Department of Civil and Environmental Engineering, Fall Semester, 2021

ENCE 353 Midterm 2, Open Notes and Open Book

Name: ________________________

E-mail (print neatly!): ________________________

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please **show all of your working**.

Question	Points	Score
1	15	
2	10	
3	15	
Total	40	

Question 1: 15 points

Elastic Curve for Beam Deflections. Figure 1 is a front elevation view of a cantilever beam carrying two external loads P. EI is constant along the cantilever beam.

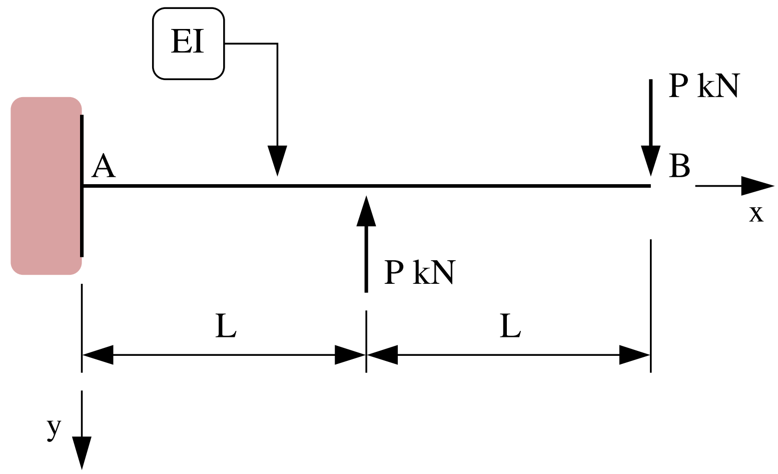

Figure 1: Cantilever beam carrying two applied loads P (kN).

[1a] (2 pts) Briefly explain how the **principle of superposition** – hint, hint, hint, hint !!! – can be applied to **this problem**.

[1b] (2 pts) Draw and label the M/EI diagrams for the two subproblems.

[1c] (6 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI} \right], \tag{1}$$

derive formulae for displacement y(x) in the two subproblems indicated in part 1a. For convenience, let's call these displacement curves $y_1(x)$ and $y_2(x)$.

Hint: Your formula should be piecewise continuous over two regions, i.e., region 1: $0 \leq x \leq L$, and region 2: $L \leq x \leq 2L$.

Question 1c continued:

[1d] (3 pts) Compute the total displacement y(x) = $y_1(x)$ + $y_2(x)$ along the beam.

[1e] (2 pts) Compute the vertical deflection of point B.

Question 2: 10 points

Analysis of a Three-Pinned Parabolic Arch. Consider the three-pinned parabolic arch shown in Figure 2

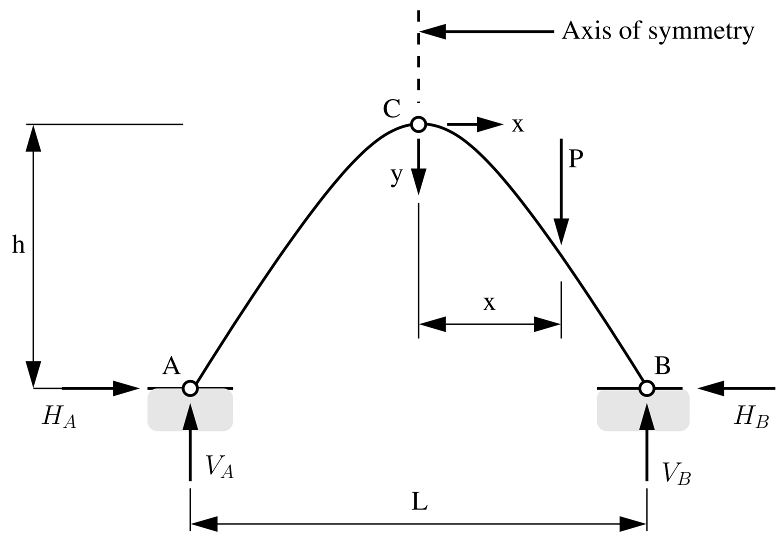

Figure 2: Three-pinned parabolic arch carrying a point load.

The height and width of the arch are h and L, respectively. A point load P is applied at a distance x, $0 \leq$ x $\leq$ L/2, from the axis of symmetry.

