Homework 5

Due: May 5, 2023

Question 1: 10 points

The cantilevered beam structure shown in Figure 1 supports a uniformly distributed load w (N/m) between points C and D.

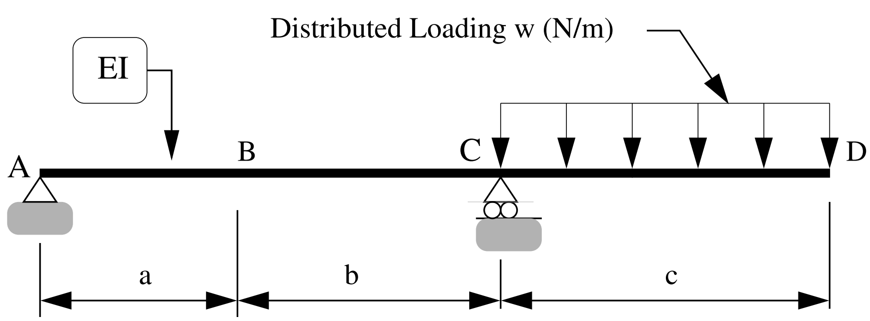

Figure 1: Front elevation view of a simple beam structure.

[1a] (4 pts) Use the method of **virtual displacements** to compute formulae for the vertical reactions at A and C. Show all of your working.

[1b] (6 pts) Use the method of **virtual displacements** to compute a formula for the bending moment at B. Show all of your working.

Question 2: 10 points

Figure 2 is a front elevation view of a bent cantilever beam carrying two external loads P. The flexural stiffness EI is constant along the beam. The axial stiffness EA is very high and, as such, axial displacements can be ignored in the analysis.

[2a] (5 pts) Use the method of **virtual forces** to compute the vertical displacement at C.

[2b] (5 pts) Use the method of **virtual forces** to compute clockwise rotation of the beam at point C.

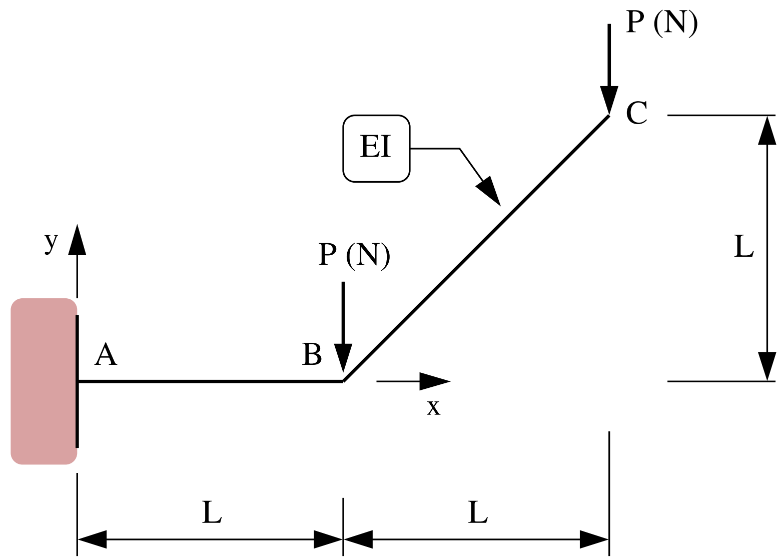

Figure 2: Cantilever beam carrying two applied loads P (N).

Question 3: 10 points

Consider the articulated cantilever beam structure shown in Figure 3.

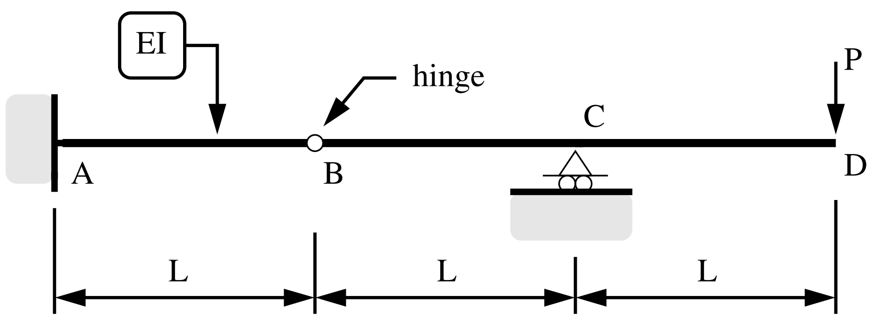

Figure 3: Elevation view of articulated cantilever beam structure.

At Point A, the cantilever is fully fixed (no movement) to a wall. Point B is a hinge. Both members have cross section properties EI. A single point load **P** (N) is applied at node D as shown in the figure.

[3a] (2 pts). Draw and label the bending moment diagram for this problem.

[3b] (2 pts). Qualitatively sketch the deflected shape. Indicate regions of tension/compression, and any points where slope of the beam is discontinuous.

