Department of Civil and Environmental Engineering, Fall Semester, 2019

ENCE 353 Midterm 2, Open Notes and Open Book

Name: AUSTIN.

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please show all of your working.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 1: 15 points

Analysis of a Supported Cantilever Beam Structure. Consider the supported cantilever beam structure shown in Figure 1.

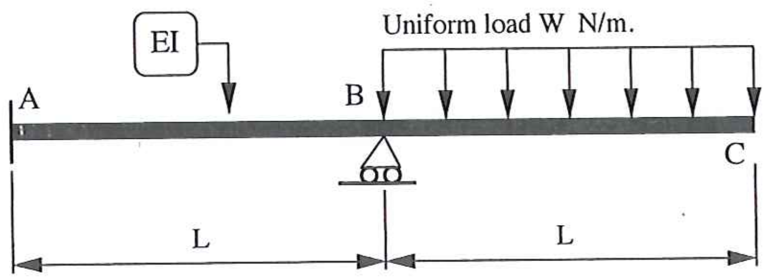
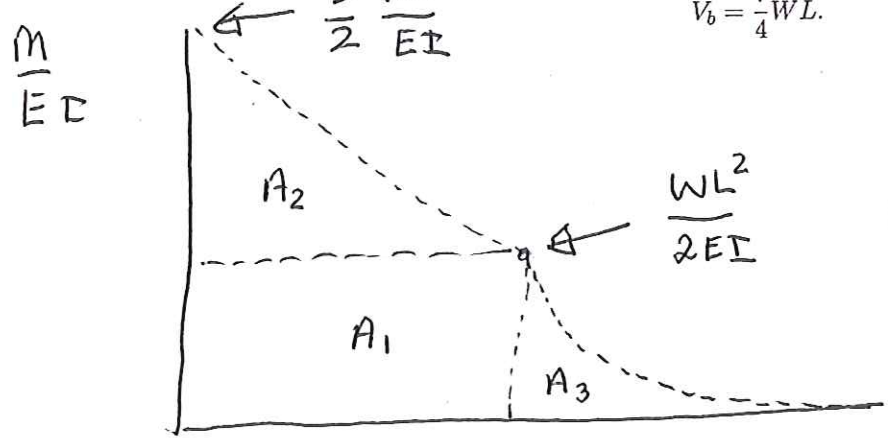


Figure 1: Front elevation view of a supported cantilever beam structure.

The cantilever is fully fixed (no rotation) at support A and is restrained against vertical displacements at B. It carries a uniform load W (N/m) along the segment length B-C.

[1a] (8 pts) Use the methods of **moment area** and **compatibility of displacements** to show that the support reaction at B is:

$$V_b = \frac{7}{4} W L. \tag{1}$$



$A_1 = A_2 = \frac{WL^3}{2EI}$

$A_3 = \frac{A_1}{3} = \frac{WL^3}{6EI}$

Release reaction at B.

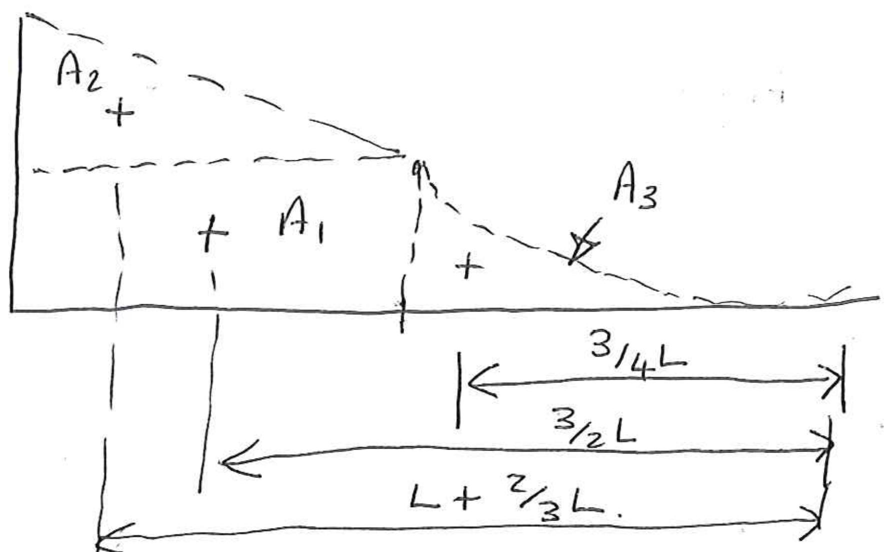
$$\Delta_B = A_1 \bar{x}_1 + A_2 \bar{x}_2 = \frac{WL^3}{2EI}\left[\frac{L}{2} + \frac{2}{3}L\right] = \frac{7}{12}\frac{WL^4}{EI}$$

