

Homework 5

Due: December 8 2023

Question 1: 10 points

The cantilevered beam structure shown in Figure 1 supports a uniformly distributed load w (N/m) between points C and D.

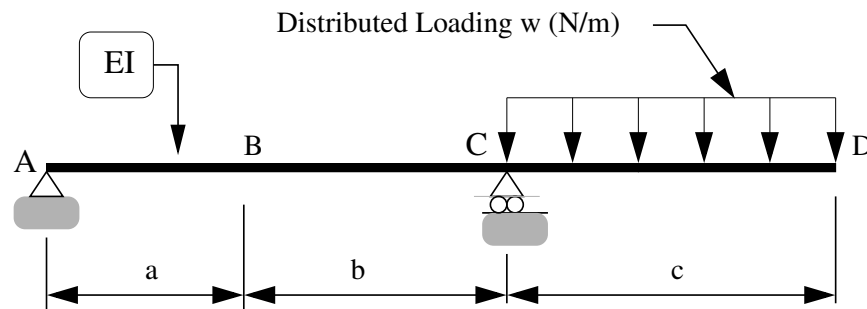


Figure 1: Front elevation view of a simple beam structure.

[1a] (4 pts) Use the method of **virtual displacements** to compute formulae for the vertical reactions at A and C. Show all of your working.

[1b] (6 pts) Use the method of **virtual displacements** to compute a formula for the bending moment at B. Show all of your working.

Question 2: 10 points

Figure 2 is a front elevation view of a bent cantilever beam carrying two external loads P . The flexural stiffness EI is constant along the beam. The axial stiffness EA is very high and, as such, axial displacements can be ignored in the analysis.

[2a] (5 pts) Use the method of **virtual forces** to compute the vertical displacement at C.

[2b] (5 pts) Use the method of **virtual forces** to compute clockwise rotation of the beam at point C.

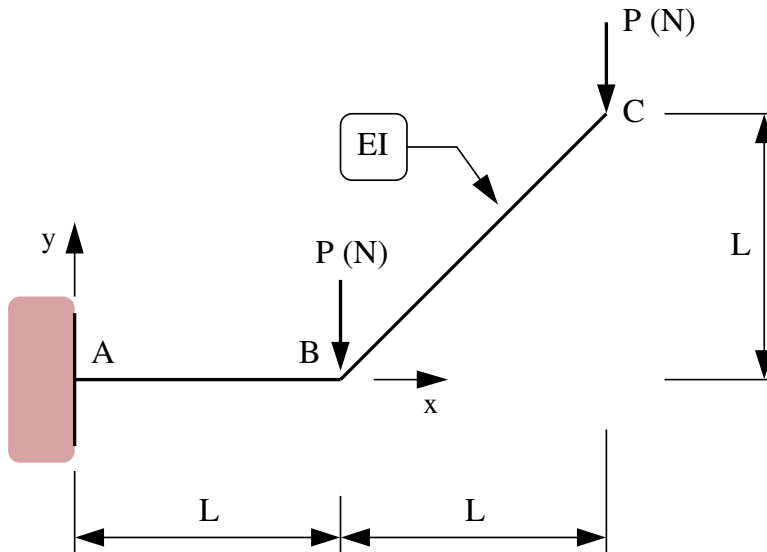


Figure 2: Cantilever beam carrying two applied loads P (N).

Question 3: 10 points

Consider the articulated cantilever beam structure shown in Figure 3.

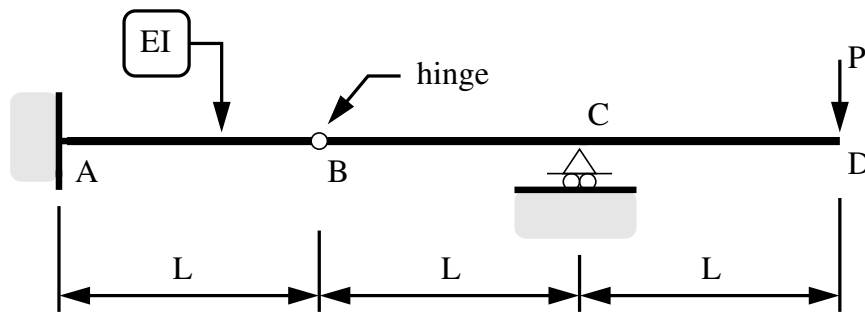


Figure 3: Elevation view of articulated cantilever beam structure.

At Point A, the cantilever is fully fixed (no movement) to a wall. Point B is a hinge. Both members have cross section properties EI . A single point load P (N) is applied at node D as shown in the figure.

[3a] (2 pts). Draw and label the bending moment diagram for this problem.

[3b] (2 pts). Qualitatively sketch the deflected shape. Indicate regions of tension/compression, and any points where slope of the beam is discontinuous.

