Department of Civil and Environmental Engineering, Spring Semester, 2024

**ENCE 353 Midterm 2, Open Notes and Open Book**

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**Exam Format and Grading.** This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are three questions. Partial credit will be given for partially correct answers, so please show all your working.

Please see the **class web page for instructions on how to submit your exam paper**.

| Question | Points | Score |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 10 | |
| Total | 40 | |

**Question 1: 15 points**

**Analysis of a Supported Cantilever Beam Structure.** Consider the supported cantilever beam structure shown in Figure 1.

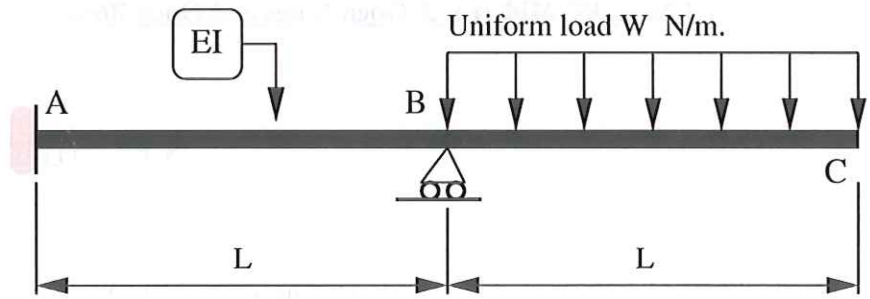
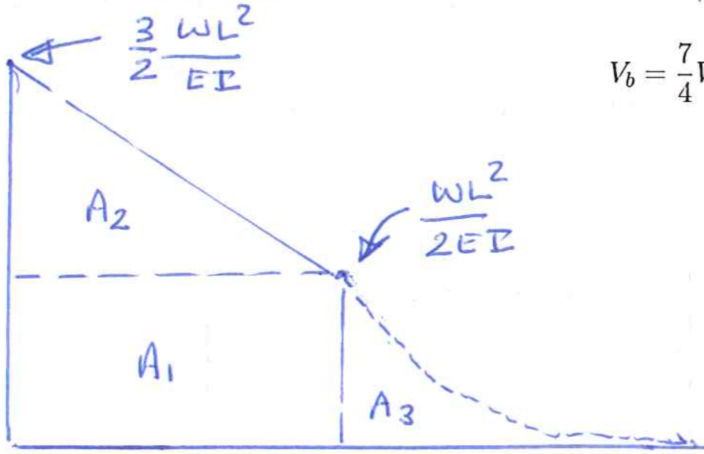



**Figure 1.** Front elevation view of a supported cantilever beam structure.

The cantilever is fully fixed (no rotation) at support A and is restrained against vertical displacements at B. It carries a uniform load $W$ (N/m) along the segment length B-C.

**[1a]** (8 pts) Use the methods of **moment area** and **compatibility of displacements** to show that the support reaction at B is:

$$V_b = \frac{7}{4} WL. \tag{1}$$




$A_1 = A_2 = \frac{WL^3}{2EI}$

$A_3 = \frac{A_1}{3} = \frac{WL^3}{6EI}.$

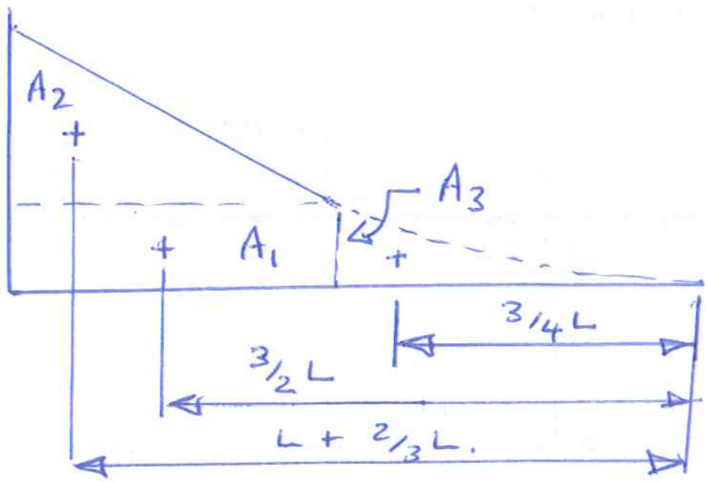
Release Reaction at B.

