Department of Civil and Environmental Engineering, Fall Semester, 2020

ENCE 353 Midterm 2, Open Notes and Open Book

Name: ________________________

E-mail (print neatly!): ________________________

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please **show all of your working**.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 1: 15 points

Analysis of a Cantilever Beam with Moment Area. Consider the cantilever shown in Figure 1.

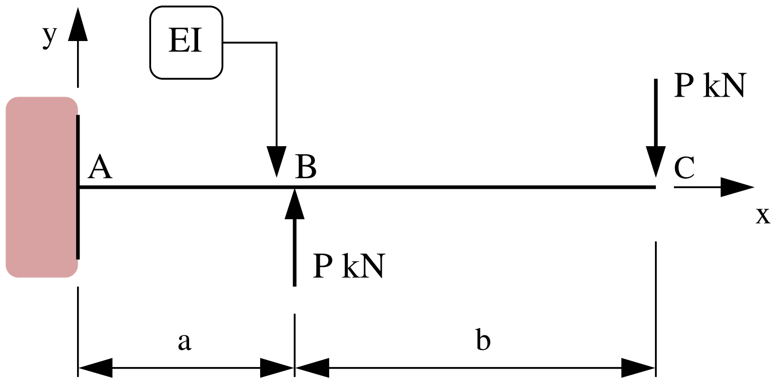

Figure 1: Front elevation view of a cantilever.

The cantilever has constant section properties, EI, along its entire length (a+b). Vertical loads of P (kN) are applied at points B and C. Notice that the y axis is pointing upwards – hence, if we apply the right-hand rule, positive rotations will be anti-clockwise.

[1a] (3 pts) Draw and label a diagram showing how the beam curvature varies along its length (i.e., $\phi(x)$).

[1b] (3 pts) Use the method of <u>**moment-area**</u> to show that the rotation of point B is:

$$\theta(a) = \left[\frac{-Pba}{EI}\right]. \tag{1}$$

[1c] (4 pts) Use the method of **moment-area** to show that the vertical displacement of point B is:

$$y(a) = \left[\frac{-Pba^2}{2EI}\right]. \tag{2}$$

[1d] (5 pts) Use the method of **moment-area** to show that the vertical displacement of point C is:

$$y(a+b) = \left[\frac{-Pb}{6EI}\right]\left[6ab + 3a^2 + 2b^2\right]. \tag{3}$$

Note: Notice that when a = 0, equation 3 simplifies to the formula we have seen many times in class.

Question 2: 15 points

Elastic Curve for a Cantilever Beam Structure. Consider the cantilever shown in Figure 2.

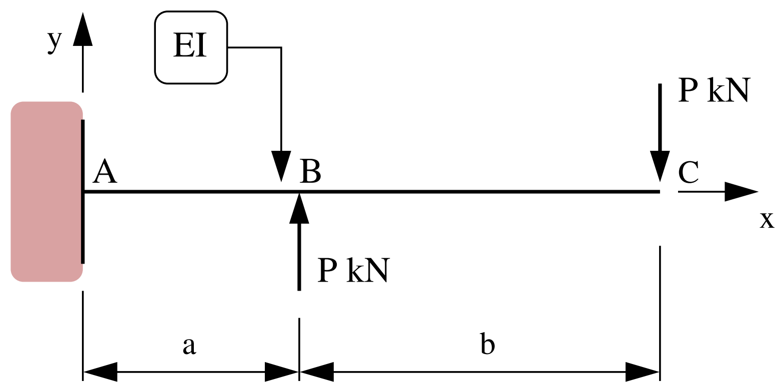

Figure 2: Front elevation view of a cantilever.

The cantilever has constant section properties, EI, along its entire length (a+b). Vertical loads of P (kN) are applied at points B and C.

[2a] (3 pts) Write a mathematical formula, M(x), for the bending moment along the beam as a function of x.

