

Solutions to Homework 1

Question 1: 20 points.

Consider the combined multi-span beam/truss structure shown in Figure 1.

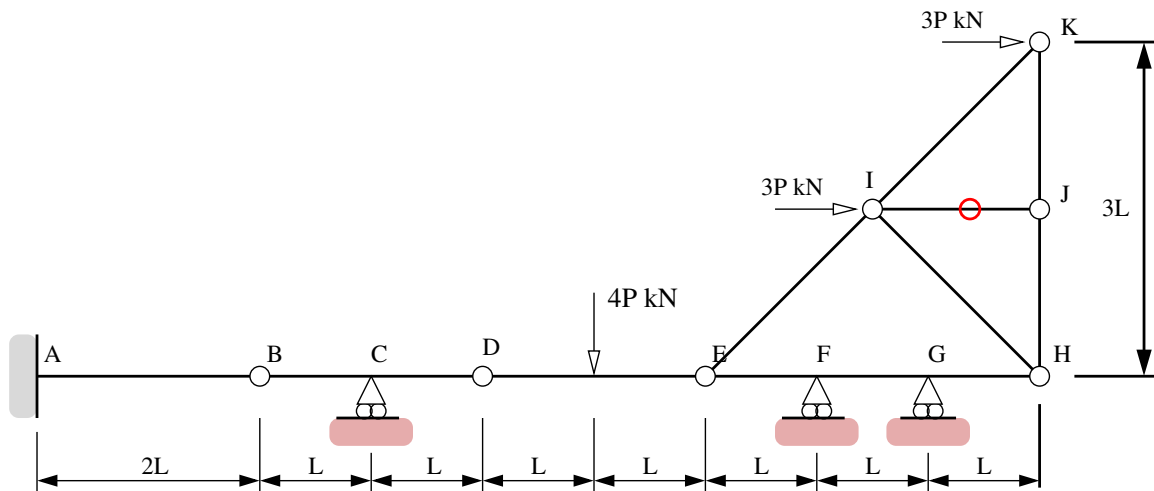


Figure 1: Front elevation view of multi-span beam structure.

The cantilever is fully-fixed to the wall at Point A. Points B, D, E and H are hinges. Horizontal and vertical point loads $3P$ (kN) and $4P$ (kN) are applied to the truss as shown in Figure 1.

Part [1a]. Compute the degree of indeterminacy for the articulated beam structure (A-B-C-D-E-F-G-H).

Sol'n: Here $f = 6$ and $r = 3$. Hence, $\hat{i} = f - 3 - r = 0$. It's statically determinate.

Part [1b]. Identify the zero-force members in the truss structure.

Sol'n: See red dots on Figure 1.

Part [1c]. Compute the distribution of forces throughout the truss structure. Draw a diagram summarizing your results.

Sol'n: Remove zero-force member from the truss, then consider equilibrium at nodes I and K. This gives:

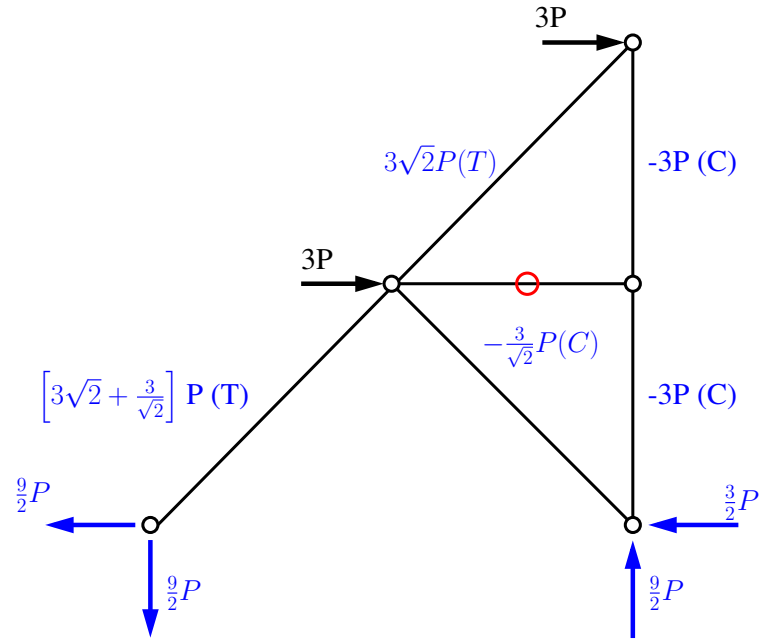


Figure 2: Summary of forces acting on truss structure

Part [1d]. Compute the vertical reaction forces at nodes F and G.

