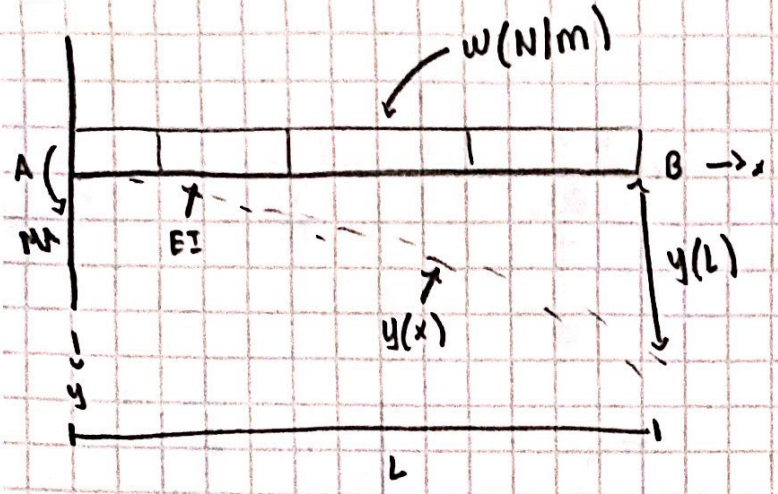
Homework 3

Homework 3
Problem 1 Solution

Question 1:



1a.) using $\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$, show that $y(x) = \frac{w}{24EI}(6L^2x^2 - 4Lx^3 + x^4)$

boundary conditions

$y(0) = 0$

$\frac{dy}{dx}(0) = 0$

$M_A = wL \cdot \frac{L}{2} = \frac{wL^2}{2}$

$V(x) = -w(L-x) = -wL + wx$

$\frac{dM}{dx} = V(x) \Rightarrow M(x) = -wLx + \frac{wx^2}{2} + M_a$

$M(x) = -wLx + \frac{wx^2}{2} + \frac{wL^2}{2}$

$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} = \frac{1}{EI}\left(-wLx + \frac{wx^2}{2} + \frac{wL^2}{2}\right)$

$\frac{dy}{dx} = \frac{1}{EI}\left(-\frac{wLx^2}{2} + \frac{wx^3}{6} + \frac{wL^2x}{2}\right) + C_1 \Rightarrow C_1 = 0$ since $\frac{dy}{dx}(0) = 0$

$y(x) = \frac{1}{EI}\left(-\frac{wLx^3}{6} + \frac{wx^4}{24} + \frac{wL^2x^2}{4}\right) = \frac{w}{24EI}(6L^2x^2 - 4Lx^3 + x^4) + C_2$

$C_2 = 0$ since $y(0) = 0$

$$\boxed{y(x) = \frac{w}{24EI}(6L^2 - 4Lx^3 + x^4)} \checkmark$$

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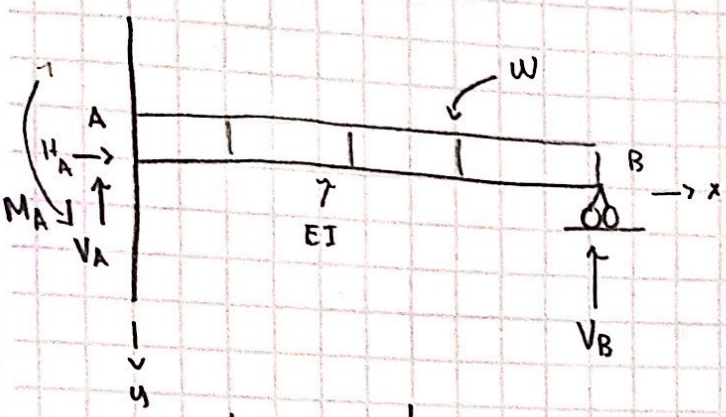
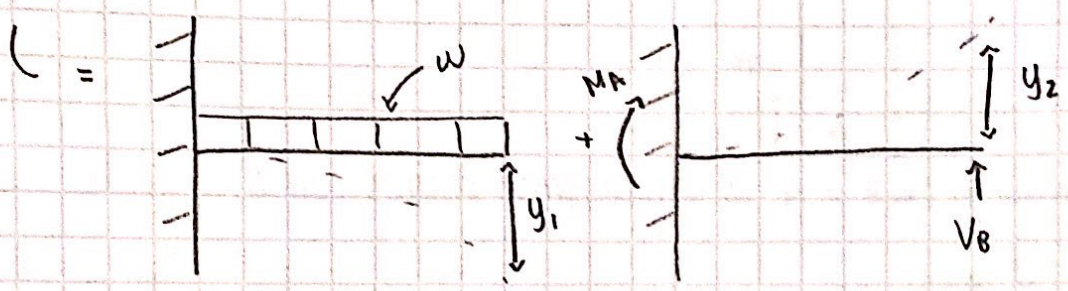
1b.- Using results of 1a, compute the reactions at A & B