[2b] (4 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{4}$$

and appropriate boundary conditions, show that in the interval $0 \leq x \leq a$, the beam displacement and rotation are:

$$y(x) = \left[\frac{-Pbx^2}{2EI}\right] \quad \text{and} \quad \theta(x) = \left[\frac{-Pbx}{EI}\right]. \tag{5}$$

Show all of your working.

[2c] (7 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{6}$$

and appropriate boundary conditions, show that within the interval $a \leq x \leq a+b$, the beam displacement is:

$$y(x) = \left[\frac{-P}{6EI}\right]\left[3(a+b)x^2 - x^3 - 3a^2x + a^3\right]. \tag{7}$$

Hint: The algebra for this question is a bit tricky so I suggest that you work out the details on paper and submit a tidy summary of your procedure. Also notice that equation 7 is consistent with equations 2 and 3 in Question 1.

Question 2c continued:

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch that carries a total snow loading of 3WL uniformly distributed over its upper section and a point loading P (kN) applied at point C.

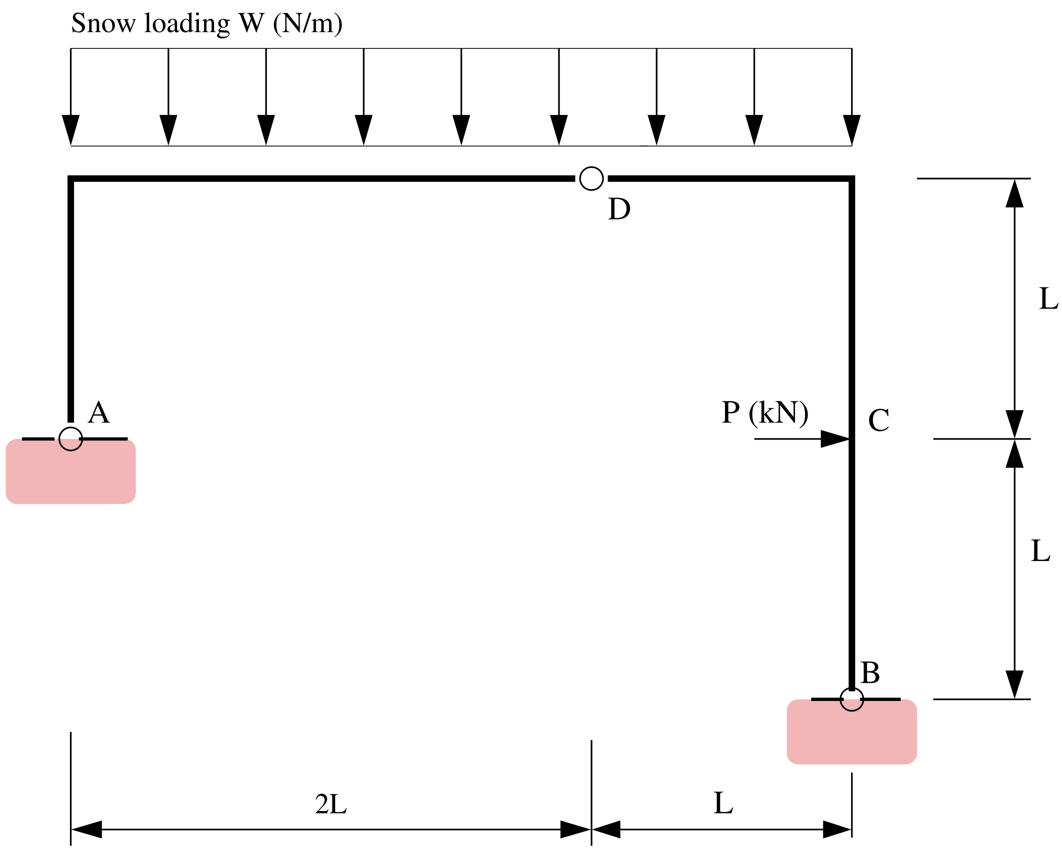

Figure 3: Front elevation view of a three-pinned arch that supports a snow loading.

[3a] (6 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of W, L and P.

Queation 3a continued:

[3b] (4 pts) Draw and label the bending moment diagram.