Homework 3
(Due: November 3, 2023)

Question 1: 10 points. The three-pin parabolic arch shown in Figure 1 has a profile shape,

$$y(x) = \left[\frac{4f}{l^2}\right] x\,(l - x)\,. \tag{1}$$

where f = 4m and l = 16m.

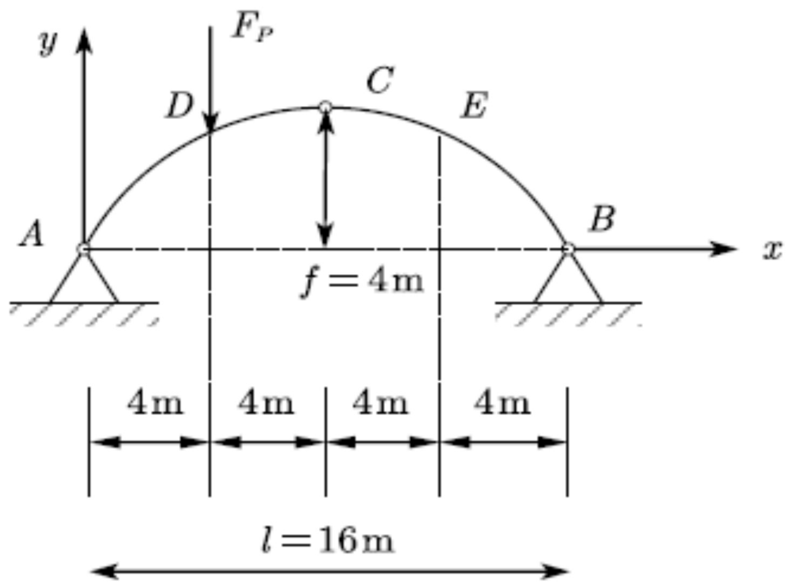


Figure 1: Elevation view of a parabolic three-pin arch.

Questions:

[1a] Calculate the horizontal and vertical components of reaction force at A and B.

[1b] Calculate the internal forces (i.e., shear, moment and axial forces) at point E.

[1c] Draw the bending moment diagram.

Question 2: 10 points

Figure 2 shows an elevation view of a pre-fabricated steel building frame that is subject to a variety of snow and wind loadings.

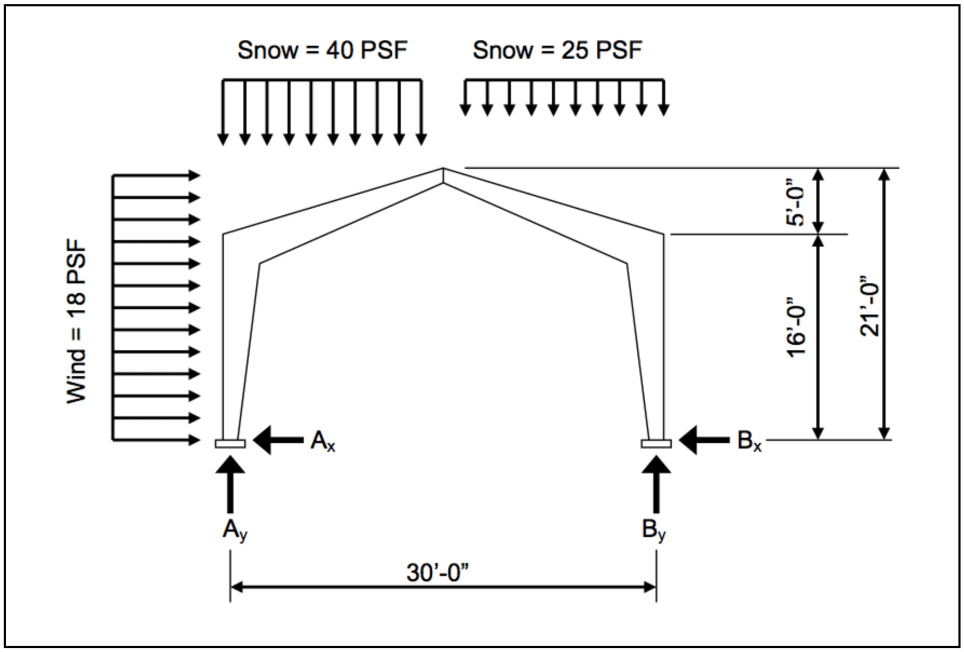


Figure 2: Elevation view of pre-fabricated steel building frame subject to snow and wind loadings.

Assuming that the frames are spaced at 20 ft centers, and that the foundation-level supports and roof apex are pinned (i.e., the frame can be modeled as a three-pinned arch), compute the vertical and horizontal reactions at the base supports.