$\Delta_B = A_1 \bar{x}_1 + A_2 \bar{x}_2 = \frac{WL^3}{2EI}\left[\frac{L}{2} + \frac{2}{3}L\right] = \frac{7}{12}\frac{WL^4}{EI}.$

Apply reaction force at B, then compatibility of displacements:

$\frac{V_B L^3}{3EI} = \Delta_B = \frac{7}{12}\frac{WL^4}{EI} \Rightarrow V_B = \frac{7}{4} WL.$

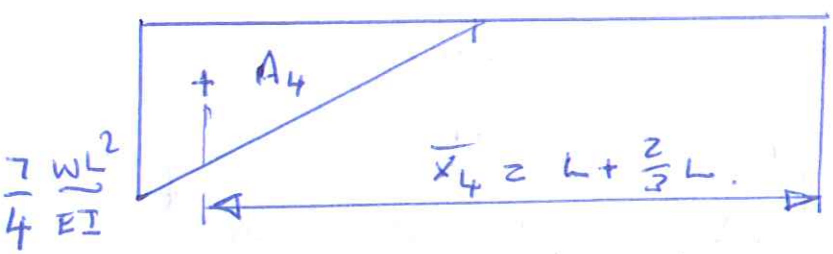
**[1b]** (7 pts) Use the method of **moment area** to compute the vertical displacement at C.




$$\Delta_C = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$$

$$= \frac{41}{24}\frac{wL^4}{EI} \downarrow \quad - (A)$$

$\Delta_C$ due to reaction force $V_B$.




$$A_4 = \left(\frac{L}{2}\right)\left(\frac{7}{4}\, wL^2\right)$$

$$\bar{x}_4 = L + \frac{2}{3}L.$$

$$\Delta_{CR} = A_4\bar{x}_4 = \frac{-35}{24}\frac{wL^4}{EI} \uparrow$$

$\uparrow$ (B)

Net displacement: (A) + (B)

$$\Delta_C = \frac{41\,wL^4}{24\,EI} - \frac{35\,wL^4}{24\,EI} = \frac{wL^4}{4\,EI} \downarrow$$

## Question 2: 15 points

Consider the cantilevered beam structure shown in Figure 3.

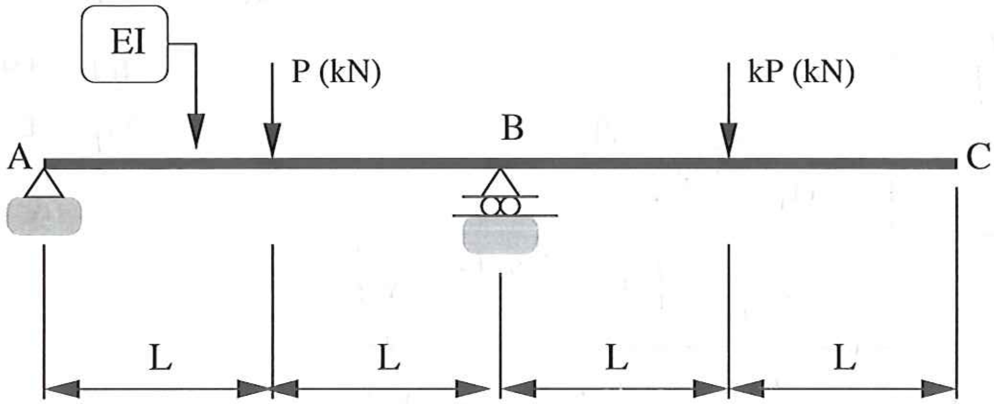
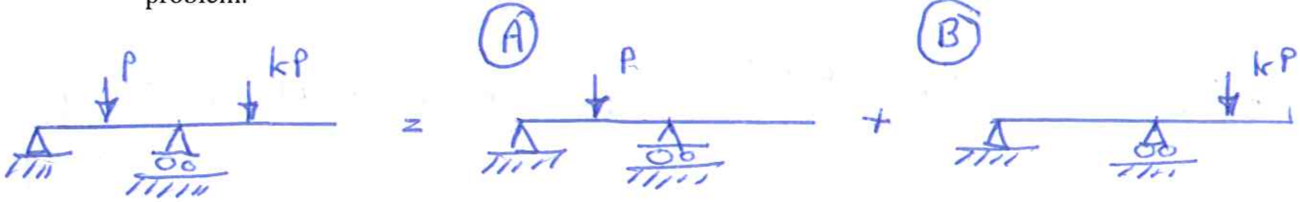



**Figure 2.** Front elevation view of a cantilevered beam structure.

Vertical loads of P (kN) and kP (kN) are applied at the mid-spans of beam segments A-B and B-C, respectively.

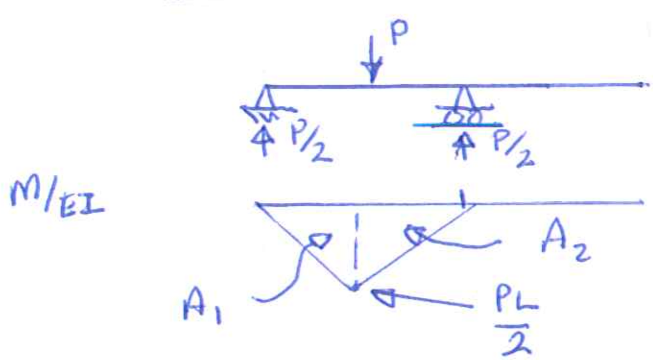
**[2a]** (3 pts) Briefly explain how the **principle of superposition** — hint, hint hint! – can be applied to this problem.