$$y_1 = y(L) = \frac{w}{24EI}\left(6L^4 - 4L^4 + L^4\right) = \frac{wL^4}{8EI}$$

$$y_2 = \frac{-RL^3}{3EI} \longrightarrow V(x) = V_B$$

$$M(x) = \int V(x) = V_B x + M_A = V_B x - V_B L$$

$$y(0) = 0 \qquad \frac{dy}{dx}(0) = 0$$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left(V_B x - V_B L\right) \Rightarrow \frac{dy}{dx} = \frac{1}{EI}\left(\frac{V_B x^2}{2} - V_B L x\right)$$

$$y(x) = \frac{1}{EI}\left(\frac{V_B x^3}{6} - \frac{V_B L x^2}{2}\right) \Rightarrow y(L) = -\frac{V_B L^3}{3EI}$$

$$\frac{wL^4}{8EI} - \frac{V_B L^3}{3EI} = 0 \qquad \frac{wL^4}{8EI} = \frac{V_B L^3}{3EI} \Rightarrow \boxed{V_B = \frac{3wL}{8}\ N}$$

$$\Sigma F_y = 0 \qquad wL = V_A + V_B \Rightarrow \boxed{V_A = \frac{5wL}{8}\ N}$$

$$\Sigma F_x = 0 \qquad \boxed{H_A = 0\ N}$$

$$M_A = -\left(\frac{3wL}{8}\right)(L) + wL\left(\frac{L}{2}\right) \Rightarrow M_A = -\frac{3wL^2}{8} + \frac{wL^2}{2} \Rightarrow \boxed{M_A = \frac{wL^2}{8}\ N\cdot m}$$

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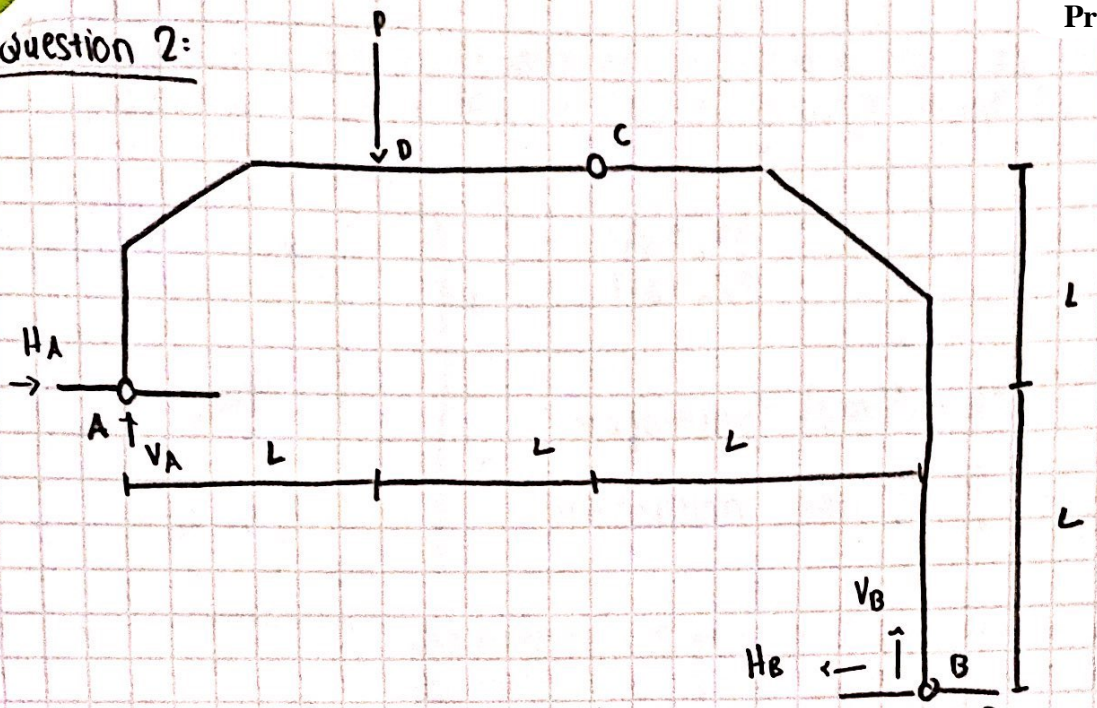
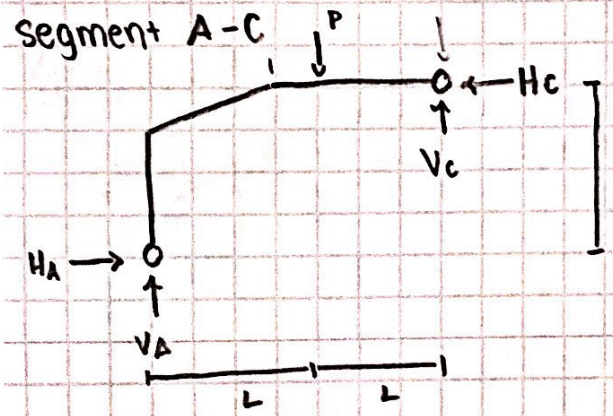
Homework 3
Problem 2 Solution