Sol'n: Isolate substructures D-E and E-F-G-H, and consider equilibrium.

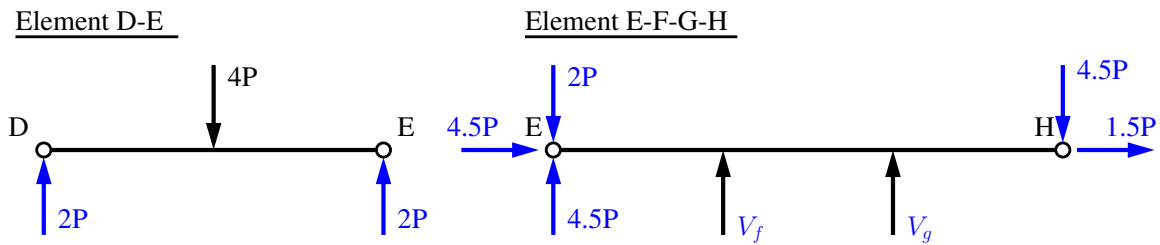


Figure 3: Summary of forces acting on truss elements D-E and E-F-G-H.

Taking moments about E for the right-most substructure:

$$\sum M_E = 0, \quad V_f L + V_g(2L) = (4.5P)(3L) \quad \longrightarrow \quad V_f + 2V_g = 13.5P. \quad (1)$$

$$\sum V = 0, \quad V_f + V_g + 4.5P = 2P + 4.5P \quad \longrightarrow \quad V_f + V_g = 2P. \quad (2)$$

From equations 1 and 2, $V_g = 11.5P$ and $V_f = -9.5P$.

Part [1e]. Draw and label diagrams showing how the **bending moment** and **axial force** vary along the beam, nodes A through H. Clearly indicate on your bending moment diagram, regions that are in tension/compression.

Sol'n: The beam is in tension for nodes A – H.

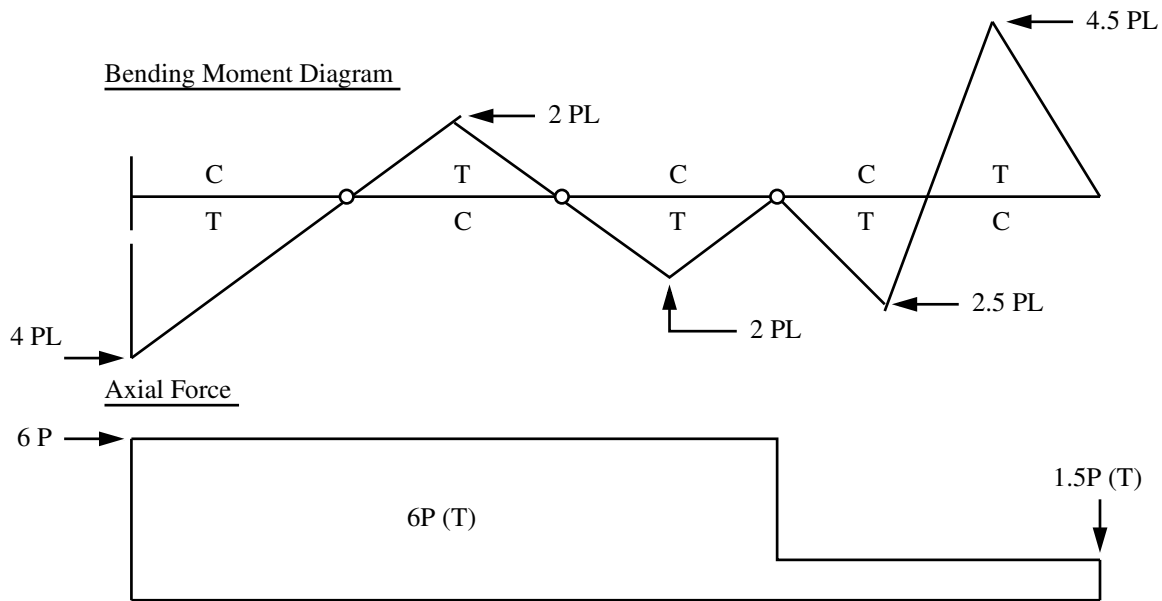


Figure 4: Summary of bending moments and axial forces for beam elements A through H.

