Statically Determinate Structures

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Overview

1. Introduction
   - Need for Mathematical Test
   - Benefits of Indeterminacy

2. Statical Determinacy of Trusses
   - Formulae and Examples

3. Statical Determinacy of Planar Structures

4. Indeterminacy of Beams

5. Indeterminacy of Frames
   - Tree and Ring Methods

6. Stability

Part 2
Introduction
Need for Mathematical Test

Three cases to consider:

Test Structure A: Determinate.

Can compute:
- Support reactions. ✓
- Member forces. ✓

Test Structure B: Indeterminate.

Can compute:
- Support reactions. ✓
- Member forces. ⬗
Need for Mathematical Test

Test Structure C: Unstable.

Can compute:
- Support reactions. ✗
- Member forces. ✗

Key Points:
- Intuition on notions of determinacy will not scale. We need a mathematical test to classify structures.
- Initial inclination is to design for A and avoid B – it’s complicated and probably won’t work. Unless, there are benefits to B?
Indeterminacy of Beams
Computing Degree of Indeterminacy

**Definition.** The degree of indeterminacy is equal to the number of additional equations needed to solve a problem uniquely.

Additional info:
- Compatibility of deformations – this is the force method.
- Equilibrium of forces – this is the displacement method.

**Beams:** \( \hat{i} = f - 3 - r \), where:
- \( f \) = total no of external forces,
- \( r \) = total no of releases (hinges),
- \( 3 \) = no of equations from statics.
**Example 1. Supported Cantilever Beam.**

We have:

\[ r = 0, \]
\[ f = \{V_A, H_A, \ldots, V_B\} = 5. \]
\[ \hat{i} = f - 3 - r = 2. \]

Need to release two restraints to create determinate structures, e.g.,
Example 2. Fixed-Fixed Beam.

We have: $r = 0,$

$f = \{ V_A, H_A, M_A, V_B, H_B, M_B \} = 6.$

$\hat{i} = f - 3 - r = 3.$
Example 3. Fixed-Fixed Beam + Hinge.

We have: \( r = 1 \),

\[ f = \{ V_A, H_A, M_A, V_B, H_B, M_B \} = 6. \]

\[ \hat{i} = f - 3 - r = 2. \]
Example 4. Two-Span Beam.

We have: \( r = 0, \)
\[
\]
\[
\hat{i} = f - 3 - r = 3.
\]
Indeterminacy of Frames
**Approach:** Systematically release redundant forces until trees are formed.

**Formula:** \( \hat{i} = f - 3t \), where:
- \( f \) = no of external forces,
- \( t \) = no of trees.

**Constraints:** Frame cannot have internal releases (no loops in trees).

**Trees:**
- A tree has one root.
- A tree cannot have a closed loop branch.
Tree Method

Example 1a.

Example 1b.
Tree Method

Example 2.

\[ \begin{align*}
\text{Cut} \\
\text{Tree 1} & \quad f = 3 \\
\text{Tree 2} & \quad f = 3 \\
& \\
\hat{i} & = 12 - 3 \times 2 = 6.
\end{align*} \]

**Example 2.**

- Tree 1: \( f = 3 \)
- Tree 2: \( f = 3 \)

\( f = 12 \)
\( t = 2 \)
\( \hat{i} = 12 - 3 \times 2 = 6 \)
**Ring Method**

Formula:  \( \hat{i} = 3n - r \), where:
- \( n \) = no of rings.
- \( r \) = no of releases (each ring has 3 degrees of indeterminacy).

**Example 1.**

\[ \hat{i} = 6 - 1 = 5. \]
Example 2.

\[ n = 4. \]
\[ r = 3 \]
\[ \hat{i} = 3 \times 4 - 3 = 9. \]