[3c] (6 pts). Use the method of **virtual forces** to compute the **vertical displacement** and **end rotation** of the beam at D.

Show all of your working.

Question 4: 10 points

Figure 4 is a front elevation view of a simple truss that supports vertical loads at nodes C and D. All of the truss members have cross section properties AE .

[4a] (5 pts). Compute the support reactions and distribution of forces throughout the structure.

[4b] (5 pts). Use the method of **virtual forces** to show that the total deflection at node C is:

$$\Delta = \frac{PL}{AE} \left[\frac{8\sqrt{10}}{3} \right]. \quad (1)$$

Question 5: 10 points

Consider the supported cantilevered beam structure shown in Figure 5. Use the principle of **virtual forces** to compute the two-by-two flexibility matrix connecting displacements at points B and C to applied loads P_b and P_c , i.e.,

$$\begin{bmatrix} \Delta_b \\ \Delta_c \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} P_b \\ P_c \end{bmatrix}. \quad (2)$$

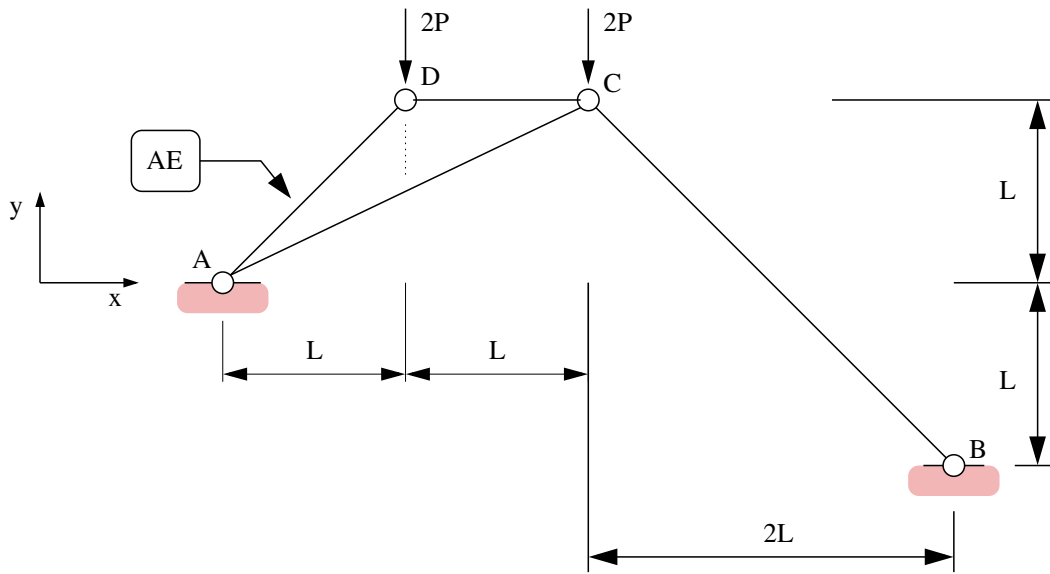


Figure 4: Front elevation view of a simple truss.

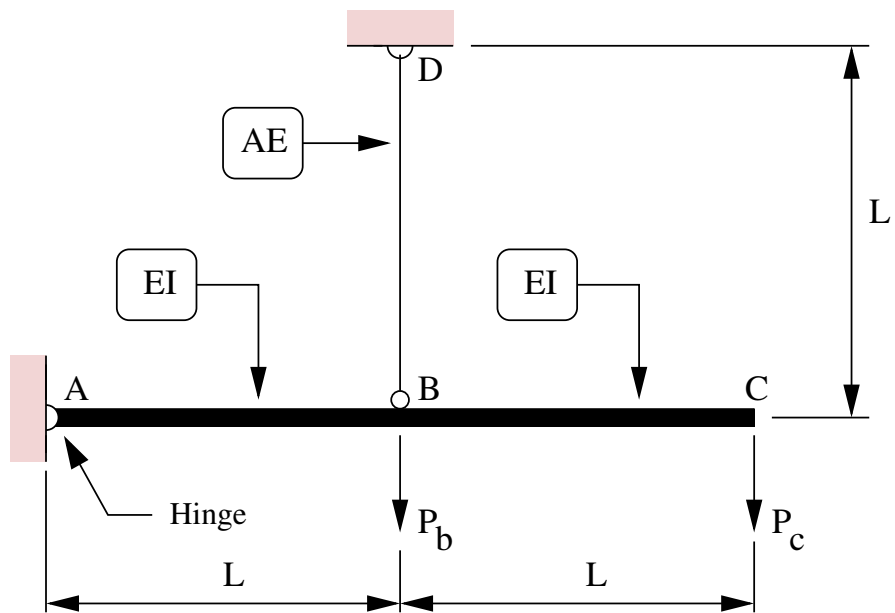


Figure 5: Front elevation view of a supported cantilevered beam structure.