Question 2: 10 points.

Classify each of the structures in Figure 5 as statically determinate, statically indeterminate, stable or unstable. For those structures that are indeterminate, specify the degree of indeterminacy.

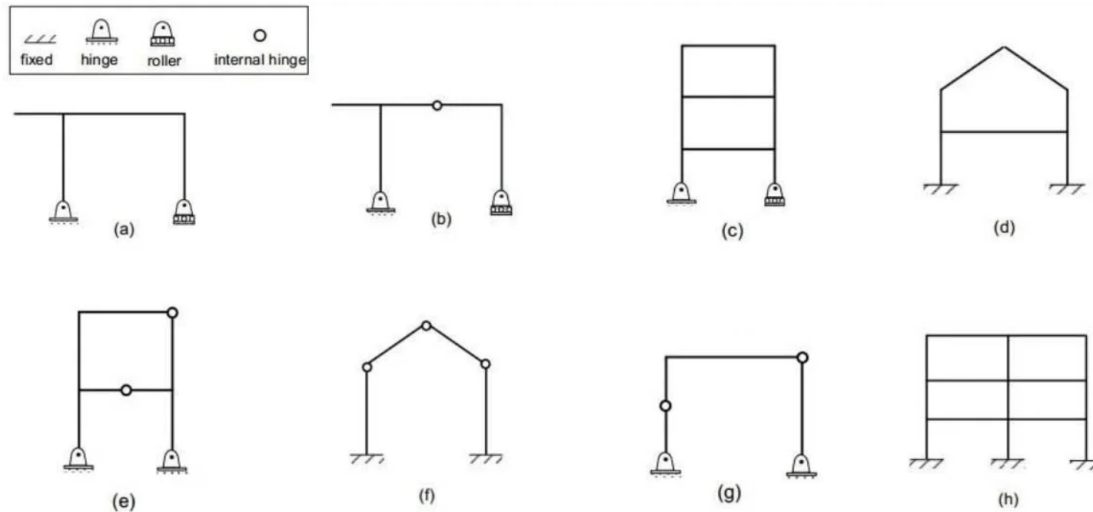


Figure 5: Assortment of statically determinate and indeterminate frame structures.

Part (a): One member, $n = 1$. No reactions $r = 3$. Test: $r - 3n = 3 - 3 = 0$, $\rightarrow$ statically determinate.

Part (b): Two members, $n = 1$. No reactions $r = 5$. Test: $r - 3n = 5 - 3 \cdot 2 = -1$, $\rightarrow$ unstable.

Part (c): Using ring method. No rings $n = 3$. No releases = 3. Determinacy $i = 3n - r = 3 \cdot 3 - 3 = 6$.

Part (d): Using ring method. No rings $n = 2$. No releases = 0. Determinacy $i = 3n - r = 3 \cdot 2 - 0 = 6$. $\rightarrow$ Statically indeterminate, stable.

Part (e): Using ring method. No rings $n = 2$. No releases = 4. Determinacy $i = 3n - r = 3 \cdot 2 - 4 = 2$, $\rightarrow$ Statically indeterminate, stable.

Part (f): Using ring method. No rings $n = 1$. No releases = 3. Determinacy $i = 3n - r = 3 \cdot 1 - 3 = 0$, $\rightarrow$ Statically determinate, stable.

Part (g): Using ring method. No rings $n = 1$. No releases = 4. Degree of indeterminacy $i = 3n - r = 3 \cdot 1 - 4 = -1$, $\rightarrow$ unstable.

Part (h): Using ring method. No rings $n = 6$. No releases = 0. Degree of indeterminacy $i = 3n - r = 3 \cdot 6 - 0 = 18$, $\rightarrow$ Statically indeterminate, stable.

Question 3: 10 points.

Consider the crane tower structure shown in Figure 6.