[3c] (6 pts). Use the method of **virtual forces** to compute the **vertical displacement** and **end rotation** of the beam at D.

Show all of your working.

Question 4: 10 points

Figure 4 is a front elevation view of a simple truss that supports vertical loads at nodes C and D. All of the truss members have cross section properties AE.

[4a] (5 pts). Compute the support reactions and distribution of forces throughout the structure.

[4b] (5 pts). Use the method of **virtual forces** to show that the total deflection at node C is:

$$\triangle = \frac{PL}{AE}\left[\frac{8\sqrt{10}}{3}\right]. \tag{1}$$

Question 5: 10 points

Consider the supported cantilevered beam structure shown in Figure 5. Use the principle of **virtual forces** to compute the two-by-two flexibility matrix connecting displacements at points B and C to applied loads P_b and P_c, i.e.,

$$\begin{bmatrix} \triangle_b \\ \triangle_c \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} P_b \\ P_c \end{bmatrix}. \tag{2}$$

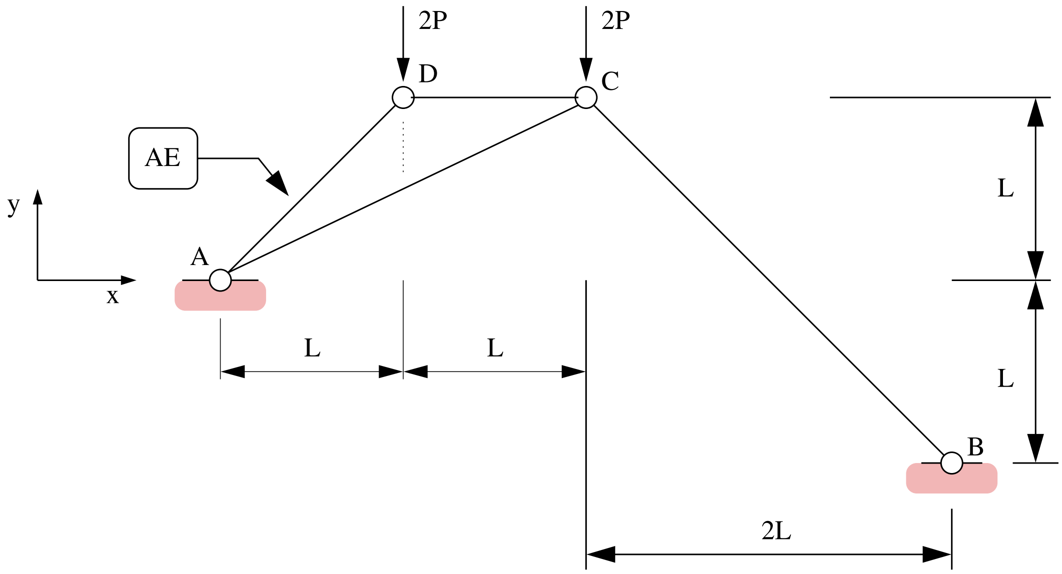

Figure 4: Front elevation view of a simple truss.

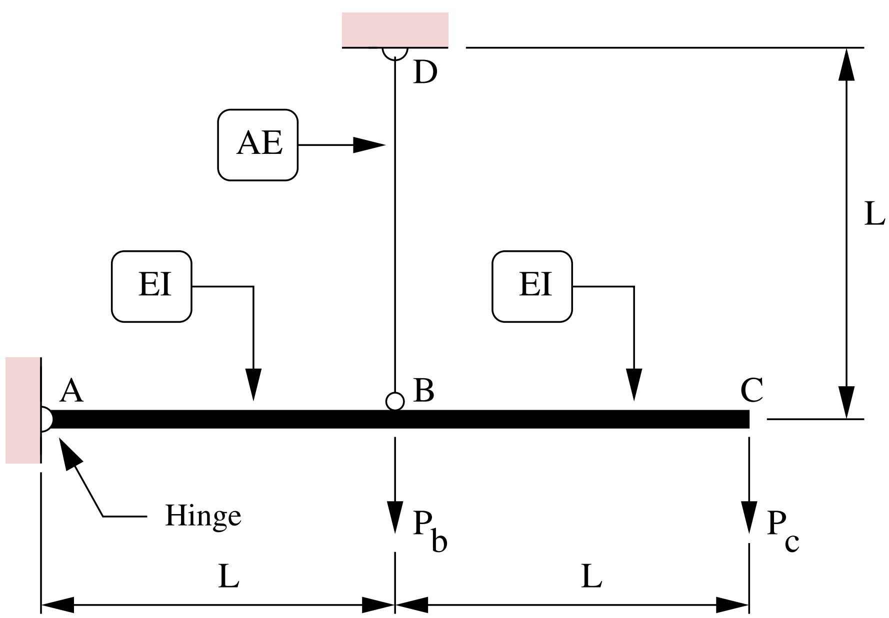

Figure 5: Front elevation view of a supported cantilevered beam structure.