Department of Civil and Environmental Engineering, Fall Semester, 2018

ENCE 353 Midterm 2, Open Notes and Open Book

Name : _____

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please show all of your working.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 1: 15 points

Deriving Formulae for the Deflection of a Cantilever Beam. The cantilever beam structure shown in Figure 1 carries a uniform load w (N/m) along its entire length.

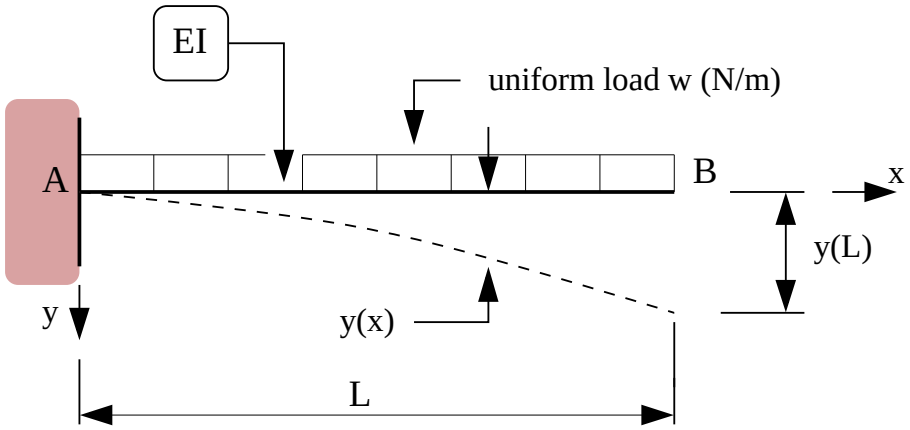

Figure 1: Front elevation view of a cantilever beam carrying a uniform load.

The beam is fully fixed at point A and the flexural stiffness EI is constant along the beam. The coordinate system is positioned at point A.

[1a] (6 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{1}$$

and appropriate boundary conditions, show that:

$$y(x) = \left(\frac{w}{24EI}\right)\left(6L^2x^2 - 4Lx^3 + x^4\right). \tag{2}$$

Question 1a continued:

[1b] (3 pts) Derive a formula for the slope of the beam as a function of x. This is a one line solution. Use your formula to verify that the beam rotation at B is:

$$\theta_B = \left(\frac{wL^3}{6EI}\right). \tag{3}$$

[1c] (6 pts) Using the results of question [1a] as a starting point, compute the support reactions at A and B for the propped cantilever shown in Figure 2

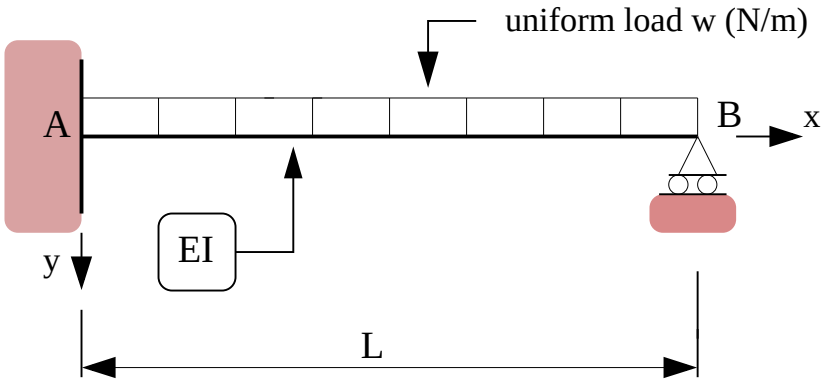

Figure 2: Propped cantilever beam carrying a uniform load.

Question 2: 15 points

Moment-Area and Rotational Deflections. The simple beam shown in Figure 3 has length L and uniform section properties EI. A point load P is applied at distance a from the left-hand support.

