

**Homework 3**  
(Due: April 7, 2023)

**Question 1: 10 points.** The three-pin parabolic arch shown in Figure 1 has a profile shape,

$$y(x) = \left[ \frac{4f}{l^2} \right] x(l-x). \quad (1)$$

where  $f = 4\text{m}$  and  $l = 16\text{m}$ .

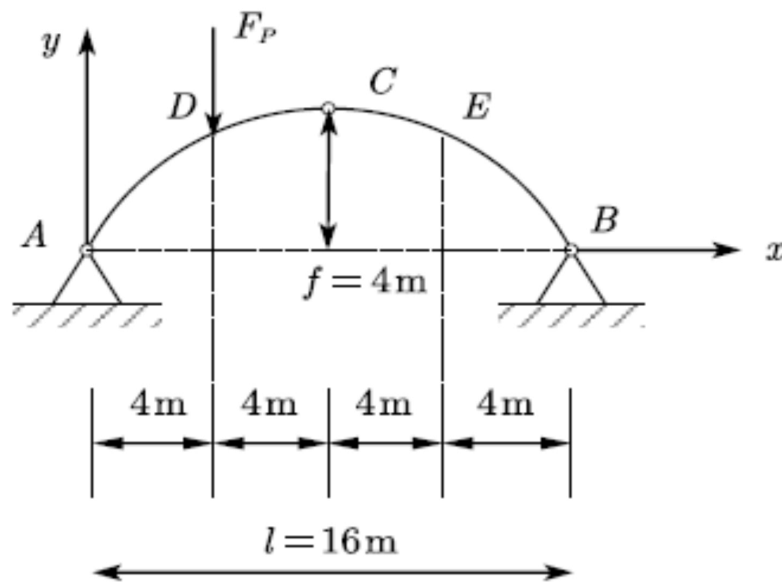


Figure 1: Elevation view of a parabolic three-pin arch.

Questions:

**[1a]** Calculate the horizontal and vertical components of reaction force at A and B.

**[1b]** Calculate the internal forces (i.e., shear, moment and axial forces) at point E.

**[1c]** Draw the bending moment diagram.

**Question 2: 10 points**

Figure 2 shows an elevation view of a pre-fabricated steel building frame that is subject to a variety of snow and wind loadings.

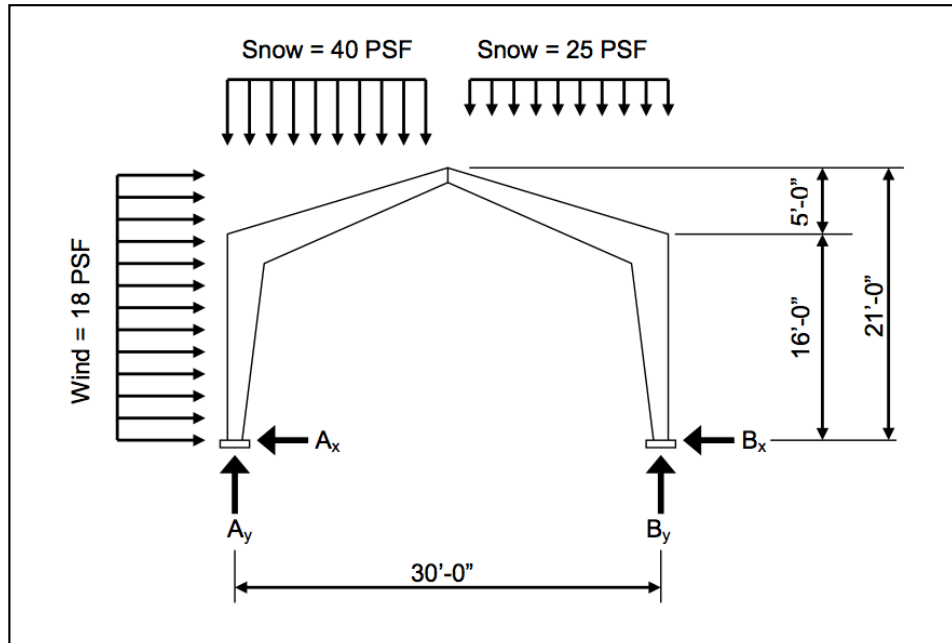


Figure 2: Elevation view of pre-fabricated steel building frame subject to snow and wind loadings.

Assuming that the frames are spaced at 20 ft centers, and that the foundation-level supports and roof apex are pinned (i.e., the frame can be modeled as a three-pinned arch), compute the vertical and horizontal reactions at the base supports.