Apply reaction force R at B: Compatibility of displacements.

$$\frac{V_B L^3}{3EI} = \Delta_B = \frac{7}{12}\frac{WL^4}{EI} \Rightarrow \boxed{V_B = \frac{7}{4} WL}$$

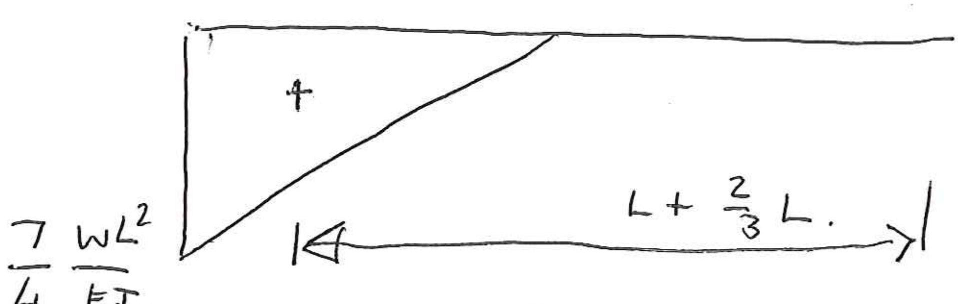
[1b] (7 pts) Use the method of **moment area** to compute the vertical displacement at C.

Δ_C with no support at B.



$$\Delta_C = A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3 = \frac{41 wL^4}{24 EI} \quad - \text{(A)}$$

Δ_C due to reaction force $V_B = \frac{7}{4} wL$.



$$\Delta_{CR} = \left(\frac{-7}{4} wL^2\right)\left(\frac{L}{2}\right)\left(L + \frac{2}{3}L\right)$$

$$= \frac{-35}{24} \frac{wL^4}{EI} \quad - \text{(B)}$$

Net displacement (A) + (B)

$$\Delta_C = \frac{41 wL^4}{24EI} - \frac{35 wL^4}{24EI} = \frac{wL^4}{4EI} \downarrow.$$

Question 2: 15 points

Consider the cantilevered beam structure shown in Figure 2.

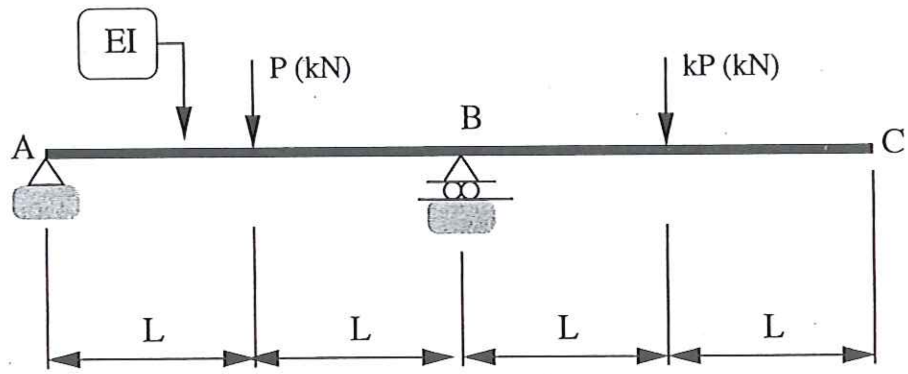
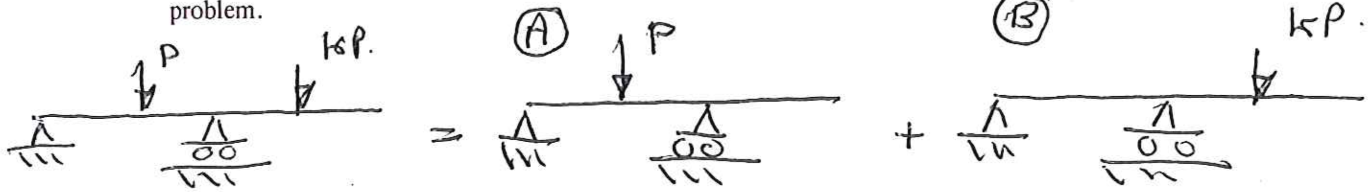


Figure 2: Front elevation view of a cantilevered beam structure.