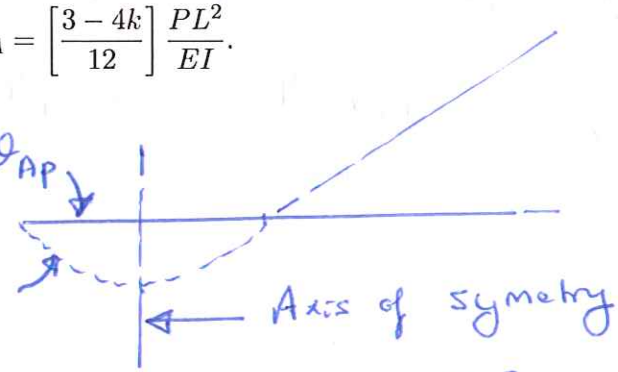



**[2b]** (8 pts) Use the **method of moment-area** to show that the clockwise rotation of point A is:

$$\theta_A = \left[\frac{3-4k}{12}\right]\frac{PL^2}{EI}. \quad (2)$$

Load Case A.

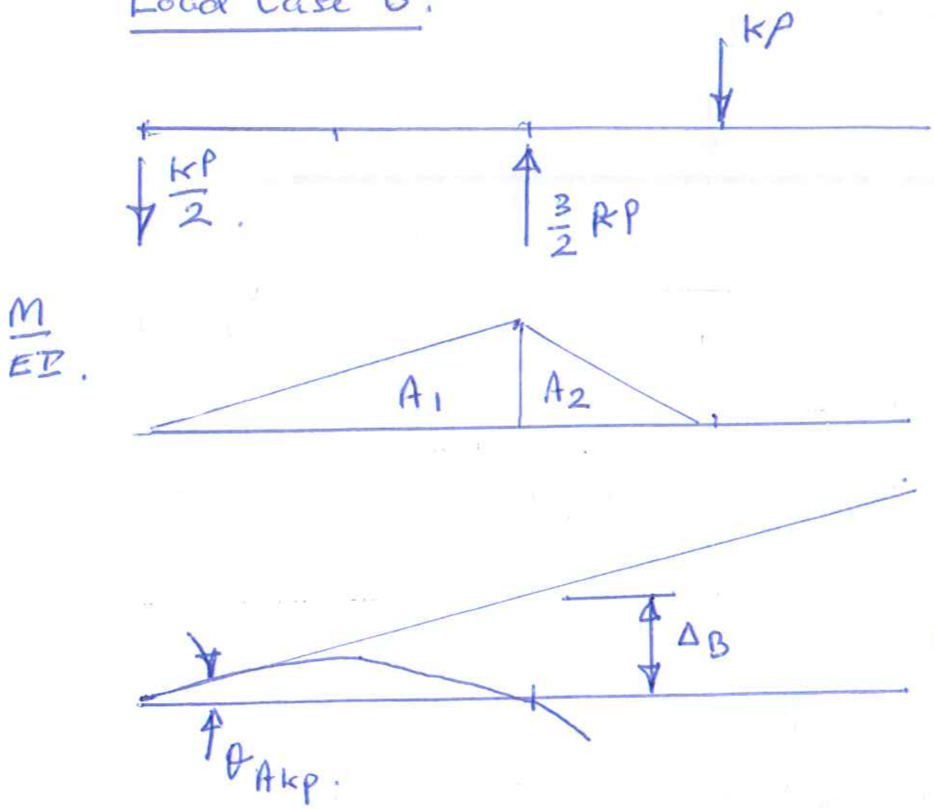



$A_1 = A_2 = \theta_{AP} = \frac{PL^2}{4EI}.$

Question 2a continued:

Load Case B:




Compute: $\theta_{Akp}$..

$$A_1 = \frac{kPL^2}{EI}.$$

$$\Delta_B = \frac{2}{3}\frac{kL^3}{EI}.$$

$$\theta_{Akp} = \frac{\Delta_B}{2L} = \frac{kPL^2}{3EI}.$$

Net Rotation:

$$\theta_A = \theta_{Ap} - \theta_{Akp}$$

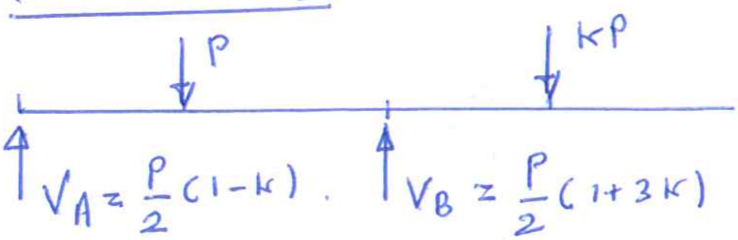
$$= \frac{PL^2}{EI}\left[\frac{1}{4} - \frac{k}{3}\right]$$

$$= \frac{PL^2}{EI}\left[\frac{3-4k}{12}\right],$$

**[2c]** (4 pts) Draw and label the deflected shape of the beam when k = 3/4. Indicate sections of beam where the fibre is in tension/compression, and where the curvature is zero.