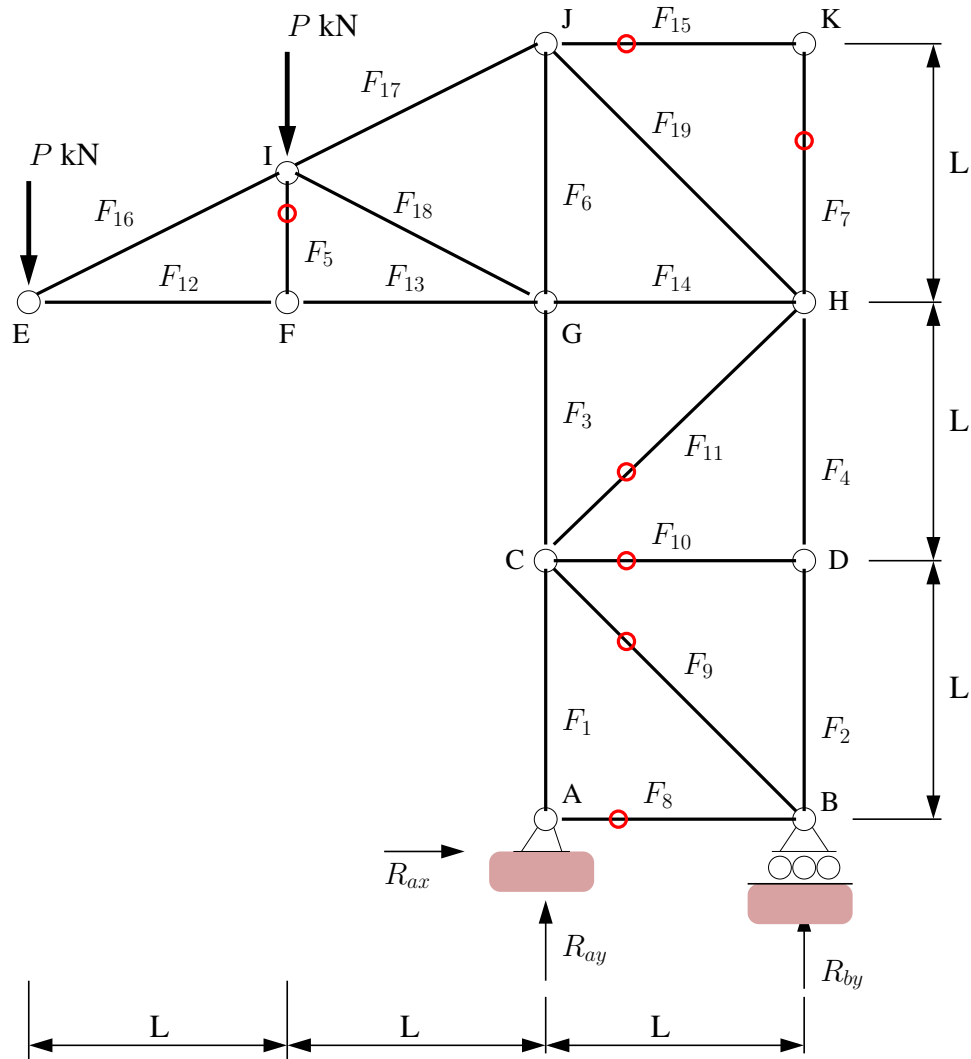


Figure 6: Elevation view of a simple crane tower.

Point loads $\mathbf{P}$ (kN) are applied at nodes E and I as shown in the figure.

Part [3a]. Compute the support reactions at A and B.

Sol'n: First, take moments about A (for the whole structure):

$$\sum M_A = 0 \longrightarrow R_{by} = -3P \text{ kN}. \quad (3)$$

Look at equilibrium in vertical direction (for the whole structure):

$$\sum V = 0 \longrightarrow R_{ay} + R_{by} = 2P \text{ kN} \longrightarrow R_{ay} = 5P \text{ kN}. \quad (4)$$

Finally, consider equilibrium in horizontal direction:

$$\sum H = 0 \longrightarrow R_{ax} = 0 \text{ kN}. \quad (5)$$

Part [3b]. Identify all of the zero-force members. If you wish, you can simply copy and annotate Figure 6.

Sol'n: Seven zero-force members (see red circles on Figure 6).