**Question 3: 15 points**

**Analysis of a Three-Pinned Parabolic Arch.** This question is inspired by the St. Louis Gateway Arch shown on the class web page. We will compute the vertical and horizontal support reactions due to **self-weight the arch** alone, and explore the validity of approximations in the analysis along the way.

Since the mathematical details of this problem are a bit complicated, I suggest that you use **Wolfram Alpha** (see: <https://www.wolframalpha.com>) for the integration, and read the web page output carefully for hints on suitable simplifications.

**Problem Setup.** Figure 3 is a front elevation view of a three-pinned parabolic arch that has a profile:  $y(x) = kx^2$ .

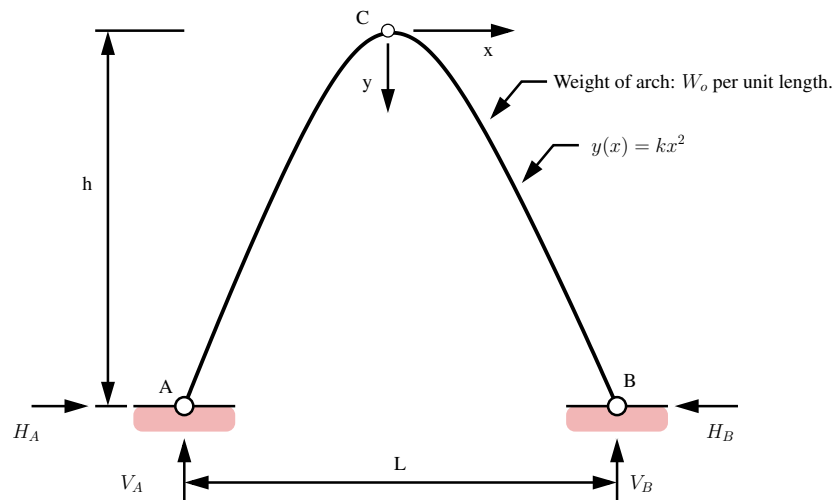


Figure 3: Front elevation view of a three-pinned parabolic arch.

The arch has height  $h$ , span  $L$ , and has self-weight  $W_o$  (N/m) along its profile. Points A, B and C are pins.

**[3a]** (3 pts) Starting from first principles of geometry, show that the equivalent loading measured in the horizontal direction is:

$$w(x) = W_o [1 + 4k^2 x^2]^{1/2}. \quad (2)$$

Show all of your working:

**[3b]** (3 pts) Show that an approximate value of  $V_A$  is:

$$V_A \approx \frac{W_o L}{2} \left[ 1 + \frac{8}{3} \left( \frac{h}{L} \right)^2 \right]. \quad (3)$$

Notice that when  $h/L = 0$ , the arch becomes a straight beam and  $V_A = \frac{W_o L}{2}$ .

**[3c]** (3 pts) Using Wolfram Alpha, or otherwise, derive a formula for the moments about C due to self-weight of the arch alone, i.e.,

$$\int_0^{L/2} w(x)x dx. \quad (4)$$

All reasonable answers will be accepted.

**[3d]** (3 pts) With equations 3 and 4 in place, write down and label the equation you would solve to compute the horizontal reaction force at A.

**[3e]** (3 pts) Now suppose that equation 3 is applied to the St. Louis Gateway Arch profile (see pic on class web page), where  $h/L = 1$ .

Does the computed value for  $V_A$  seem reasonable to you, or not? And if not, how you would correct the analysis? Either way, justify your answer.

**Question 4: 10 points**

The cable structure shown in Figure 4 carries a uniform load  $w_o$  N/m along its entire length.

