Department of Civil and Environmental Engineering, Spring Semester, 2020

ENCE 353 Midterm 2, Open Notes and Open Book

Name: _____

E-mail (print neatly!): _____

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please **show all of your working**.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 1: 15 points

Analysis of a Supported Cantilever Beam Structure. Figure 1 is a front elevation view of a cantilever beam carrying two external loads P. EI is constant along the cantilever beam.

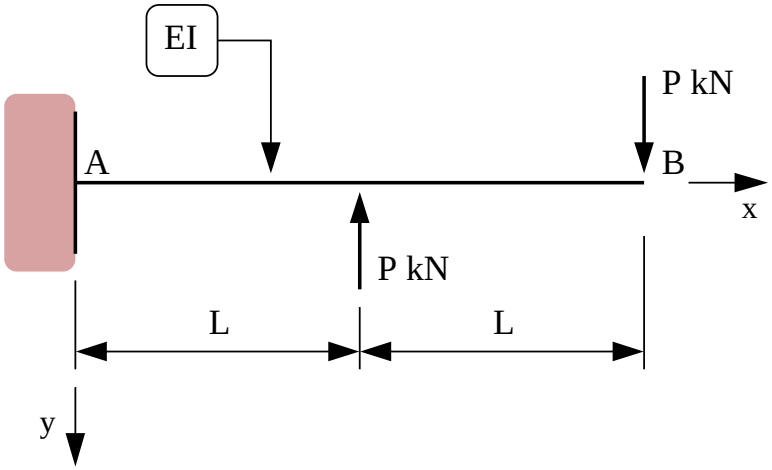

Figure 1: Cantilever beam carrying two applied loads P (kN).

[1a] (3 pts) Briefly explain how the **principle of superposition** can be applied to **this problem**.

[1b] (5 pts) Use the method of **moment-area** to show that the clockwise rotation of point B is:

$$\theta_B = \left[\frac{3}{2}\right] \frac{PL^2}{EI}. \tag{1}$$

[1c] (7 pts) Use the method of **<u>moment-area</u>** to show that the vertical displacement at B is:

$$y(2L) = \left[\frac{11}{6}\right] \frac{PL^3}{EI}. \tag{2}$$

Question 2: 15 points

Elastic Curve for Beam Deflections. Figure 2 is a front elevation view of a simply supported beam that carries a triangular load.

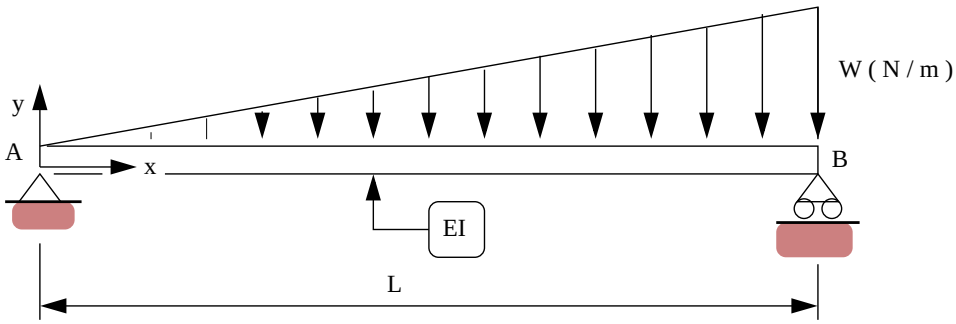

Figure 2: Simply supported beam carrying a triangular load.

The load increases from zero at point A to W (N/m) at point B. Thus, the total beam loading is WL/2.

[2a] (5 pts). Starting from first principles of engineering (i.e., equilibrium of a substructure extracted from Figure 2), show that the bending moment at point x is:

$$M(x) = \left[\frac{W}{6L}\right] x \left(L^2 - x^2\right). \tag{3}$$

[2b] (5 pts). Show that the elastic curve for beam deflection is given by (notice that in Figure 2, the y axis is pointing upwards):

$$y(x) = \left[\frac{-W}{6LEI}\right]\left[\frac{L^2x^3}{6} - \frac{x^5}{20} - \frac{14L^4x}{120}\right] \tag{4}$$

[2c] (5 pts). Show that the maximum beam curvature occurs at x = $L/\sqrt{3}$.

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch that carries a total snow loading of 3WL uniformly distributed over its upper section.

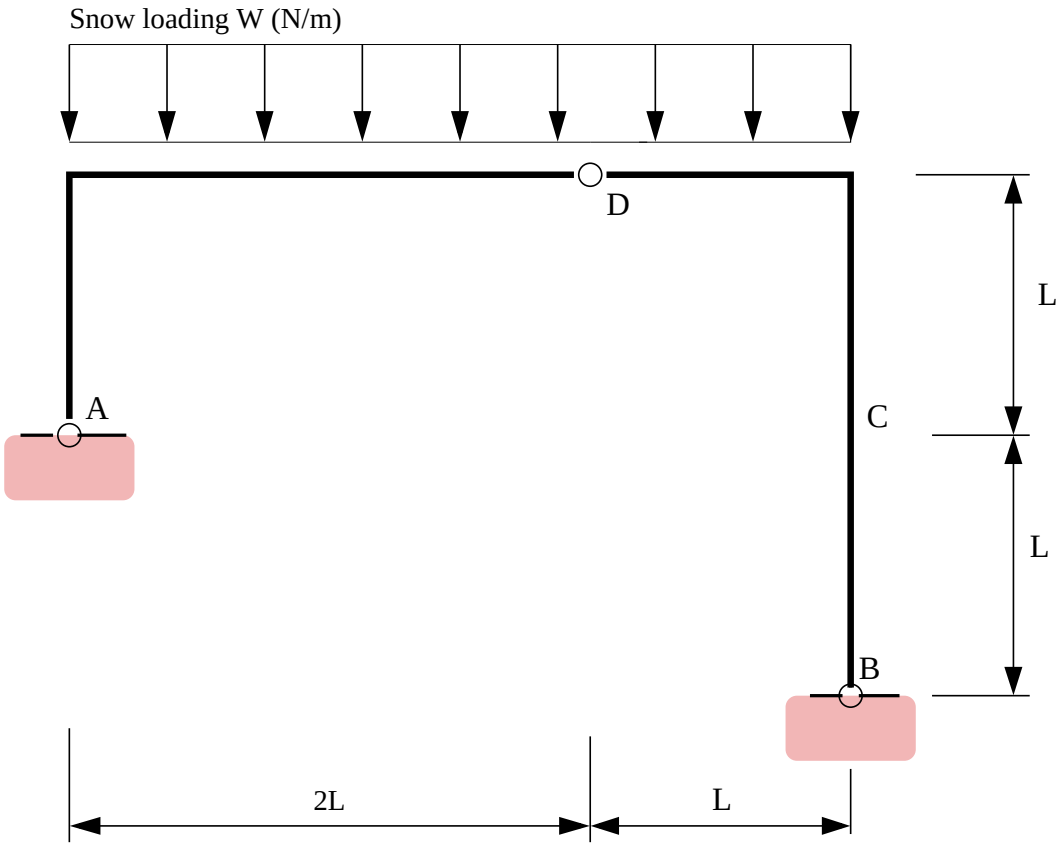

Figure 3: Front elevation view of a three-pinned arch that supports a snow loading.

[3a] (6 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of W and L.

Queation 3a continued:

[3b] (4 pts) Draw and label the bending moment diagram.