Vertical loads of P (kN) and kP (kN) are applied at the mid-spans of beam segments A-B and B-C, respectively.

[2a] (3 pts) Briefly explain how the **principle of superposition** — hint, hint hint! – can be applied to this problem.

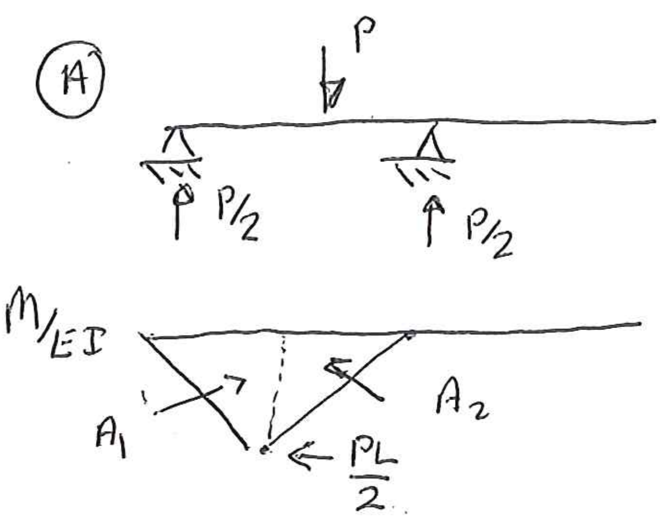


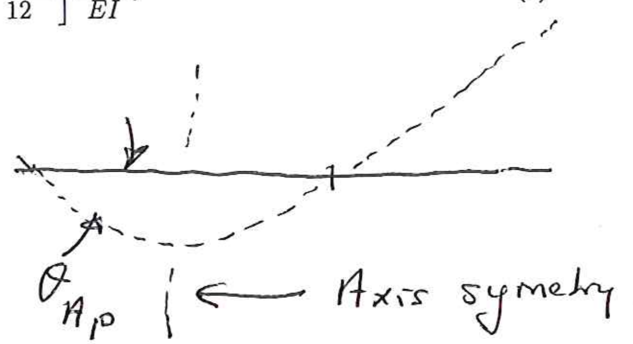
[2b] (8 pts) Use the **method of moment-area** to show that the clockwise rotation of point A is:

Load case

$$\theta_A = \left[\frac{3-4k}{12}\right]\frac{PL^2}{EI}. \quad (2)$$

(A)

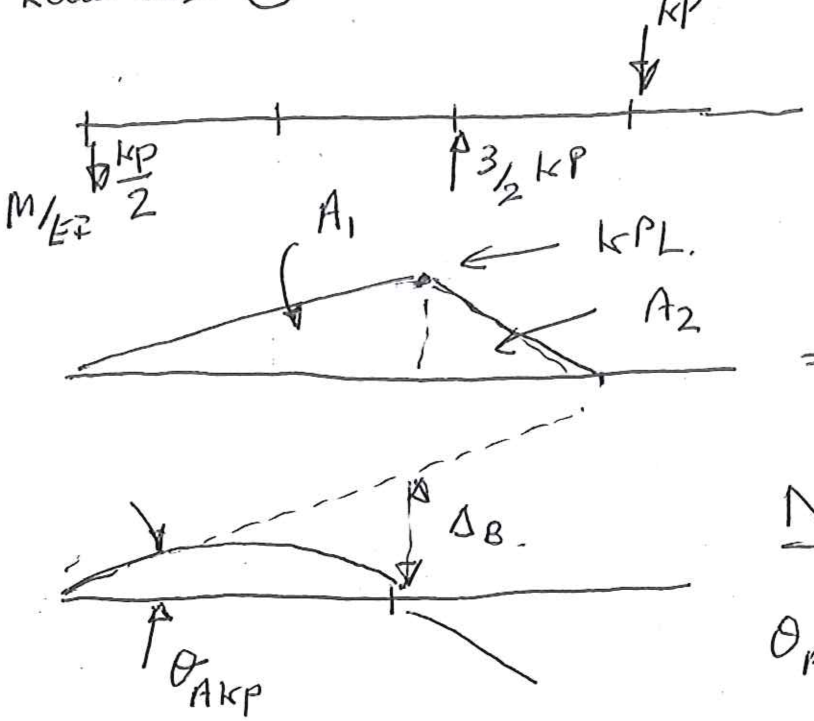




$A_1 = A_2 = \theta_{AP} = \dfrac{PL^2}{4EI}.$

Question 2a continued:

Load Case (B).



$$A_1 = \frac{kPL^2}{EI}, \quad \Delta_B = \frac{+2}{3}\frac{kL^3}{EI}$$

$$\Rightarrow \theta_{Akp} = \frac{\Delta_B}{2L} = \frac{+kPL^2}{3EI}.$$

Net rotation

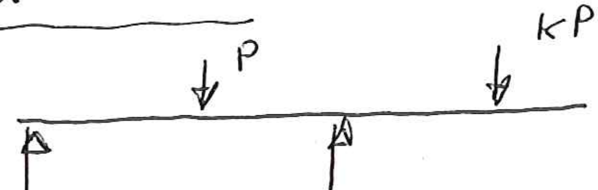
$$\theta_A = \theta_{Ap} - \theta_{Akp}$$

$$= \frac{PL^2}{EI}\left[\frac{1}{4} - \frac{k}{3}\right] = \frac{PL^2}{EI}\left[\frac{3-4k}{12}\right]$$

[2c] (4 pts) Draw and label the deflected shape of the beam when k = 3/4. Indicate sections of beam where the fibre is in tension/compression, and where the curvature is zero.

When $k = 3/4 \Rightarrow 3-4k = 0 \Rightarrow \theta_A = 0.$

Net reactions

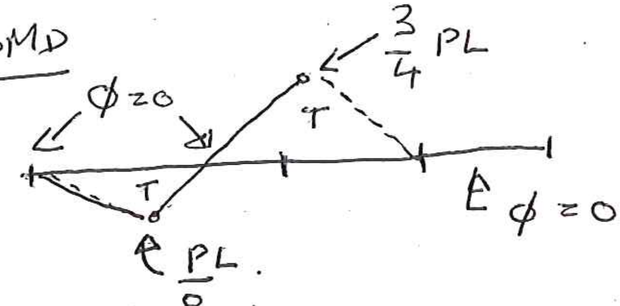


$$V_A = \frac{P}{2}(1-k) \qquad V_B = \frac{P}{2}(1+3k)$$

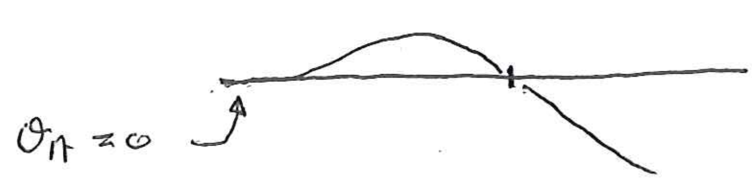
$k = 3/4$

$$V_A = P/8 \qquad V_B = \frac{13}{8}P.$$

BMD



Deflected Shape



Question 3: 10 points

The cable structure shown in Figure 3 carries a triangular load that is zero at the left-hand support and increases to w_o N/m at the right-hand support.

