ENCE 353 Introduction to Structural Analysis,

Spring Semester, 2020

Homework <u>3</u> (Due: Wednesday, March 25, 2020)

Question 1: 6 points

The cantilever beam structure shown in Figure 1 carries a uniform load w (N/m) along its entire length.



Figure 1: Cantilever beam carrying a uniform load.

The beam is fully fixed at point A and the flexural stiffness EI is constant along the beam. The coordinate system is positioned at point A.

[1a] (3 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right],\tag{1}$$

and appropriate boundary conditions, show that:

$$y(x) = \left(\frac{w}{24EI}\right) \left(6L^2 x^2 - 4Lx^3 + x^4\right).$$
 (2)

[1b] (3 pts) Using the results of question [1a] as a starting point, compute the support reactions at A and B for the propped cantilever shown in Figure 2



Figure 2: Propped cantilever beam carrying a uniform load.

Question 2: 6 points

Figure 3 is a front elevation view of a simple three-pinned arch. A vertical load P is applied at node D.



Figure 3: Simple three-pinned arch.

- [2a] (3 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of P.
- [2b] (3 pts) Compute the magnitude and orientation of the <u>total reaction force vector</u> at support B. Show that it passes through the hinge at C. You can annotate Figure 3 if you think it will help to explain your solution.

Question 3: 8 points

For the three-pin arc structure shown below (Figure 4), the profile is given by:

$$y = \frac{4f}{l^2} * x * (l - x)$$
(3)

where f = 4 m and l = 16 m.

- [3a] (2 pts) Calculate the reactions at A and B.
- [3b] (3 pts) Calculate the internal forces at point E (i.e. axial force F_N , shear force F_Q and bending moment M).
- [3c] (3 pts) Draw the moment diagram.



Figure 4: Simple three-pinned arch.