Department of Civil and Environmental Engineering, Fall Semester, 2017

ENCE 353 Midterm 2, Open Notes and Open Book

Name : AUSTIN

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please show all of your working.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

fix!!

Question 1: 15 points

Deriving Formulae for the Deflection of a Cantilever Beam. Consider the cantilever beam structure shown in Figure 2.

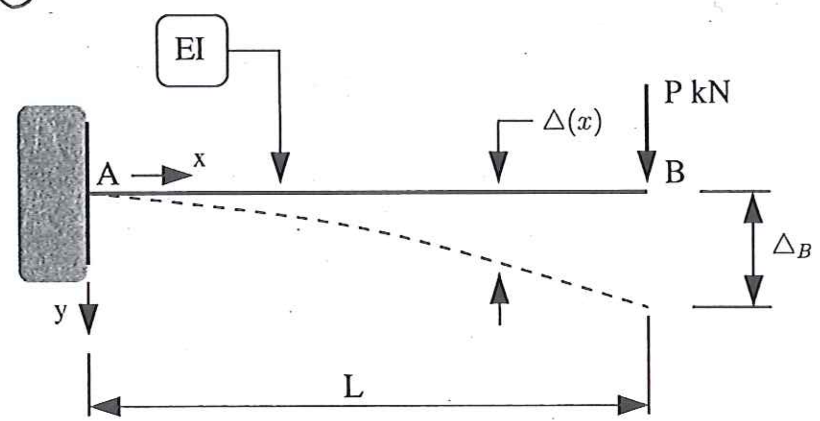


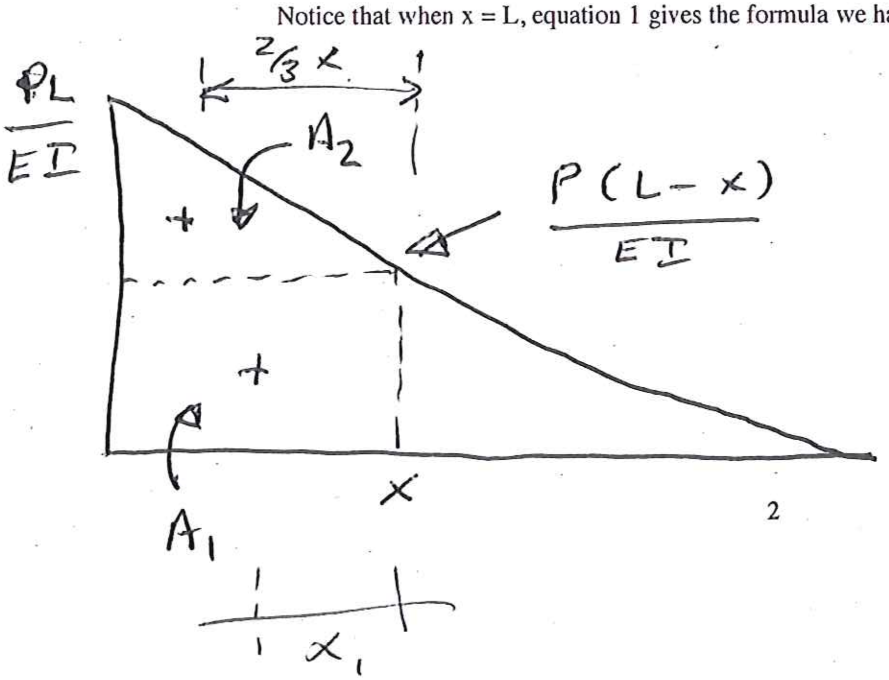
Figure 1: Front elevation view of a cantilever.

In this problem setup: (1) the beam is fully fixed at point A and the flexural stiffness EI is constant along the beam, (2) there is a vertical load P kN acting downwards at point B, and (1) the coordinate system is positioned at point A.

[1a] (6 pts) Use the method of <u>**moment area**</u> to show that:

$$\triangle(x) = \left(\frac{P}{6EI}\right) x^2 (3L - x). \tag{1}$$

Notice that when x = L, equation 1 gives the formula we have used in class.



$$A_1 = \frac{P}{EI}(L-x)\,x$$

$$A_2 = \frac{x}{2}\left(\frac{PL}{EI} - \frac{P(L-x)}{EI}\right)$$

$$= \frac{Px^2}{2EI}.$$

$$x_1 = x/2$$

$$x_2 = 2/3\,x.$$

Question 1a continued ....