When $k = 3/4$, $3 - 4k = 0$, $\Rightarrow \theta_A = 0$.

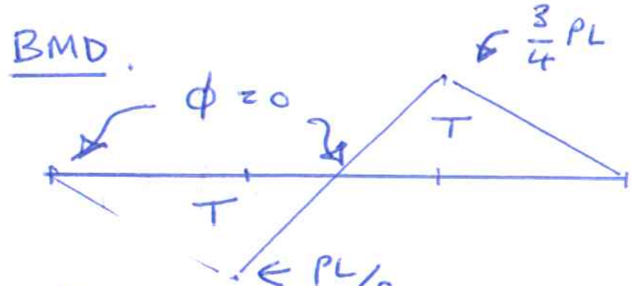
Net Reactions:



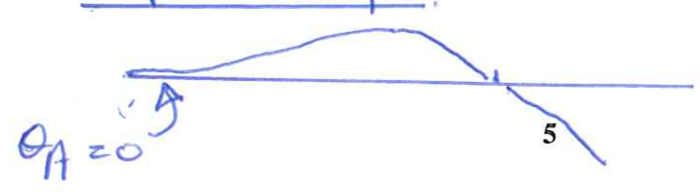

When $k = 3/4$,

$$V_A = P/8, \quad V_B = \frac{13}{8}P.$$

BMD.




Deflected Shape.



**Question 3: 10 points**

Consider the simply supported beam structure shown in Figure 3.

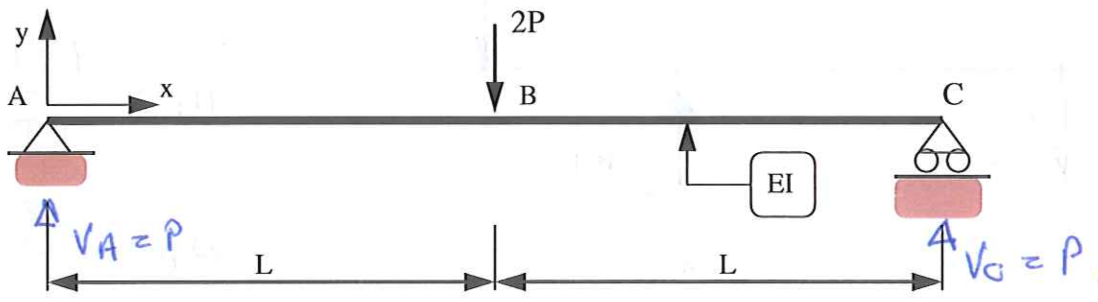



**Figure 3.** Simply supported elastic beam.

The beam has constant flexural stiffness EI along its entire length (2L). A vertical point load P is applied at the beam midpoint.

**[3a]** (3 pts) Write down the bending moment, M(x), as a piecewise linear function.

$$M(x) = \begin{cases} Px & \text{for } 0 \le x \le L. \\ P(2L - x) & \text{for } L \le x \le 2L. \end{cases}$$

**[3b]** (7 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{3}$$

and appropriate boundary conditions, derive a formula for the beam displacement y(x) over the interval $0 \le x \le L$. What is the location and value of the maximum vertical displacement?

Show all of your working.

Within A – B, $\dfrac{d^2y}{dx^2} = \dfrac{M(x)}{EI}$

Integrating $EI \dfrac{dy}{dx} = \dfrac{Px^2}{2} + A.$

$EI\, y(x) = \dfrac{Px^3}{6} + Ax + B.$

Question 3b continued ...

<u>Boundary conditions:</u> $y(0) = 0 \Rightarrow B = 0$

$$\frac{dy}{dx} = 0 \text{ at } x = L$$

$$\Rightarrow \frac{PL^2}{2} + A = 0$$

$$\Rightarrow A = \frac{-PL^2}{2}$$

<u>Displacements.</u>

$$EI\, y(x) = \frac{Px^3}{6} - \frac{PL^2 x}{2}$$

$$\Rightarrow y(x) = \frac{Px(x^2 - 3L^2)}{6EI}.$$

<u>Max downwards deflection</u> at $x = L$,

$$y(L) = \frac{PL(L^2 - 3L^2)}{6EI} = \frac{-PL^3}{3EI}$$