Question 2:



2a., compute vertical & horizontal components of A & B as a function of P.

Segment A-C



$\Sigma M_A = 0$

$2V_c L + H_c L = PL$

$2V_c + H_c = P$

$\Sigma F_y = 0$

$V_A + V_c = P$

$\Sigma F_x = 0$

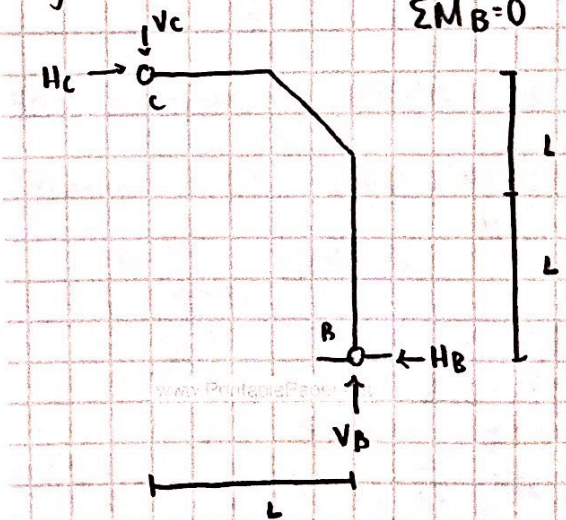
$V_A = V_c$

For A-B

$\Sigma F_y = 0 \quad V_A + V_B = P$

$\Sigma F_x = 0 \quad H_A = H_B$

Segment A-B



$\Sigma M_B = 0 \quad 2H_c L = V_c L$

$2H_c = V_c$

$\hookrightarrow 2H_B = V_B$

$\Sigma F_y = 0$

$V_B = V_c$

$\Sigma F_x = 0$

$H_B = H_c$

$\boxed{V_B = .4P}$

$2(2H_c) + H_c = P$

$H_c = .2P$

$\boxed{H_B = .2P = H_A}$

$2V_c + .5V_c = P \quad V_c = .4P$

$V_A = P - V_B \Rightarrow \boxed{V_A = .6P}$

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Homework 3
Problem 2 Solution

2B.) compute magnitude and orientation of the total reaction force vector at B. Show that it passes through hinge C.

$V_B = .4P$
$H_B = .2P$

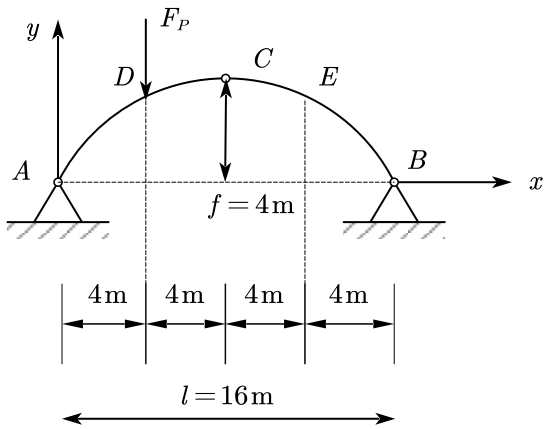
magnitude $= ((.4P)^2 + (.2P)^2)^{1/2} = \boxed{0.447P}$

orientation $= \tan^{-1}\left(\frac{.4P}{.2P}\right) = \boxed{63.43°}$

the force passes through C because the orientation of the force vector is the same as the length vector