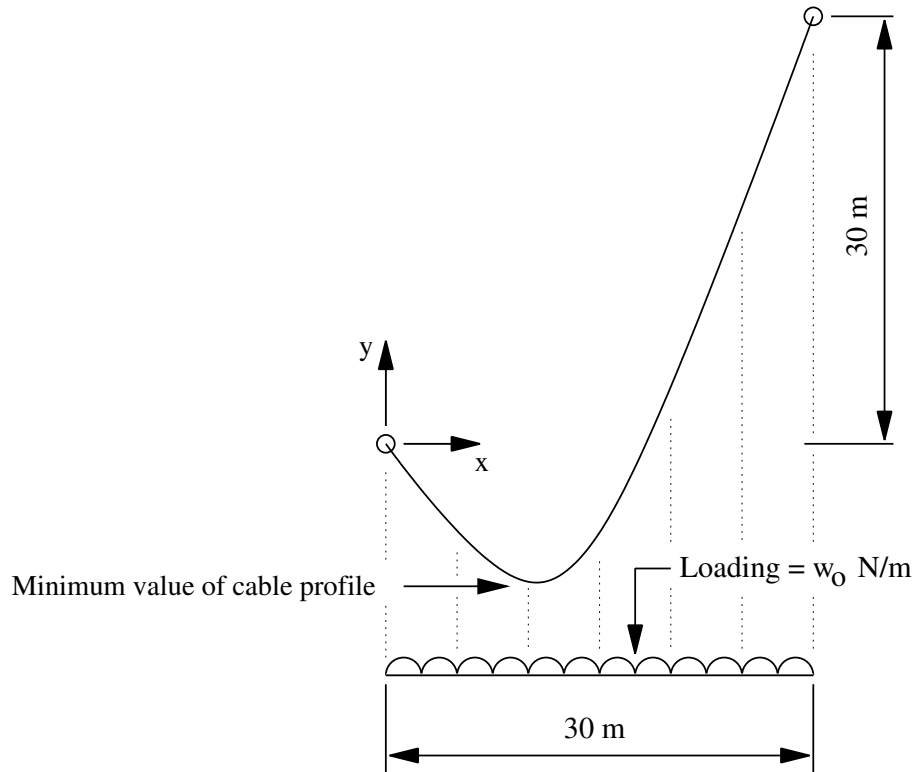


Figure 4: Elevation view of a pedestrian swing bridge.

**[4a]** Starting from first principles (i.e., the differential equation), show that cable profile is given by the equation

$$y(x) = \frac{w_o x^2}{2H} + \left(1 - \frac{15w_o}{H}\right) x. \quad (5)$$

Now let us assume that the minimum value of the cable profile occurs at  $x = 10$ .

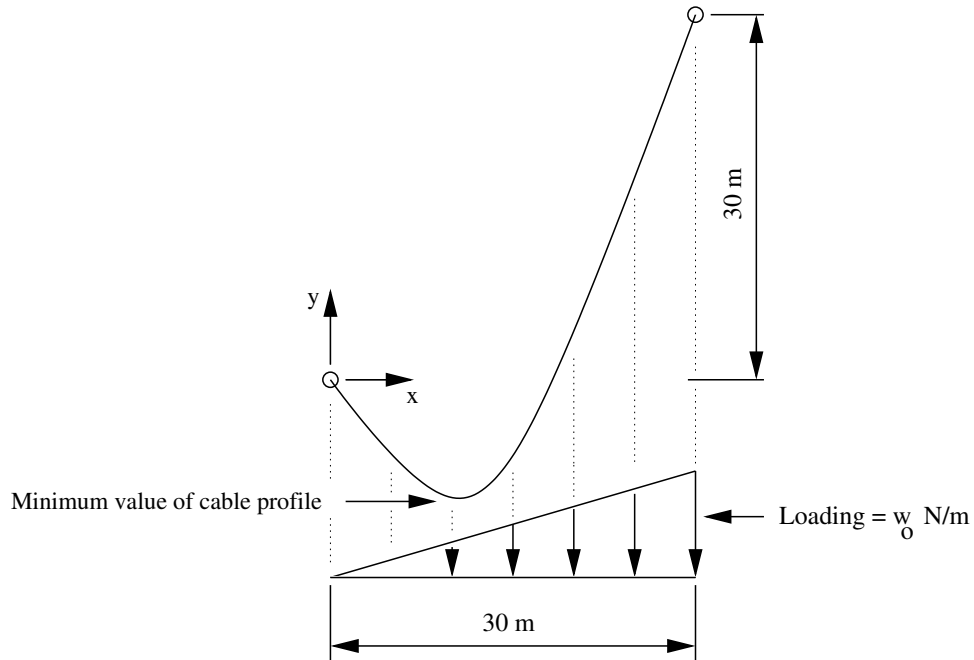
**[4b]** Show that the horizontal cable force is:

$$H = 5w_o. \quad (6)$$

**[4c]** Derive a simple expression for the maximum tensile force in the cable.

**Question 5: 10 points**

The cable structure shown in Figure carries a triangular load that is zero at the left-hand support and increases to  $w_o$  N/m at the right-hand support.



**[5a]** Starting from first principles (i.e., the differential equation), show that cable profile is given by the equation

$$y(x) = \frac{w_o x^3}{180H} + \left(1 - \frac{5w_o}{H}\right) x. \quad (7)$$

Now let us assume that the minimum value of the cable profile occurs at  $x = 10$ .

**[5b]** Show that the horizontal cable force is:

$$H = \frac{20w_o}{6}. \quad (8)$$

**[5c]** Draw and label a diagram showing the horizontal and vertical components of reaction force at the left- and right-hand cable supports.