[2a] (5 pts) Starting from first principles of engineering (i.e., equations of equilibrium), show that the vertical and horizontal reaction forces at A and B are:

$$V_A(x) = P \left[\frac{1}{2} - \frac{x}{L} \right] \quad (2)$$

$$V_B(x) = P \left[\frac{1}{2} + \frac{x}{L} \right] \quad (3)$$

$$H_A(x) = H_B(x) = \frac{P}{2h} \left[\frac{L}{2} - x \right] \quad (4)$$

Question 2a continued:

Now suppose that the point load P is replaced by a uniform load w_o (N/m) along the right-hand side of the arch, i.e.,

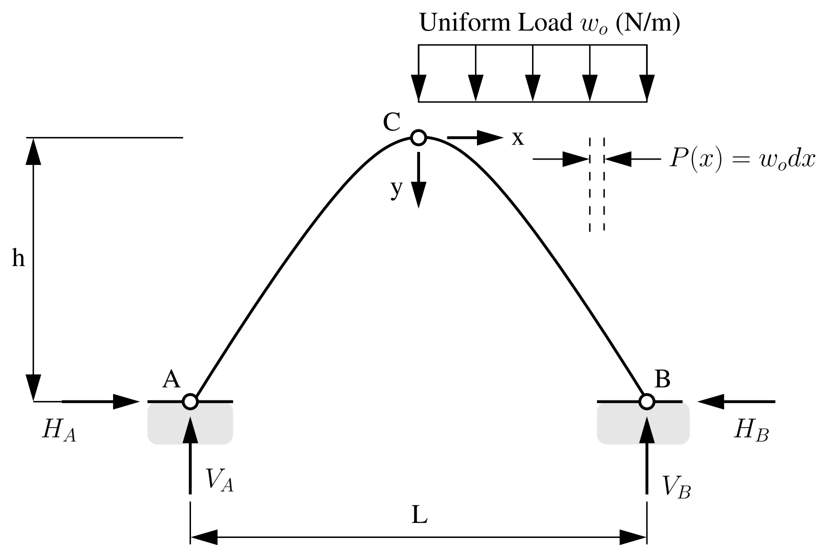

Figure 3: Three-pinned parabolic arch carrying a uniform load.

[2b] (5 pts) By using equations 2 through 4 as a starting point, and noting that a tiny increment of uniform loading can be written P(x) = w_0 dx, show that:

$$V_A = \left[\frac{w_o L}{8} \right], \quad V_B = \left[\frac{3 w_o L}{8} \right] \quad \text{and} \quad H_A = H_B = \left[\frac{w_o L^2}{16h} \right]. \tag{5}$$

Question 2b continued:

Question 3: 15 points

Analysis of a Cantilevered Beam with Moment Area. Figure 4 is a front elevation view of a cantilevered beam carrying two external loads P. EI is constant along the beam structure A-B-C-D.

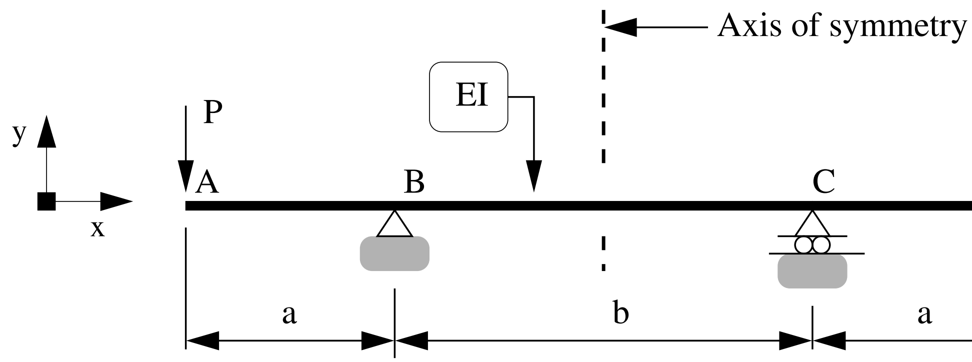

Figure 4: Cantilevered beam carrying two applied loads P.

Notice that the beam geometry and load pattern are symmetric – hint, hint, hint!! – about the beam midpoint.

[3a] (2 pts) Draw and label the M/EI diagram.

[3b] (2 pts) Create a qualitative sketch the deflection shape. Indicate where the maximum upwards deflection and maximum downward deflections will occur.

[3c] (2 pts) Draw and label the **moment area diagram** (i.e., with rotations, tangents, displacements, etc) for this problem.

[3d] (2 pts) Use the **method of moment area** to show that the maximum upward deflection is:

$$y_{\text{maximum upward}} = \frac{Pab^2}{8EI}. \tag{6}$$

[3e] (3 pts) Use the **method of moment area** to show that the maximum downward deflection is:

$$y_{\text{maximum downward}} = \frac{Pa^2}{6EI}\left[2a + 3b\right]. \tag{7}$$

[3f] (4 pts) Show that the maximum upward and maximum downward deflections will have equal magnitude when

$$\left[\frac{a}{b}\right] = \left[\frac{\sqrt{15}-3}{4}\right]. \tag{8}$$