Department of Civil and Environmental Engineering, Fall Semester, 2022

ENCE 353 Midterm 2, Open Notes and Open Book

Name: ____________________

E-mail (print neatly!): ____________________

Exam Format and Grading. This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are three questions. Partial credit will be given for partially correct answers, so please show all your working.

Please see the **class web page for instructions on how to submit your exam paper**.

Question	Points	Score
1	15	
2	10	
3	15	
Total	40	

Question 1: 15 points

Moment-Area Method. Figure 1 is a front elevation view of a cantilevered beam carrying a single point load P. EI is constant along the beam structure A-B-C-D.

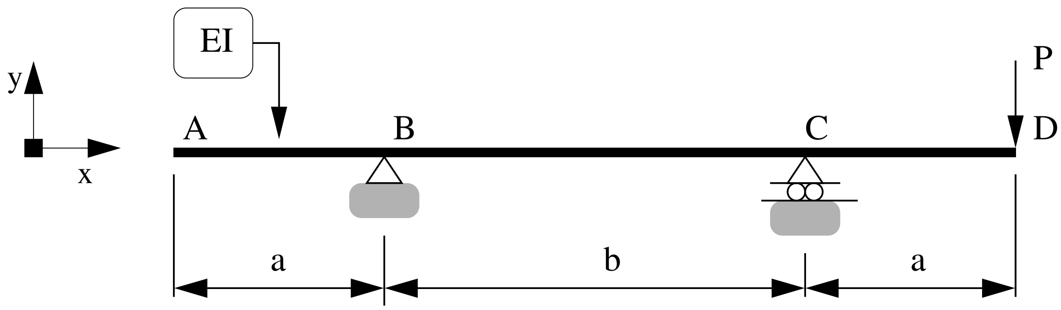

Figure 1: Cantilevered beam carrying a single applied load P.

[1a] (2 pts) Draw and label the M/EI diagram.

[1b] (2 pts) Draw and label the **moment area analysis diagram** (i.e., with rotations, tangents, displacements, etc) for this problem.

[1c] (3 pts) Use the method of **moment area** to show that the anticlockwise rotation of the beam at B is:

$$\theta_B = \frac{Pab}{6EI} \tag{1}$$

and the clockwise rotation of the beam at C is:

$$\theta_C = \frac{Pab}{3EI}. \tag{2}$$

Show all of your working.

[1d] (3 pts) Use the method of **moment area** to show that the vertical deflection of the beam at points A and D (measured downwards) is:

$$y_A = \frac{Pa^2 b}{6EI} \tag{3}$$

and

$$y_D = \frac{Pa^2}{3EI}(a+b). \tag{4}$$

Show all of your working.

[1e] (5 pts) Show that the maximum upwards deflection of the beam occurs at a distance $b/\sqrt{3}$ from B, and that its value is:

$$y_{\text{maximum upward}} = \frac{Pab^2}{9\sqrt{3}EI}. \tag{5}$$

Show all of your working.

Question 2: 10 points

Compute Cable Profile and Tension in a Suspension Bridge. Figure 2 is a front elevation view of a cable span in a suspension bridge.

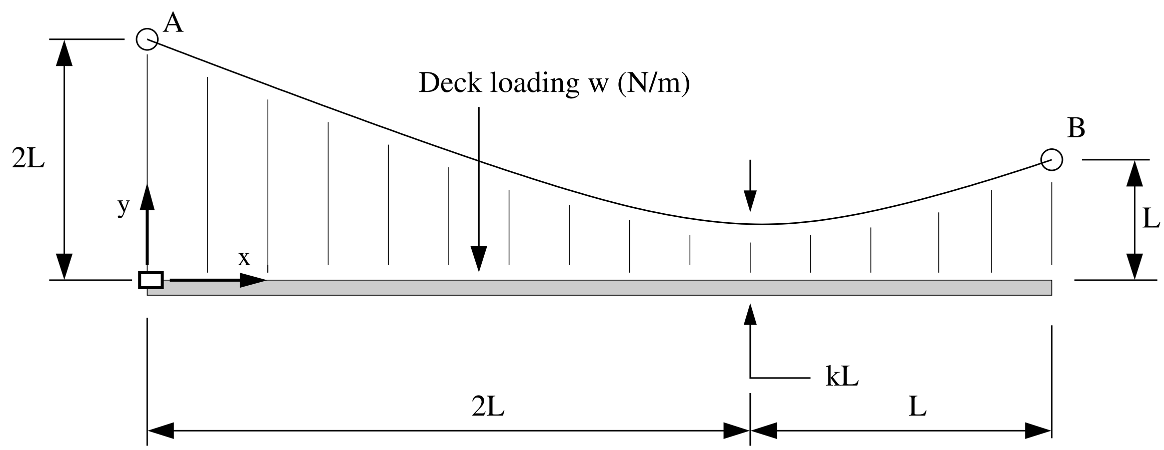

Figure 2: Elevation view of a single cable span in a suspension bridge (not to scale).