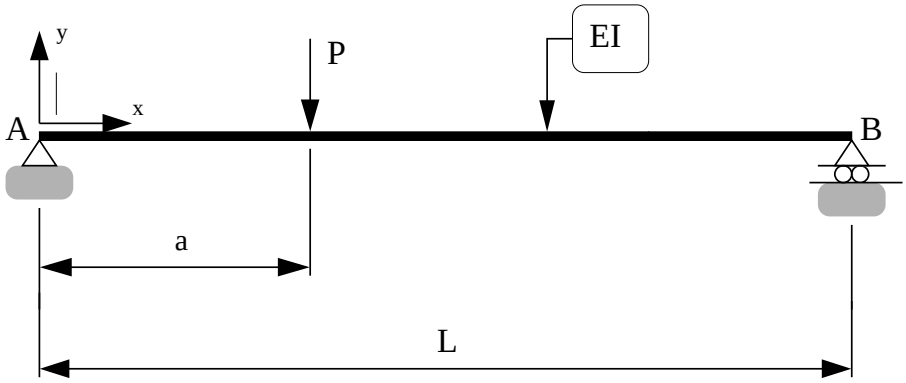

Figure 3: Front elevation view of a simple beam structure.

[2a] (3 pts) Draw and label the M(x)/EI diagram in terms of the problem parameters (i.e., P, EI, L and a).

[2b] (8 pts) Use the **method of moment-area** to show that the beam rotation at A is:

$$\theta_A = \left[\frac{P}{6EI}\right]\left[\frac{a\,(L-a)\,(2L-a)}{L}\right]. \tag{4}$$

[2c] (4 pts) Show that the **maximum value** of beam rotation at A occurs when:

$$a = L\left[1 - \frac{\sqrt{12}}{6}\right]. \tag{5}$$

Question 3: 10 points

Simple Three-Pinned Arch. Figure 4 is a front elevation view of a simple three-pinned arch. A vertical load P is applied at node D.

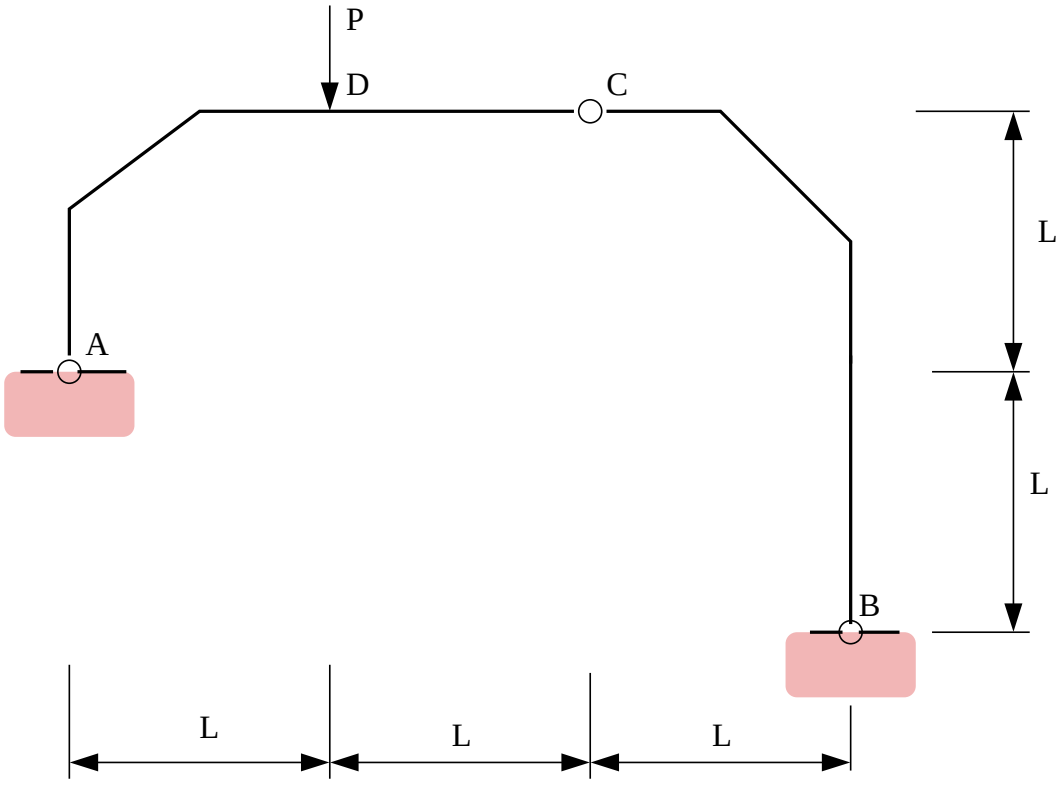

Figure 4: Front elevation view of a simple three-pinned arch.

[3a] (5 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of P.

[3b] (5 pts) Compute the magnitude and orientation of the **total reaction force vector** at support B. Show that it passes through the hinge at C. You can annotate Figure 4 if you think it will help to explain your solution.