$$A(x) = A_1 x_1 + A_2 x_2$$

$$= \frac{P}{EI}(L-x).\, x\left(\frac{x}{2}\right) + \frac{Px^2}{2EI}\left(\frac{2}{3}x\right)$$

$$= \frac{Px^2}{6EI}(3L-x).$$

[1b] (6 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{2}$$

and appropriate boundary conditions, show that:

elastic curve

$$\Delta(x) = \left(\frac{P}{6EI}\right) x^2 (3L - x). \tag{3}$$

$$EI \frac{d^2y}{dx^2} = PL - Px.$$

BC. $y(0) = 0$ & $\left.\frac{dy}{dx}\right|_{x=0} = 0.$

$$\Rightarrow \cancel{\frac{dy}{dx}}\, EI \frac{dy}{dx} = PLx - \frac{Px^2}{2} + A.$$

$$\Rightarrow EI\, y(x) = \frac{PLx^2}{2} - \frac{Px^3}{6} + Ax + B.$$

Apply BC $\Rightarrow A = B = 0$

$$\Rightarrow y(x) = \frac{Px^2}{6EI}(3L-x).$$

Notice: $\boxed{y(L) = \frac{PL^3}{3EI}}$

[1c] (3 pts) Derive a formula for the slope of the beam as a function of x. This is a one line solution. Use your formula to verify that the beam rotation at B is:

$$\theta_B = \left(\frac{PL^2}{2EI}\right). \tag{4}$$

$$EI \frac{dy}{dx} = EI\,\theta(x) = PLx - \frac{Px^2}{2}$$

$$\Rightarrow \boxed{EI\,\theta_B = PL^2 - \frac{PL^2}{2} = \frac{PL^2}{2}}$$

Note: $\tan(\theta(x)) = \dfrac{dy}{dx} \doteq \theta$

but. $\tan(\theta) = \theta + \dfrac{\theta^3}{3} + O(\theta^5)$

$\Rightarrow$ small $\theta \rightarrow \tan(\theta) \doteq \theta$.

Question 2: 15 points

Moment-Area and Deflections. Consider the cantilevered beam structure shown in Figure 2.

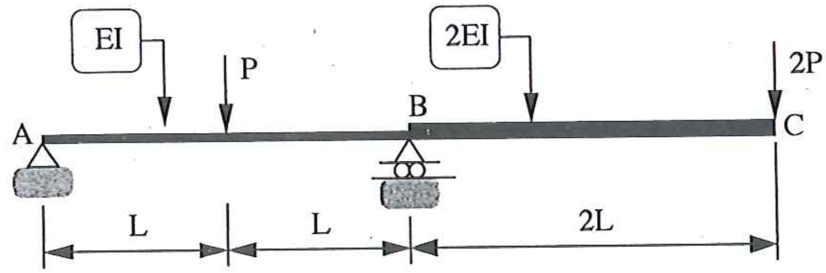
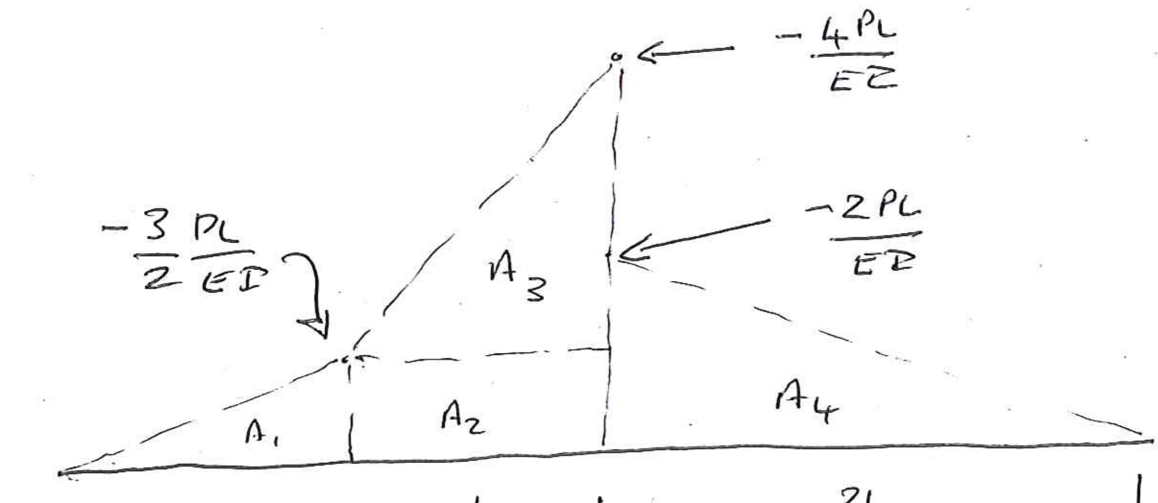


Figure 2: Front elevation view of a cantilevered beam structure.

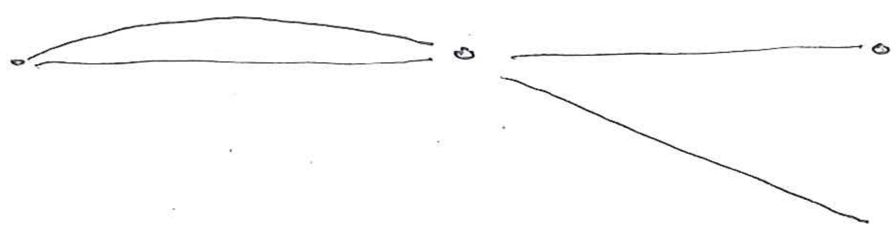
Notice that segments A-B and B-C have cross-sectional properties EI and 2EI, respectively.

