Department of Civil and Environmental Engineering, Spring Semester, 2022

ENCE 353 Midterm 2, Open Notes and Open Book

Name: ________________________

E-mail (print neatly!): ________________________

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please **show all of your working**.

Question	Points	Score
1	15	
2	10	
3	15	
Total	40	

Question 1: 15 points

Analysis of a Cantilevered Beam with Moment-Area Method. Consider the cantilevered beam structure shown in Figure 1.

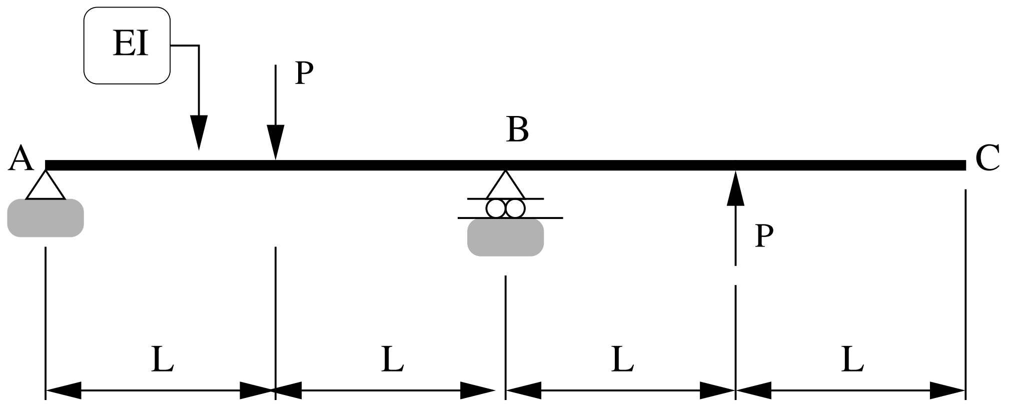

Figure 1: Front elevation view of a cantilevered beam structure.

[1a] (3 pts) Briefly explain how the **principle of superposition** – hint, hint, hint, hint !!! – can be applied to **this problem**.

[1b] (2 pts) Draw and label the M/EI diagrams for the two subproblems.

[1c] (4 pts) Use the method of **moment area** to compute the rotation at point A.

[1d] (4 pts) Use the method of **moment area** to compute the vertical deflection of the beam at point C.

[1e] (2 pts) Draw the deflected shape of the beam. Indicate the sections of beam where the curvature is constant.

Question 2: 10 points

Analysis of a Three-Pinned Parabolic Arch. Consider the three-pinned parabolic arch shown in Figure 2

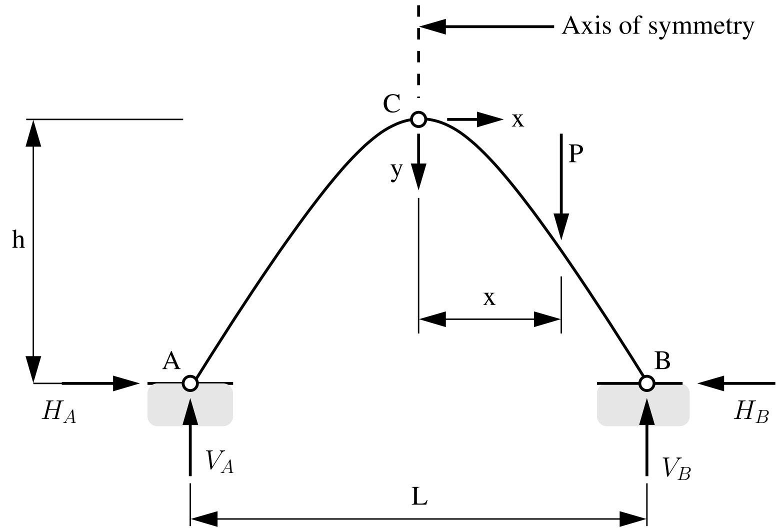

Figure 2: Three-pinned parabolic arch carrying a point load.

The height and width of the arch are h and L, respectively. A point load P is applied at a distance x, $0 \le x \le L/2$, from the axis of symmetry.