$\tan^{-1}\left(\frac{2L}{L}\right) = \tan^{-1}\left(\frac{.4P}{.2P}\right) = \tan^{-1}(2) = 63.43°$

Problem 3. For the three-pin arc structure shown below, the profile is given by $y = \frac{4f}{l^2}x(l-x)$, where $f = 4$ m and $l = 16$ m.



(a) Calculate the reactions at A and B;

(b) Calculate the internal forces at point E (i.e. axial force F_N, shear force F_Q and bending moment M);

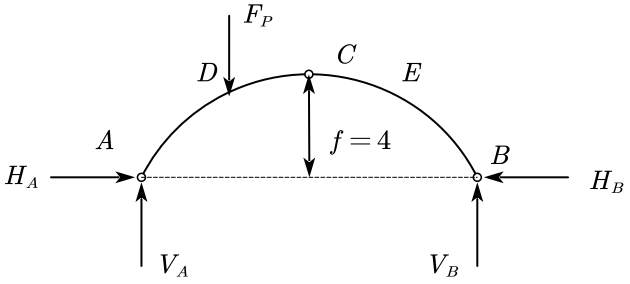
(c) Draw the moment diagram.

(d) if $f = 8$ m, redo the calculation (a), (b), and (c).

Not part of our assignment

Solution:

(a) Draw the free body diagram of the arc structure:



$$\sum F_x = 0,\ H_A = H_B$$

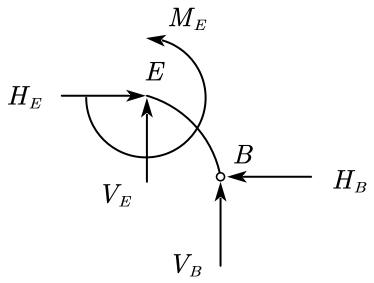
$$\sum F_y = 0,\ V_A + V_B = F_p$$

$$\sum M_A = 0,\ F_P \cdot 4 - V_B \cdot 16 = 0 \Rightarrow V_B = \frac{F_P}{4}(\uparrow) \Rightarrow V_A = \frac{3F_P}{4}(\uparrow)$$

Slice the arc structure at point C, and write the moment equilibrium about point C (left part) would get:

$$\sum M_C = 0,\ -F_P \cdot 4 + V_A \cdot 8 - H_A \cdot 4 = 0 \Rightarrow H_A = \frac{F_P}{2}(\rightarrow) \Rightarrow H_B = \frac{F_P}{2}(\leftarrow)$$

(b) Slice the arc structure at point E, and draw the free body diagram of the right part:

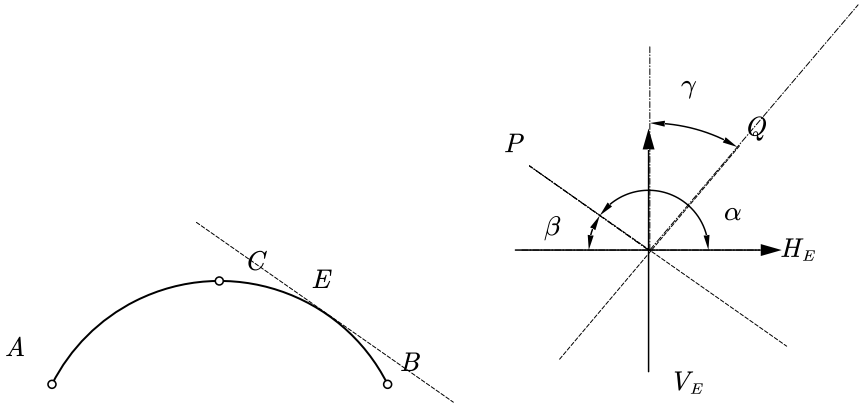


$$\sum F_x = 0,\ H_E = H_B \Rightarrow H_E = \frac{F_P}{2}(\rightarrow)$$

$$\sum F_y = 0,\ V_E + V_B = 0 \Rightarrow V_E = -\frac{F_P}{4}(\downarrow)$$

$$\sum M_E = 0,\ -M_E - V_B \cdot 4 + H_B \cdot y(12) = 0 \Rightarrow M_E = \frac{F_P}{2}(\curvearrowleft)$$

