Department of Civil and Environmental Engineering, Spring Semester, 2021

ENCE 353 Midterm 2, Open Notes and Open Book

Name: ________________________

E-mail (print neatly!): ________________________

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please **show all of your working**.

Question	Points	Score
1	15	
2	10	
3	15	
Total	40	

Question 1: 15 points

Analysis of a Cantilever with Moment Area. Consider the cantilever shown in Figure 1.

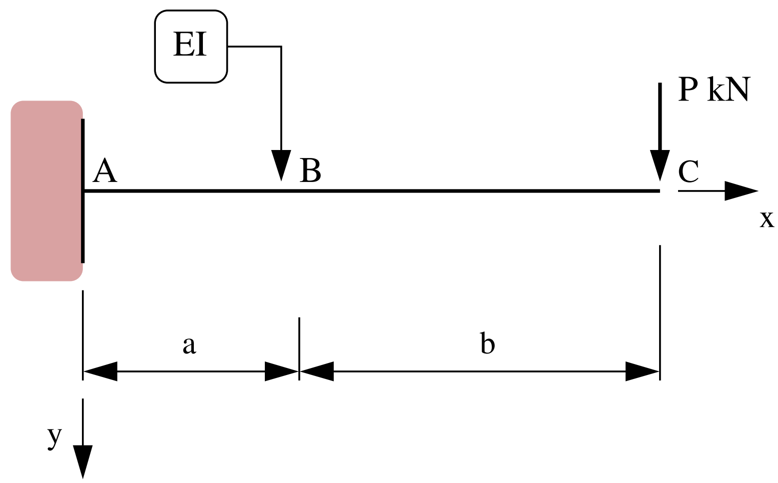

Figure 1: Front elevation view of a cantilever.

The cantilever has constant section properties, EI, along its entire length (a+b). A vertical load P (kN) is applied at point C.

[1a] (3 pts) Use the method of **moment area** to show that the vertical deflection of the cantilever at point C is:

$$y_C = \frac{P(a+b)^3}{3EI}. \tag{1}$$

[1b] (3 pts) Use the method of **moment area** to show that the vertical deflection of the cantilever at point B is:

$$y_B = \frac{Pa^2}{6EI}[3b + 2a]. \tag{2}$$

Now suppose that a roller support is inserted below point B as follows:

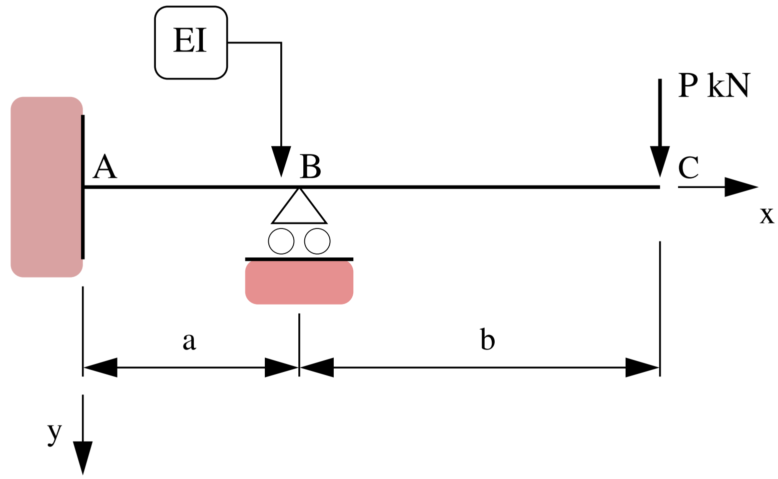

Figure 2: Front elevation view of a cantilever supported by a roller at point B.

[1c] (3 pts) Show that the vertical support reaction at B is:

$$V_b = \frac{P}{2}\left[\frac{3b + 2a}{a}\right]. \tag{3}$$

[1d] (2 pts) Hence, derive a simple expression for the bending moment at A.

Finally, let's replace the roller support below point B with a spring.

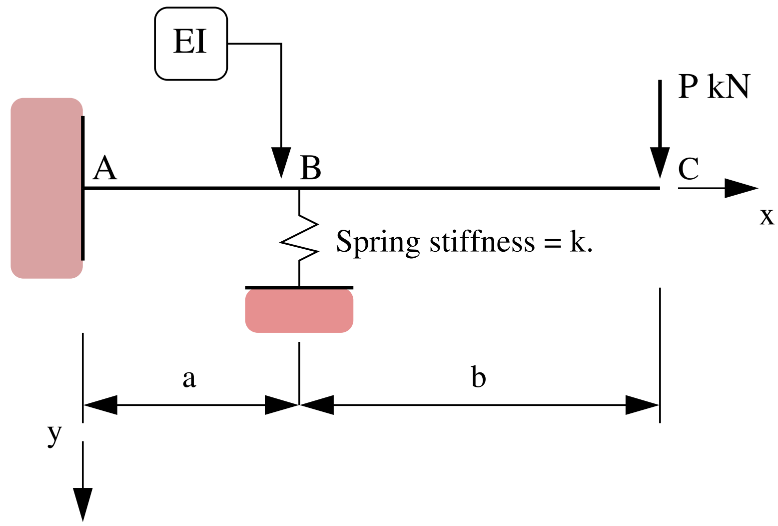

Figure 3: Cantilever supported by a spring at point B.