Part [3c]. Using the method of joints (or otherwise) compute the distribution of tension and compression forces throughout the crane structure. Draw and label a diagram showing the distribution of forces in the simplified crane tower structure.

Sol'n: There are 11 joints, therefore 22 equations of equilibrium to consider. Note, however, that joint K only connects to zero-force elements. So, we will only look at equilibrium for the ten remaining joints:

At Joint A:

$$\sum H = 0 \quad R_{ax} + F_8 = 0 \longrightarrow F_8 = 0 \text{ kN}. \quad (6)$$

$$\sum V = 0, \quad R_{ay} + F_1 = 0 \longrightarrow F_1 = -5P \text{ kN}(C). \quad (7)$$

At Joint B:

$$\sum H = 0, \quad F_8 + \frac{1}{\sqrt{2}}F_9 = 0 \longrightarrow F_9 = 0 \text{ kN}. \quad (8)$$

$$\sum V = 0, \quad F_2 + \frac{1}{\sqrt{2}}F_9 + R_{by} = 0 \longrightarrow F_2 = 3P \text{ kN}(T). \quad (9)$$

At Joint C:

$$\sum H = 0, \quad \frac{1}{\sqrt{2}}F_9 + F_{10} + \frac{1}{\sqrt{2}}F_{11} = 0 \longrightarrow F_{11} = 0 \text{ kN}. \quad (10)$$

$$\sum V = 0, \quad F_1 - F_3 + \frac{1}{\sqrt{2}}F_9 - \frac{1}{\sqrt{2}}F_{11} = 0. \longrightarrow F_3 = -5P \, kN(C). \quad (11)$$

At Joint D:

$$\sum H = 0, \quad F_{10} = 0 \, kN. \quad (12)$$

$$\sum V = 0, \quad F_2 - F_4 = 0 \longrightarrow F_4 = 3P \, kN(T). \quad (13)$$

At Joint E:

$$\sum V = 0, \quad \frac{F_{16}}{\sqrt{5}} - P = 0 \longrightarrow F_{16} = \sqrt{5}P \, kN(T). \quad (14)$$

$$\sum H = 0, \quad \frac{2F_{16}}{\sqrt{5}} + F_{12} = 0 \longrightarrow F_{12} = -2P \, kN(T). \quad (15)$$

At Joint F:

$$\sum V = 0, \quad F_8 = 0 \, kN(T). \quad (16)$$

$$\sum H = 0, \quad F_{12} - F_{12} = 0 \longrightarrow F_{13} = -2P \, kN(T). \quad (17)$$

At Joint I:

$$\sum H = 0, \quad \frac{2}{\sqrt{5}}F_{16} - \frac{2}{\sqrt{5}}F_{17} - \frac{2}{\sqrt{5}}F_{18} = 0. \longrightarrow F_{17} = \frac{3\sqrt{5}}{2}P \, kN(T). \quad (18)$$

$$\sum V = 0, \quad P + F_5 + \frac{1}{\sqrt{5}}F_{16} - \frac{1}{\sqrt{5}}F_{17} + \frac{1}{\sqrt{5}}F_{18} = 0, \longrightarrow F_{18} = -\frac{\sqrt{5}}{2}P \, kN(C). \quad (19)$$

At Joint G:

$$\sum H = 0, \quad F_{13} - F_{14} + \frac{2}{\sqrt{5}}F_{18} = 0. \longrightarrow F_{14} = -3P \, kN(C). \quad (20)$$

$$\sum V = 0, \quad -F_3 + F_6 + \frac{1}{\sqrt{5}}F_{18} = 0. \longrightarrow F_6 = -\frac{9}{2}P \text{ kN}(C). \quad (21)$$

At Joint H:

$$\sum H = 0, \quad \frac{1}{\sqrt{2}}F_{11} + F_{14} + \frac{1}{\sqrt{2}}F_{19} = 0. \longrightarrow F_{14} = -3P \text{ kN}(C). \quad (22)$$

$$\sum V = 0, \quad F_4 - F_7 + \frac{1}{\sqrt{2}}F_{11} - \frac{1}{\sqrt{2}}F_{19} = 0 \longrightarrow F_{19} = 3\sqrt{2}P \text{ kN}(T). \quad (23)$$

At Joint J:

$$\sum H = 0, \quad F_{15} - \frac{2}{\sqrt{5}}F_{17} + \frac{1}{\sqrt{2}}F_{19} = 0. \quad (\text{check equilibrium}) \quad (24)$$

$$\sum V = 0, \quad F_6 + \frac{1}{\sqrt{5}}F_{17} + \frac{1}{\sqrt{2}}F_{19} = 0 \quad (\text{check equilibrium}) \quad (25)$$

Part [3d]. If the maximum force any member can support is 10 kN in tension and 7 kN in compression, determine the maximum value of P that the crane tower can safely carry.