Question 3: 15 points

Analysis of a Three-Pinned Parabolic Arch. This question is inspired by the St. Louis Gateway Arch shown on the class web page. We will compute the vertical and horizontal support reactions due to **self-weight the arch** alone, and explore the validity of approximations in the analysis along the way.

Since the mathematical details of this problem are a bit complicated, I suggest that you use **Wolfram Alpha** (see: https://www.wolframalpha.com) for the integration, and read the web page output carefully for hints on suitable simplifications.

Problem Setup. Figure 3 is a front elevation view of a three-pinned parabolic arch that has a profile: $y(x) = kx^2$.

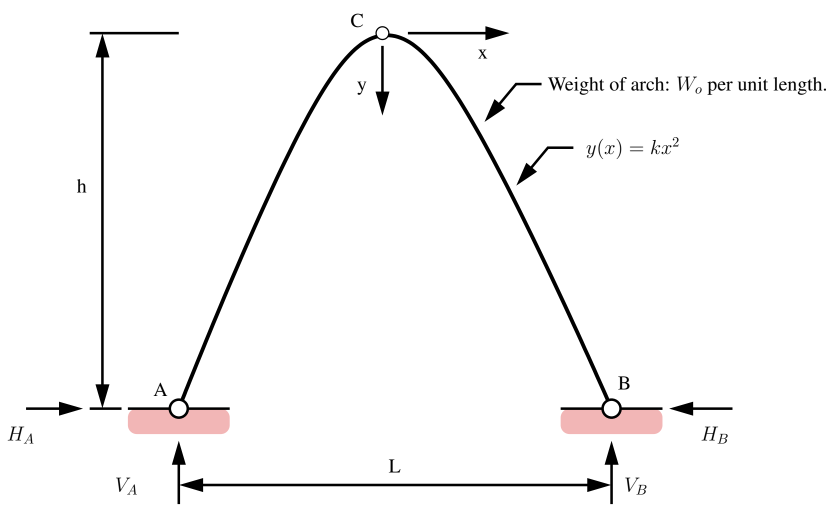


Figure 3: Front elevation view of a three-pinned parabolic arch.

The arch has height h, span L, and has self-weight W_o (N/m) along its profile. Points A, B and C are pins.

[3a] (3 pts) Starting from first principles of geometry, show that the equivalent loading measured in the horizontal direction is:

$$w(x) = W_o\big[1 + 4k^2x^2\big]^{1/2}. \tag{2}$$

Show all of your working:

[3b] (3 pts) Show that an approximate value of V_A is:

$$V_A \approx \frac{W_oL}{2}\left[1 + \frac{8}{3}\left(\frac{h}{L}\right)^2\right]. \tag{3}$$

Notice that when h/L = 0, the arch becomes a straight beam and $V_A = \frac{W_o L}{2}$.

[3c] (3 pts) Using Wolfram Alpha, or otherwise, derive a formula for the moments about C due to self-weight of the arch alone, i.e.,

$$\int_0^{L/2} w(x) x dx. \tag{4}$$

All reasonable answers will be accepted.

[3d] (3 pts) With equations 3 and 4 in place, write down and label the equation you would solve to compute the horizontal reaction force at A.

[3e] (3 pts) Now suppose that equation 3 is applied to the St. Louis Gateway Arch profile (see pic on class web page), where h/L = 1.

Does the computed value for V_A seem reasonable to you, or not? And if not, how you would correct the analysis? Either way, justify your answer.

Question 4: 10 points

The cable structure shown in Figure 4 carries a uniform load w_o N/m along its entire length.