The axial force at point E is along with the tangent of the arc profile at point E and the shear force is perpendicular to the tangent of the arc profile. Thus, to calculate the axial and shear force, decomposition of H_E and V_E is needed:



$$\frac{dy}{dx}|_{x=12} = \frac{4f}{l^2}(l-2x)|_{x=12} = -\frac{1}{2}$$

$$\tan\alpha = -\frac{1}{2} \Rightarrow \alpha = 153.43(\text{deg}) \Rightarrow \beta = 26.57(\text{deg}) \Rightarrow \gamma = \beta = 26.57(\text{deg})$$

$$\sin\beta = 0.447, \cos\beta = 0.894$$

$$P = V_E\sin\beta + H\cos\beta = \frac{F_P}{4}\cdot 0.447 + \frac{F_P}{2}\cdot 0.894 = 0.559F_P(\text{compression})$$

$$Q = -V_E\cos\beta + H\sin\beta = -\frac{F_P}{4}\cdot 0.894 + \frac{F_P}{2}\cdot 0.447 = 0$$

(c) Assume the moment at point x=x is positive in clockwise:

For section 0<=x<=4, write the moment equilibrium about point at x=x:

$$\sum M_{x=x} = 0,\ V_A \cdot x - H_A \cdot y(x) + M(x) = 0$$

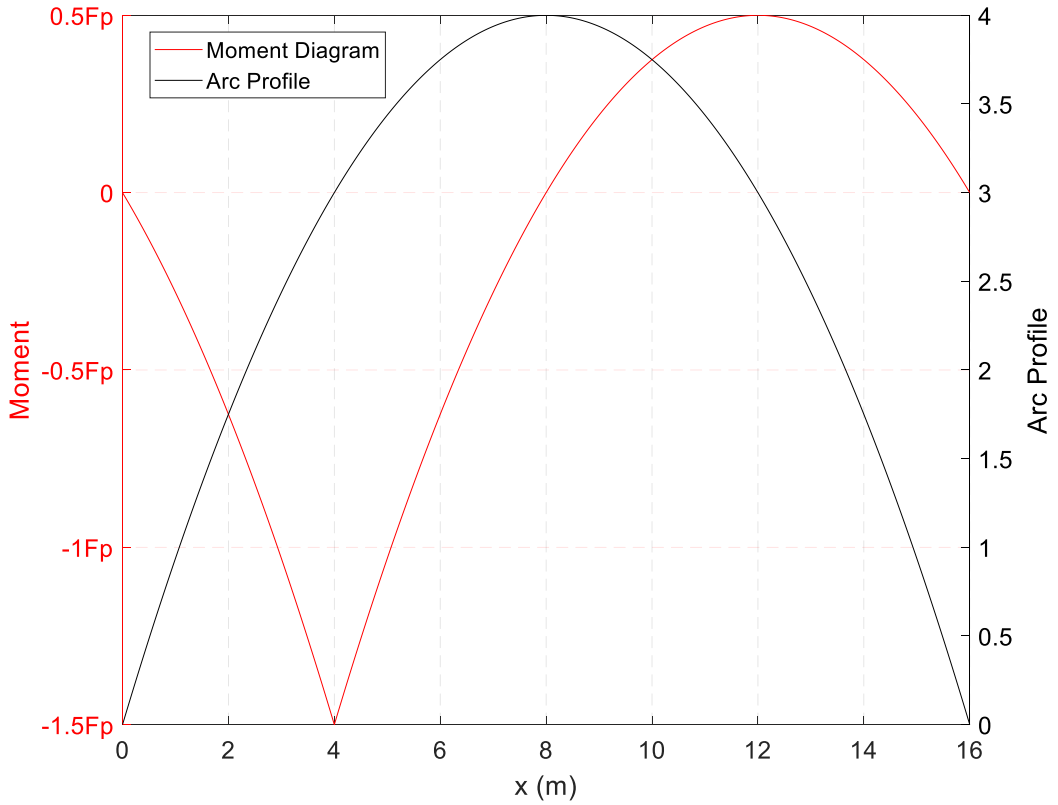
$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right) = -\frac{F_P}{32}x^2 - \frac{F_P}{4}x$$