Sol'n: Find limiting cases:

Maximum tensile force is $3\sqrt{2}P$ (kN).

Maximum compressive force is $-4.5P$ (kN).

Limiting constraint is: $-4.5P = 7$ kN, therefore $P_{max} = 1.55$ kN.

Question 4: 20 points. Consider the leaning tower structure shown in Figure 8.

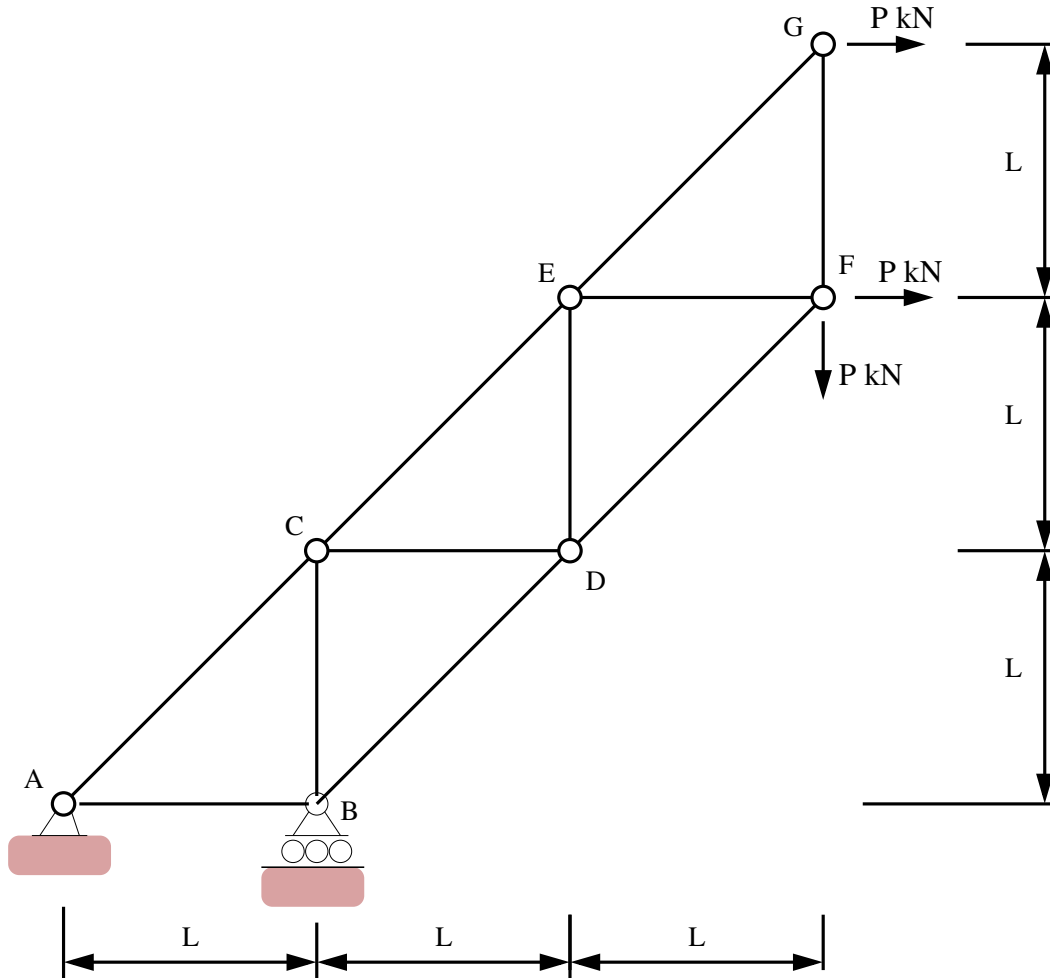


Figure 7: Elevation view of a leaning tower structure.