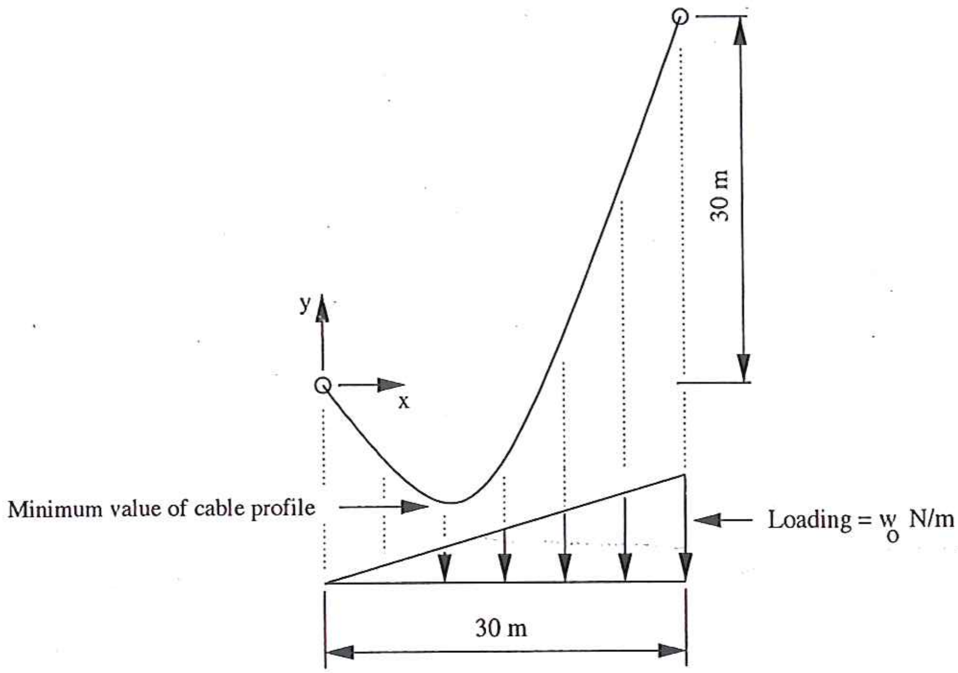


Figure 3: Elevation view of a swing bridge carrying a triangular loading.

[3a] (4 pts). Starting from first principles (i.e., the differential equation), show that cable profile is given by the equation

$$y(x) = \frac{w_o x^3}{180H} + \left(1 - \frac{5w_o}{H}\right) x. \tag{3}$$

Loading $\quad w(x) = \frac{w_o}{30} x.$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{w(x)}{H} = \frac{w_o x}{30H}.$$

$$\frac{dy}{dx} = \frac{w_o x^2}{60H} + A$$

$$y(x) = \frac{w_o x^3}{180H} + Ax + B.$$

BC.

$y(0) = 0 \Rightarrow B = 0$

$y(30) = 30 \Rightarrow$

$$30 = \frac{w_o 30^3}{180H} + A \cdot 30$$

$$\Rightarrow A = \left(1 - \frac{5w_o}{H}\right)$$

$$\therefore y(x) = \frac{w_o x^3}{180H} + \left(1 - \frac{5w_o}{H}\right) x.$$

Now let us assume that the minumum value of the cable profile occurs at x = 10.

[3b] (4 pts). Show that the horizontal cable force is:

$$H = \frac{20w_o}{6}. \tag{4}$$

$$\left.\frac{dy}{dx}\right|_{x=10} = \frac{W_0\, 160}{60\, H} + \left(1 - \frac{5w_0}{H}\right) = 0$$

$$\Rightarrow H = \frac{20}{6} W_0 .$$

Also note, $A = -\frac{1}{2}$.

[3c] (2 pts). Draw and label a diagram showing the horizontal and vertical components of reaction force at the left- and right-hand cable supports.

$$V = H \frac{dy}{dx}.$$

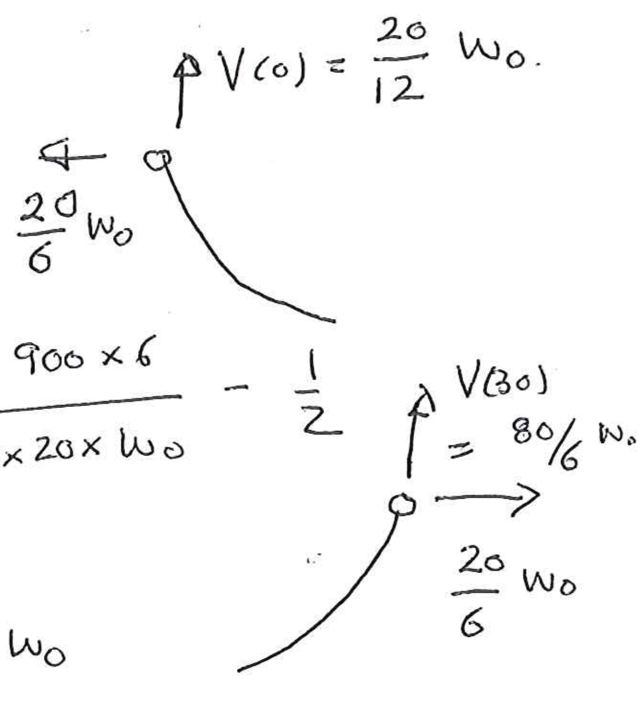
At x = 0, $\left.\frac{dy}{dx}\right|_{x=0} = A = -\frac{1}{2}$

At x = 30

$$\left.\frac{dy}{dx}\right|_{x=30} = \frac{W_0 \; 900 \times 6}{60 \times 20 \times W_0} - \frac{1}{2}$$

$$= 4.$$

$$\Rightarrow V(30) = \frac{80}{6} W_0$$



Check Equilibrium

$$V(0) + V(30) = \frac{20}{12} W_0 + \frac{160}{12} W_0 = 15 W_0 \checkmark$$