[1e] (2 pts) Show that the support reaction, V_b, is now given by the equation:

$$V_b\left[\frac{1}{k} + \frac{a^3}{3EI}\right] = \frac{Pa^2}{6EI}[3b + 2a]. \tag{4}$$

[1f] (2 pts) Explain why V_b for spring support (i.e., equation 4) is always lower than for roller support (i.e., equation 3).

Question 2: 10 points

Derive Elastic Curve for Beam Deflection. Figure 4 shows a beam structure that carries a uniform load W (N/m) along its length.

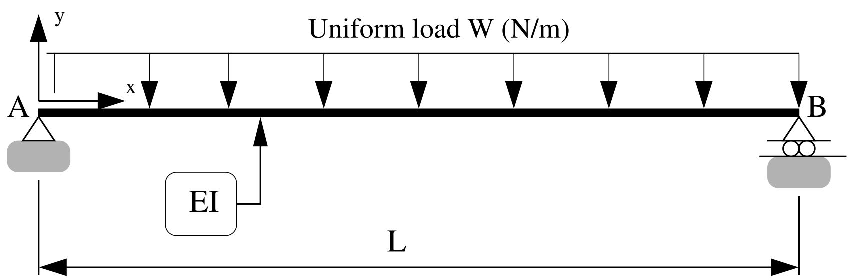

Figure 4: Front elevation view of a simple beam.

[2a] (10 pts). Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{-M(x)}{EI}\right], \tag{5}$$

(notice the minus sign on M(x)) and appropriate boundary conditions, show that that vertical beam deflection is given by:

$$y(x) = -\frac{Wx}{24EI}\big(x^3 - 2Lx^2 + L^3\big). \tag{6}$$

Question 2 continued:

Question 3: 15 points

Analysis of a Three-Pinned Parabolic Arch. This question is inspired by the St. Louis Gateway Arch shown on the class web page. We will compute the vertical and horizontal support reactions due to **self-weight the arch** alone, and explore the validity of approximations in the analysis along the way.

Since the mathematical details of this problem are a bit complicated, I suggest that you use **Wolfram Alpha** (see: https://www.wolframalpha.com) for the integration, and read the web page output for hints on suitable simplifications.

Problem Setup. Figure 5 is a front elevation view of a three-pinned parabolic arch that has a profile: $y(x) = kx^2$.

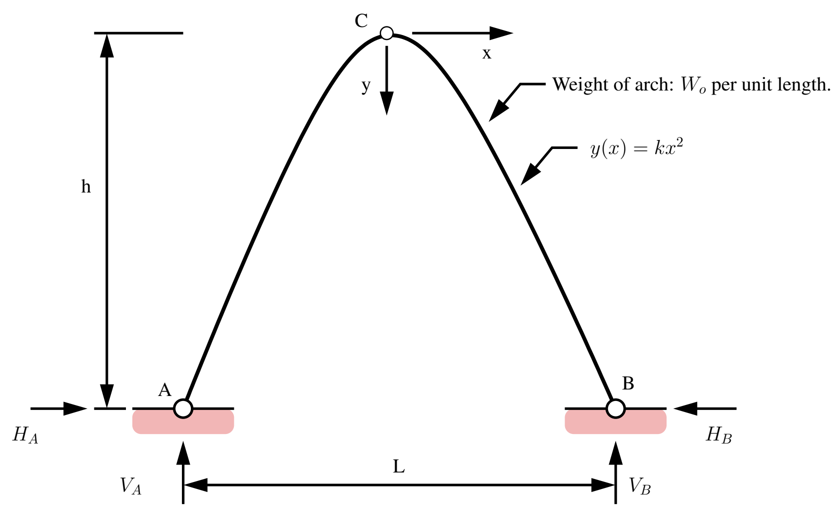

Figure 5: Front elevation view of a three-pinned parabolic arch.

The arch has height h, span L, and has self-weight W_o (N/m) along its profile. Points A, B and C are pins.

[3a] (3 pts) Starting from first principles of geometry, show that the equivalent loading measured in the horizontal direction is:

$$w(x) = W_o\big[1 + 4k^2x^2\big]^{1/2}. \quad (7)$$

Show all of your working:

Question 3a continued:

[3b] (3 pts) Show that an approximate value of V_A is:

$$V_A \approx \frac{W_o L}{2}\left[1 + \frac{8}{3}\left(\frac{h}{L}\right)^2\right]. \tag{8}$$

Notice that when h/L = 0, the arch becomes a straight beam and $V_A = \frac{W_o L}{2}$.

[3c] (3 pts) Using Wolfram Alpha, or otherwise, derive a formula for the moments about C due to self-weight of the arch alone, i.e.,

$$\int_0^{L/2} w(x)xdx. \tag{9}$$

All reasonable answers will be accepted.

[3d] (3 pts) With equations 8 and 9 in place, write down and label the equation you would solve to compute the horizontal reaction force at A.

[3e] (3 pts) Now suppose that equation 8 is applied to the St. Louis Gateway Arch (see dimensions on class web page), where h/L = 1. Does the computed value for V_A seem reasonable to you, or not? And if not, how you would fix the problem?

Either way, justify your answer.