Horizontal and vertical loads $\mathbf{P}$ (kN) are applied at nodes F and G as shown in the figure.

Note: No joints $j = 7$, no members $m = 11$, and no reactions $r = 3$. Hence $m + r = 2j \rightarrow$ statically determinate.

Part [4a]. Compute the **total support reactions** at A and B.

Sol'n: First, take moments about A (for the whole structure):

$$\sum M_A = 0 \rightarrow R_{by}L = 3PL + 2PL + 3PL = 8PL \rightarrow R_{by} = 8P \text{ kN}. \quad (26)$$

Next, look at equilibrium in vertical direction (for the whole structure):

$$\sum V = 0 \longrightarrow R_{ay} + R_{by} = P \text{ kN} \longrightarrow R_{ay} = -7P \text{ kN}. \quad (27)$$

Summing forces in the horizontal direction, $R_{ax} = -2P \text{ kN}$. Hence, the total reaction force at A is: $[2^2 + 7^2]^{1/2} = \sqrt{53} \text{ kN}$.

Part [4b]. Using the method of joints (or otherwise) compute the distribution of tension and compression forces throughout the structure. Show all of your working.

Sol'n: Systematically look at equilibrium at nodes A, B, C, D, G and F.

At Joint A:

$$\sum V = 0, \quad \frac{F_{ac}}{\sqrt{2}} = 7P \longrightarrow F_{ac} = 7\sqrt{2}P \text{ kN}(T). \quad (28)$$

$$\sum H = 0 \quad \frac{F_{ac}}{\sqrt{2}} + F_{ab} = 2 \longrightarrow F_{ab} = -5P \text{ kN}(C). \quad (29)$$

At Joint B:

$$\sum H = 0, \quad \frac{F_{bd}}{\sqrt{2}} + 5P = 0 \longrightarrow F_{bd} = -5\sqrt{2}P \text{ kN}(C). \quad (30)$$

$$\sum V = 0, \quad R_{by} + F_{bc} - \frac{F_{bd}}{\sqrt{2}} = 0 \longrightarrow F_{bc} = -3P \text{ kN}(C). \quad (31)$$

At Joint C:

$$\sum V = 0, \quad \frac{F_{ce}}{\sqrt{2}} + 3P - 7P = 0 \longrightarrow F_{ce} = 4\sqrt{2}P \text{ kN}(T). \quad (32)$$

$$\sum H = 0, \quad \frac{F_{ce}}{\sqrt{2}} + F_{cd} = 7P \longrightarrow F_{cd} = 3P \text{ kN}(T). \quad (33)$$

At Joint D:

$$\sum H = 0, \quad \frac{F_{df}}{\sqrt{2}} = 2P - 3P \longrightarrow F_{df} = -2\sqrt{2}P \text{ kN}(C). \quad (34)$$

$$\sum V = 0, \quad F_{de} + \frac{F_{df}}{\sqrt{2}} + 3P = 0 \longrightarrow F_{de} = -3P \text{ kN}(C). \quad (35)$$

At Joint G:

$$\sum H = 0, \quad \frac{F_{eg}}{\sqrt{2}} = P \longrightarrow F_{eg} = \sqrt{2}P \text{ kN}(T). \quad (36)$$

$$\sum V = 0, \quad \frac{F_{eg}}{\sqrt{2}} + F_{fg} = 0 \longrightarrow F_{fg} = -P \text{ kN}(C). \quad (37)$$

At Joint F:

$$\sum V = 0, \quad P + \frac{F_{df}}{\sqrt{2}} = 0 \longrightarrow F_{df} = -2\sqrt{2}P \text{ kN}(C). \quad (38)$$

$$\sum H = 0, \quad F_{ef} + \frac{F_{df}}{\sqrt{2}} = P \longrightarrow F_{ef} = 3P \text{ kN}(T). \quad (39)$$

At Joint E: Can validate equilibrium by checking $\sum V = \sum H = 0$.

Part [4c]. Now suppose that the maximum tensile force any member can support is 10 kN, and that the maximum allowable compressive force is:

$$P_{ci} = 8 \left(\frac{L}{L_i} \right)^2 \text{ kN}, \quad (40)$$

where L_i is the length of the i-th element, and P_{ci} is the maximum allowable compressive force of the i-th element before buckling.