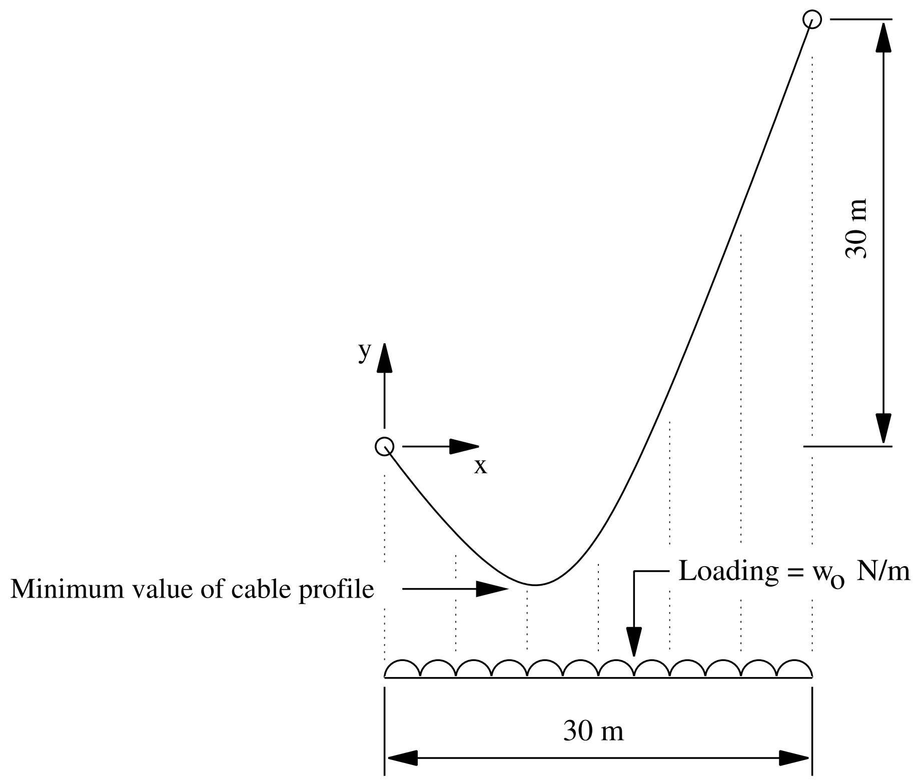


Figure 4: Elevation view of a pedestrian swing bridge.

[4a] Starting from first principles (i.e., the differential equation), show that cable profile is given by the equation

$$y(x) = \frac{w_o x^2}{2H} + \left(1 - \frac{15 w_o}{H}\right) x. \tag{5}$$

Now let us assume that the minumum value of the cable profile occurs at x = 10.

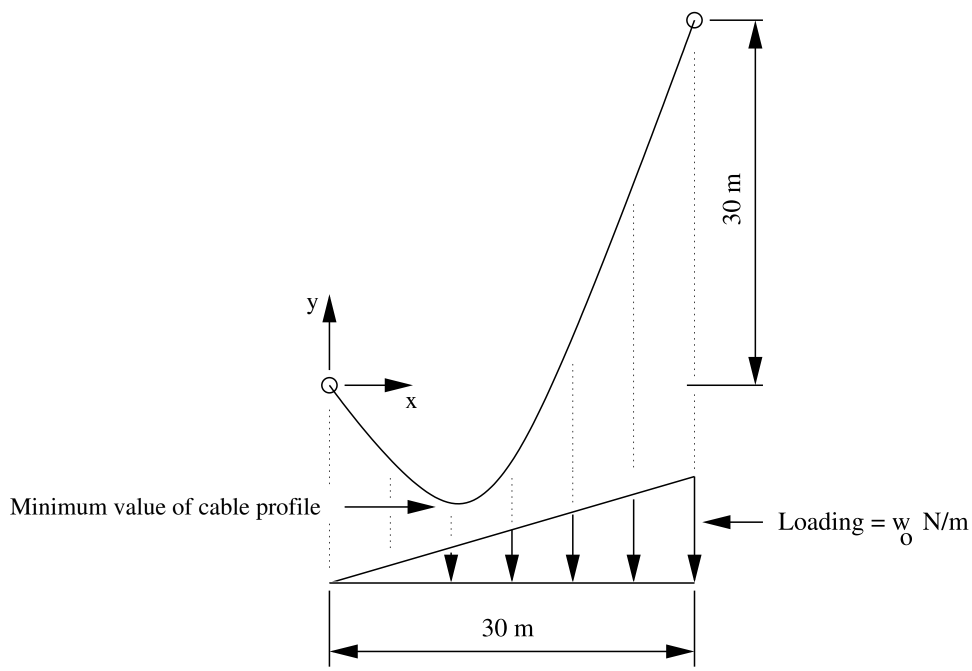
[4b] Show that the horizontal cable force is:

$$H = 5 w_o. \tag{6}$$

[4c] Derive a simple expression for the maximum tensile force in the cable.

Question 5: 10 points

The cable structure shown in Figure carries a triangular load that is zero at the left-hand support and increases to w_o N/m at the right-hand support.



[5a] Starting from first principles (i.e., the differential equation), show that cable profile is given by the equation

$$y(x) = \frac{w_o x^3}{180H} + \left(1 - \frac{5w_o}{H}\right)x. \tag{7}$$

Now let us assume that the minumum value of the cable profile occurs at x = 10.

[5b] Show that the horizontal cable force is:

$$H = \frac{20w_o}{6}. \tag{8}$$

[5c] Draw and label a diagram showing the horizontal and vertical components of reaction force at the left- and right-hand cable supports.