The span has length 3L and carries a uniform load w (N/m). The lowest point of the cable profile is kL above the bridge deck.

[2a] (4 pts) If the lowest point in the cable profile is at distance 2L from the left-hand side, show that:

$$k = \left[\frac{2}{3} \right]. \tag{6}$$

[2b] (3 pts) Hence, show that the horizontal component of cable force is:

$$H = \left[\frac{3}{2}\right] wL. \tag{7}$$

[2c] (3 pts) Compute the vertical components of cable force at supports A and B.

Question 3: 15 points

Analysis of a Three-Pinned Parabolic Arch. This question is inspired by the St. Louis Gateway Arch shown on the class web page. We will compute the vertical and horizontal support reactions due to **self-weight the arch** alone, and explore the validity of approximations in the analysis along the way.

Since the mathematical details of this problem are a bit complicated, I suggest that you use **Wolfram Alpha** (see: https://www.wolframalpha.com) for the integration, and read the web page output carefully for hints on suitable simplifications.

Problem Setup. Figure 3 is a front elevation view of a three-pinned parabolic arch that has a profile: $y(x) = kx^2$.

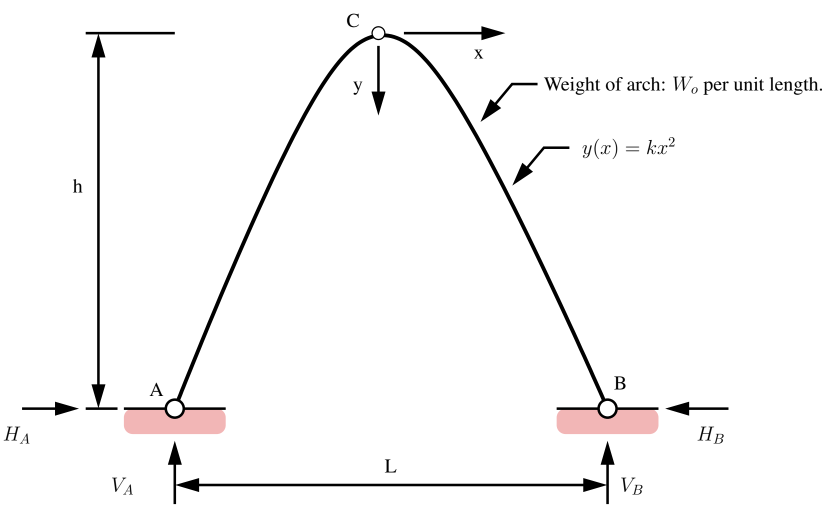

Figure 3: Front elevation view of a three-pinned parabolic arch.

The arch has height h, span L, and has self-weight W_o (N/m) along its profile. Points A, B and C are pins.

[3a] (3 pts) Starting from first principles of geometry, show that the equivalent loading measured in the horizontal direction is:

$$w(x) = W_o\big[1 + 4k^2x^2\big]^{1/2}. \tag{8}$$

Show all of your working:

Question 3a continued:

[3b] (3 pts) Show that an approximate value of V_A is:

$$V_A \approx \frac{W_o L}{2}\left[1 + \frac{8}{3}\left(\frac{h}{L}\right)^2\right]. \tag{9}$$

Notice that when h/L = 0, the arch becomes a straight beam and $V_A = \frac{W_o L}{2}$.

[3c] (3 pts) Using Wolfram Alpha, or otherwise, derive a formula for the moments about C due to self-weight of the arch alone, i.e.,

$$\int_0^{L/2} w(x)xdx. \tag{10}$$

All reasonable answers will be accepted.

[3d] (3 pts) With equations 9 and 10 in place, write down and label the equation you would solve to compute the horizontal reaction force at A.

[3e] (3 pts) Now suppose that equation 9 is applied to the St. Louis Gateway Arch (see dimensions on class web page), where h/L = 1.

Does the computed value for V_A seem reasonable to you, or not? And if not, how you would fix the problem? Either way, justify your answer.