Determine the maximum value of P (kN) that the leaning tower can safely carry.

Sol'n: From the analysis:

Maximum tensile force = $7\sqrt{2}P$ (T).

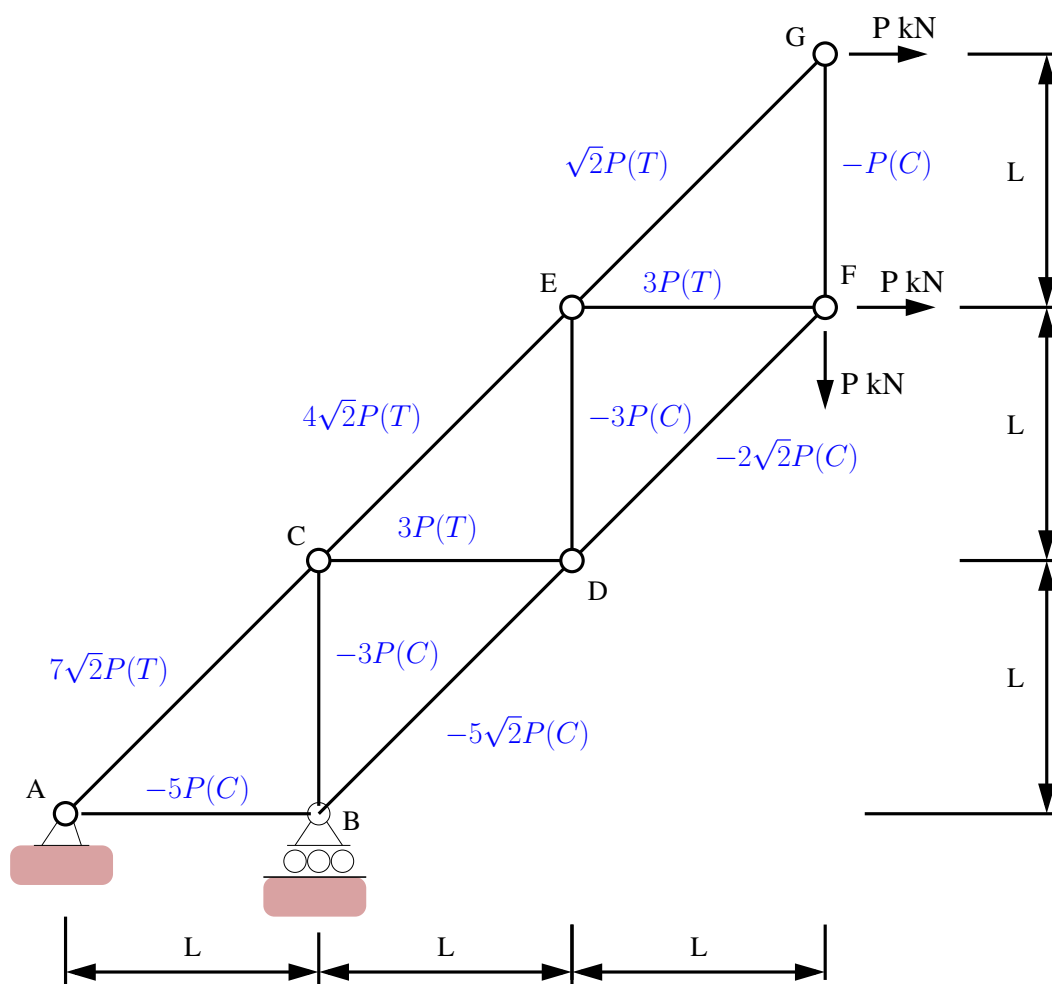


Figure 8: Elevation view of a leaning tower structure.

Maximum compressive force = $-5\sqrt{2}P$ (C) in element BD.

Limiting constraint in tension:

$$7\sqrt{2}P \leq 10kN \longrightarrow P \leq \frac{10}{7\sqrt{2}}P \approx 1.01kN. \quad (41)$$

Limiting constraint in compression:

$$5\sqrt{2}P \leq \frac{8}{2} = 4kN \longrightarrow P \leq \frac{4}{5\sqrt{2}}P \approx 0.56kN. \quad (42)$$

Compression case limits allowable load.