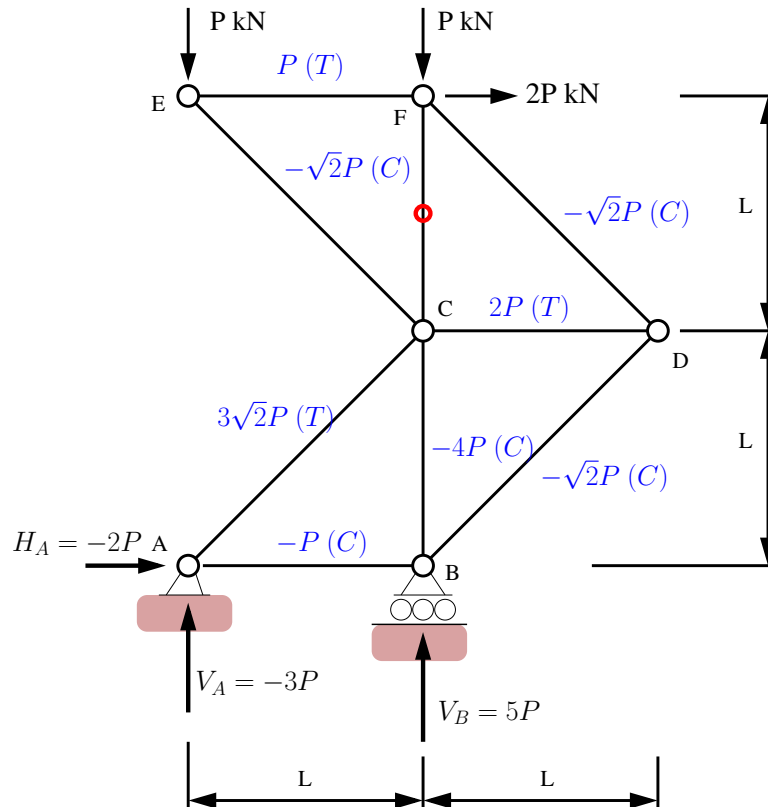
Joint F:

$$\sum H = 0, \quad -F_{ef} + \frac{F_{df}}{\sqrt{2}} + 2 = 0. \quad (14)$$

$$\sum V = 0, \quad -F_{cf} - \frac{F_{df}}{\sqrt{2}} - P = 0, \rightarrow F_{cf} = 0. \quad (15)$$

Joint C: Can use this joint to verify the truss is in equilibrium.

Summary: Distribution of tensile, compressive, and zero-force members:



Part [2d] (3 pts). Now suppose that the maximum tensile force any member can support is 10 kN, and that the maximum allowable compressive force is:

$$P_{ci} = 5 \left(\frac{L}{L_i} \right)^2 kN, \quad (16)$$

where L_i is the length of the i -th element, and P_{ci} is the maximum allowable compressive force of the i -th element before buckling. Determine the maximum value of P (kN) that the tower can safely carry.

Sol'n: From the analysis: (a) Maximum tensile force is: $3\sqrt{2}P$ (T), (b) Maximum compressive force in elements of length $\sqrt{2}L$ is: $-\sqrt{2}P$ (C), (c) Maximum compressive force in elements of length L is: $-4P$ (C).

Case (a): Tension constraint:

$$3\sqrt{2}P \leq 10\text{kN} \quad \longrightarrow \quad P \leq 2.35\text{kN}. \quad (17)$$

Case (b): Compression constraint:

$$\sqrt{2}P \leq \frac{5}{2}\text{kN} \quad \longrightarrow \quad P \leq 1.76\text{kN}. \quad (18)$$

Case (c): Compression constraint:

$$4P \leq 5\text{kN} \quad \longrightarrow \quad P \leq 1.25\text{kN}. \quad (19)$$

The most restrictive constraint is buckling of element BC. The structure will fail once P exceeds 1.25 kN.

Question 3 (8 points): Method of Sections.

The frame tower (see Figure 7) carries horizontal and vertical loads P kN and $2P$ kN at joints C, E, G and H.

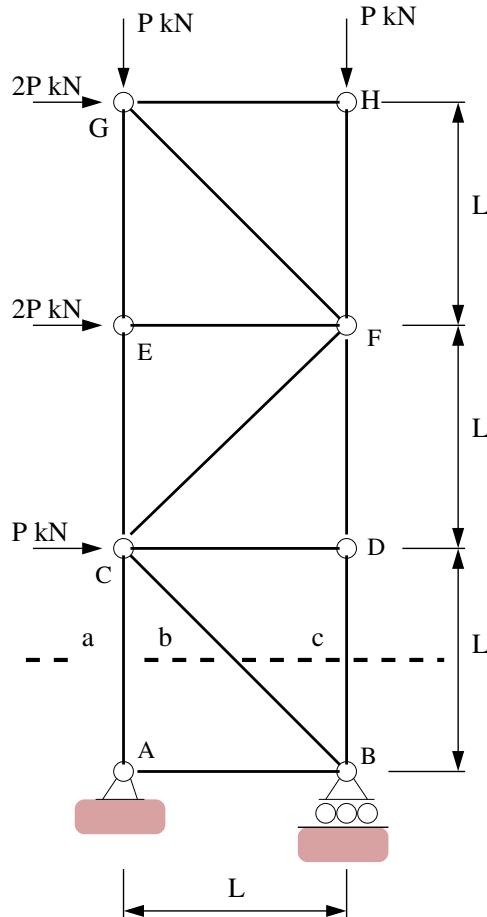


Figure 7. Elevation view of frame tower.

Part [3a]: (8 pts). Use the **method of sections** to determine the forces in members a, b and c, as a function of the problem parameters L and P . Clearly indicate if the members are in tension or compression.

Soln: Cut the structure at a-b-c, then take moments about joint C.

$$\sum M_C = 0, \longrightarrow \overline{BD} \cdot L + PL + 2PL + 4PL = 0 \longrightarrow c = \overline{BD} = -7P(C). \quad (20)$$

Similarly, taking moments about joint B,

$$\sum M_B = 0, \longrightarrow \overline{AC} \cdot L + PL - PL - 4PL - 6PL = 0 \longrightarrow a = \overline{AC} = 10P(T). \quad (21)$$

Summing forces in the vertical direction,

$$\sum V = 0, \longrightarrow \overline{AC} + \frac{\overline{BC}}{\sqrt{2}} + \overline{BD} = -2P, \longrightarrow b = \overline{BC} = -5\sqrt{2}P(C). \quad (22)$$

Also note that:

1. The vertical reactions forces at A and B are $V_A = -10P$ and $V_B = 12P$.
2. $\overline{AB} = 5P(T)$ and that for the whole structure, the sum of forces in the horizontal direction is zero.