For section 4<=x<=16,

$$\sum M_{x=x} = 0,\ V_A \cdot x - H_A \cdot y(x) - F_P \cdot (x-4) + M(x) = 0$$

$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right) = -\frac{F_P}{32}x^2 + \frac{3F_P}{4}x - 4F_P$$

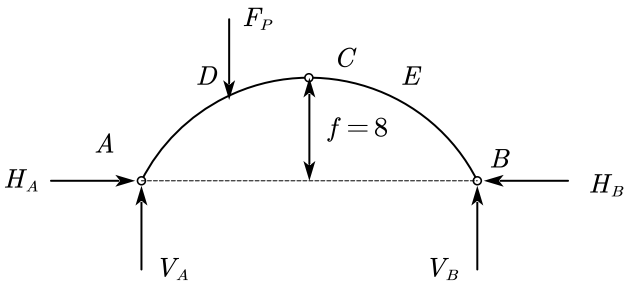
Draw the moment diagram (Assume the value of F_p >=0):



***Our assignment solution ends here**

(d-a)

Draw the free body diagram of the arc structure:



$$\sum F_x = 0,\ H_A = H_B$$

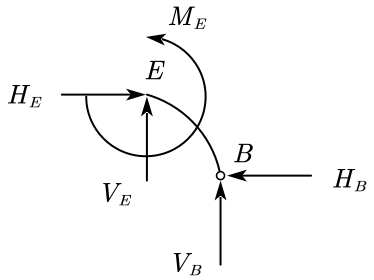
$$\sum F_y = 0,\ V_A + V_B = F_p$$

$$\sum M_A = 0,\ F_P \cdot 4 - V_B \cdot 16 = 0 \Rightarrow V_B = \frac{F_P}{4}(\uparrow) \Rightarrow V_A = \frac{3F_P}{4}(\uparrow)$$

Slice the arc structure at point C, and write the moment equilibrium about point C (left part) would get:

$$\sum M_C = 0,\ -F_P \cdot 4 + V_A \cdot 8 - H_A \cdot 8 = 0 \Rightarrow H_A = \frac{F_P}{4}(\rightarrow) \Rightarrow H_B = \frac{F_P}{4}(\leftarrow)$$

(d-b) Slice the arc structure at point E, and draw the free body diagram of the right part:

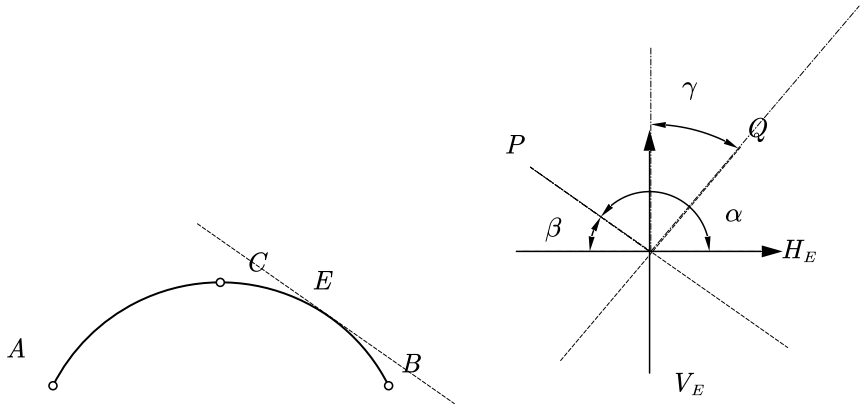


$$\sum F_x = 0,\ H_E = H_B \Rightarrow H_E = \frac{F_P}{4}(\rightarrow)$$

$$\sum F_y = 0,\ V_E + V_B = 0 \Rightarrow V_E = -\frac{F_P}{4}(\downarrow)$$

$$\sum M_E = 0,\ -M_E - V_B \cdot 4 + H_B \cdot y(12) = 0 \Rightarrow M_E = \frac{F_P}{2}(\curvearrowleft)$$