[2a] (3 pts) Starting from first principles of engineering (i.e., equations of equilibrium), show that the vertical and horizontal reaction forces at A and B are:

$$V_A(x) = P\left[\frac{1}{2} - \frac{x}{L}\right] \tag{1}$$

$$V_B(x) = P\left[\frac{1}{2} + \frac{x}{L}\right] \tag{2}$$

$$H_A(x) = H_B(x) = \frac{P}{2h}\left[\frac{L}{2} - x\right] \tag{3}$$

Question 2a continued:

Now suppose that the point load P is replaced by a uniform load w_o (N/m) along the complete width of the arch, i.e.,

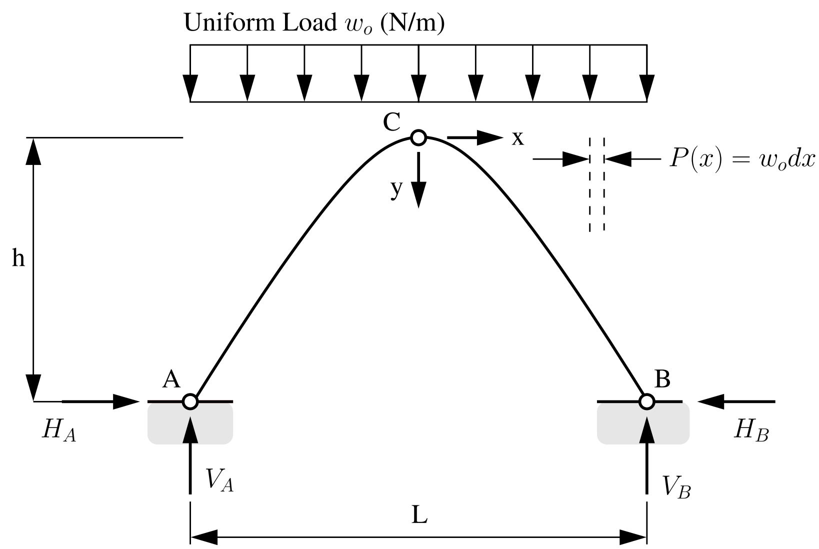

Figure 3: Three-pinned parabolic arch carrying a uniform load.

[2b] (3 pts) Draw and label a picture showing how the principle of superposition can be used in this problem.

[2c] (4 pts) By **using equations 1 through 3 as a starting point**, and noting that a tiny increment of uniform loading can be written P(x) = w_0 dx, show that:

$$V_A = V_B = \left[\frac{w_o L}{2}\right] \quad \text{and} \quad H_A = H_B = \left[\frac{w_o L^2}{8h}\right]. \tag{4}$$

Question 2c continued:

Question 3: 15 points

Elastic Curve for Cantilevered Beam. Figure 4 is a front elevation view of a cantilevered beam carrying two external loads P. EI is constant along the beam structure A-B-C-D.

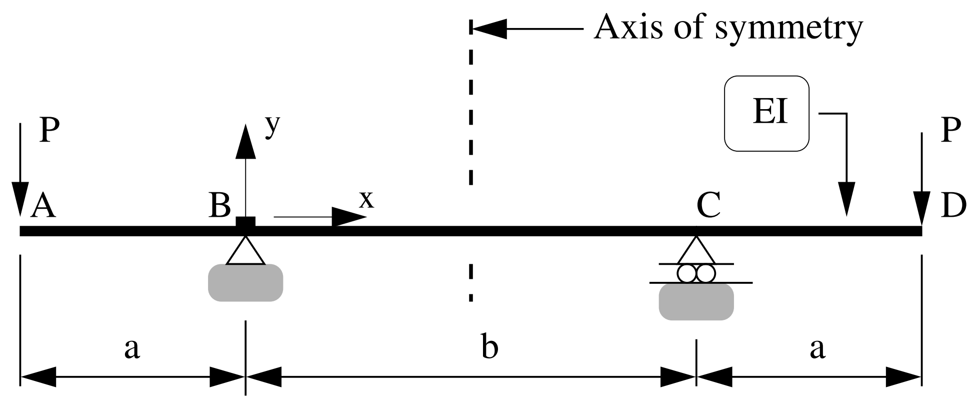

Figure 4: Cantilevered beam carrying two applied loads P.

Notice that the coordinate system is positioned at point B.

[3a] (2 pts) Draw and label the M/EI diagram.

[3b] (1 pt) Create a qualitative sketch the deflection shape. Indicate where the maximum upwards deflection and maximum downward deflections will occur.

[3c] (4 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{5}$$

derive a formula for displacement y(x) within the segment B-C. Clearly indicate the boundary conditions that apply.

[3d] (2 pts) Use your result from part 3c to show that the maximum upward deflection is:

$$y_{\text{maximum upward}} = \frac{Pab^2}{8EI}. \tag{6}$$

[3e] (3 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{7}$$

derive a formula for displacement y(x) within the segment A-B. Clearly indicate the boundary conditions that apply.

[3f] (2 pts) Use your result from part 3e to show that the maximum downwards deflection is:

$$y_{\text{maximum downward}} = \frac{-Pa^2}{6EI}\left[2a + 3b\right]. \tag{8}$$