The axial force at point E is along with the tangent of the arc profile at point E and the shear force is perpendicular to the tangent of the arc profile. Thus, to calculate the axial and shear force, decomposition of H_E and V_E is needed:



$$\frac{dy}{dx}|_{x=12} = \frac{4f}{l^2}(l-2x)|_{x=12} = -1$$

$$\tan\alpha = -1 \Rightarrow \alpha = 135\,(\text{deg}) \Rightarrow \beta = 45\,(\text{deg}) \Rightarrow \gamma = \beta = 45\,(\text{deg})$$

$$\sin\beta = 0.707, \cos\beta = 0.707$$

$$P = V_E\sin\beta + H\cos\beta = \frac{F_P}{4}\cdot 0.707 + \frac{F_P}{4}\cdot 0.707 = 0.353F_P\,(\text{compression})$$

$$Q = -V_E\cos\beta + H\sin\beta = -\frac{F_P}{4}\cdot 0.707 + \frac{F_P}{4}\cdot 0.707 = 0$$

(d-c) Assume the moment at point x=x is positive in clockwise:

For section 0<=x<=4, write the moment equilibrium about point at x=x:

$$\sum M_{x=x} = 0,\ V_A\cdot x - H_A\cdot y(x) + M(x) = 0$$

$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right) = -\frac{F_P}{32}x^2 - \frac{F_P}{4}x$$

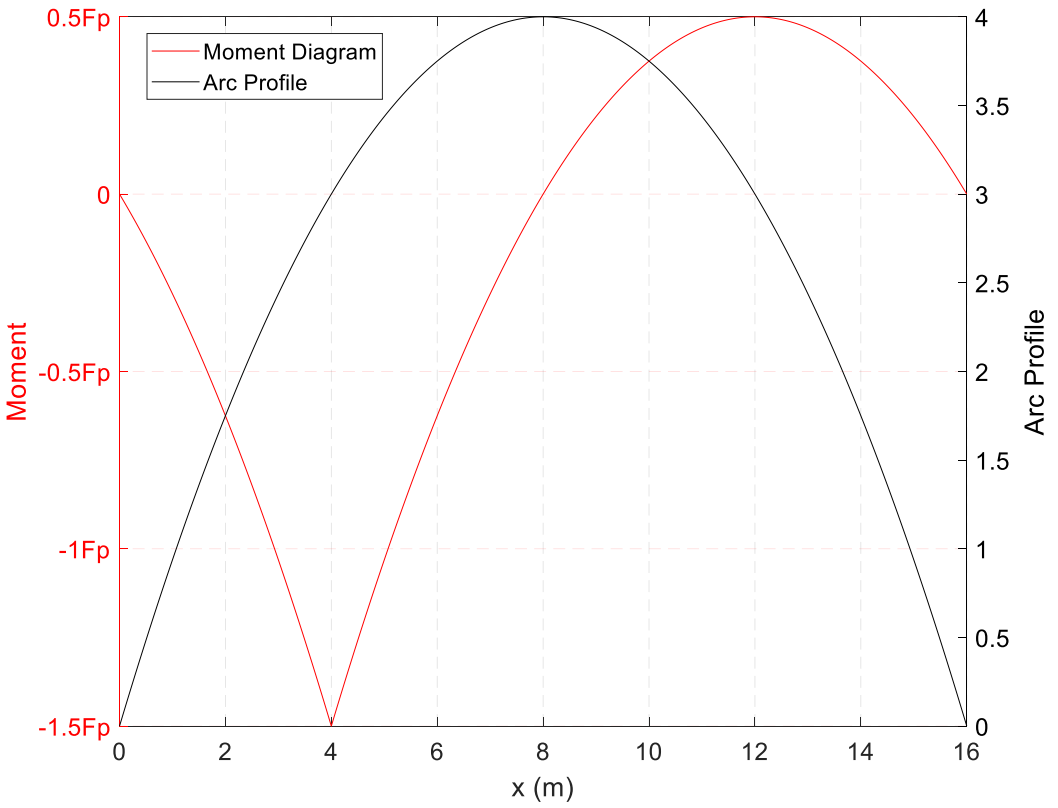
For section 4<=x<=16,

$$\sum M_{x=x} = 0,\ V_A\cdot x - H_A\cdot y(x) - F_P\cdot(x-4) + M(x) = 0$$

$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right) - F_P\cdot(x-4)$$

$$= -\frac{F_P}{32}x^2 + \frac{3F_P}{4}x - 4F_P$$

Draw the moment diagram (Assume the value of F_p >=0):


Moment Diagram
Arc Profile
0.5Fp
0
-0.5Fp
-1Fp
-1.5Fp
Moment
4
3.5
3
2.5
2
1.5
1
0.5
0
Arc Profile
0
2
4
6
8
10
12
14
16
x (m)