[2a] (3 pts) Compute and draw the M(x)/EI diagram for the complete beam A-B-C.

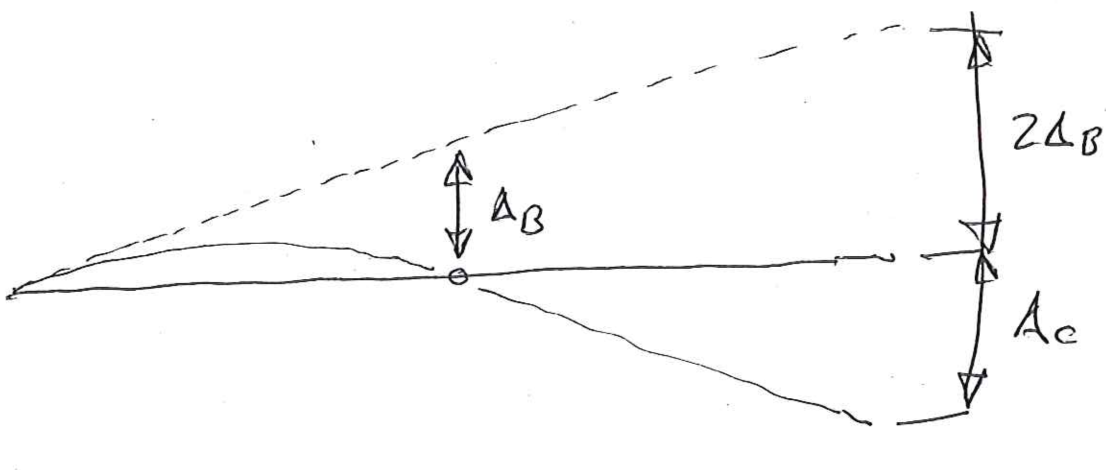
From equilibrium, $V_B = \frac{9}{2}P$, $V_A = -\frac{3}{2}P$.



[2b] (4 pts) Draw and label a diagram of the deflected shape. Clearly indicate on your diagram regions (or points) of the beam having zero curvature.



[2c] (4 pts) Draw and label a diagram showing how the rotation at A is related to the beam deflections at points B and C.



Aushn

[2d] (4 pts) Use the method of moment-area to compute the vertical deflection of the beam at point C.

$$A_1 = \left(\frac{3}{2} \cdot \frac{PL}{EI}\right) \cdot L \cdot \frac{1}{2} = \frac{3}{4}\frac{PL^2}{EI}$$

$$A_2 = \left(\frac{3}{2}\frac{PL}{EI}\right) \cdot L = \frac{3}{2}\frac{PL^2}{EI}$$

$$A_3 = \left(\frac{5}{2}\frac{PL}{EI}\right) . L \cdot \frac{1}{2} = \frac{5}{4} \cdot \frac{PL^2}{EI}$$

$$A_4 = \left(\frac{2PL}{EI}\right) . 2L \cdot \frac{1}{2} = \frac{2PL^2}{EI}.$$

For Δ_B calculation

$$\bar{x}_1 = \frac{4}{3}L, \quad \bar{x}_2 = \frac{L}{2}, \quad \bar{x}_3 = \frac{L}{3}$$

For Δ_c calculation

$$\bar{x}_1 = \frac{10}{3}L, \quad \bar{x}_2 = \frac{5}{2}L, \quad \bar{x}_3 = \frac{7}{3}L, \quad \bar{x}_4 = \frac{4}{3}L.$$

$$\Delta_B = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$$

$$= \frac{PL^3}{EI}\left[\frac{3}{4} \cdot \frac{4}{3} + \frac{3}{2} \cdot \frac{1}{2} + \frac{5}{4} \cdot \frac{1}{3}\right] = \frac{26}{12}\frac{PL^3}{EI}$$

$$2\Delta_B + \Delta_c = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 + A_4\bar{x}_4$$

$$= \frac{PL^3}{EI}\left[\frac{3}{4} \cdot \frac{10}{3} + \frac{3}{2} \cdot \frac{5}{2} + \frac{5}{4} \cdot \frac{7}{3} + 2 \cdot \frac{4}{3}\right] = \frac{142}{12}\frac{PL^3}{EI}$$

$$\Rightarrow \Delta_c = \left(\frac{142-52}{12}\right)\frac{PL^3}{EI} = \frac{15}{2}\frac{PL^3}{EI}.$$

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch. A vertical load P is applied at node D.

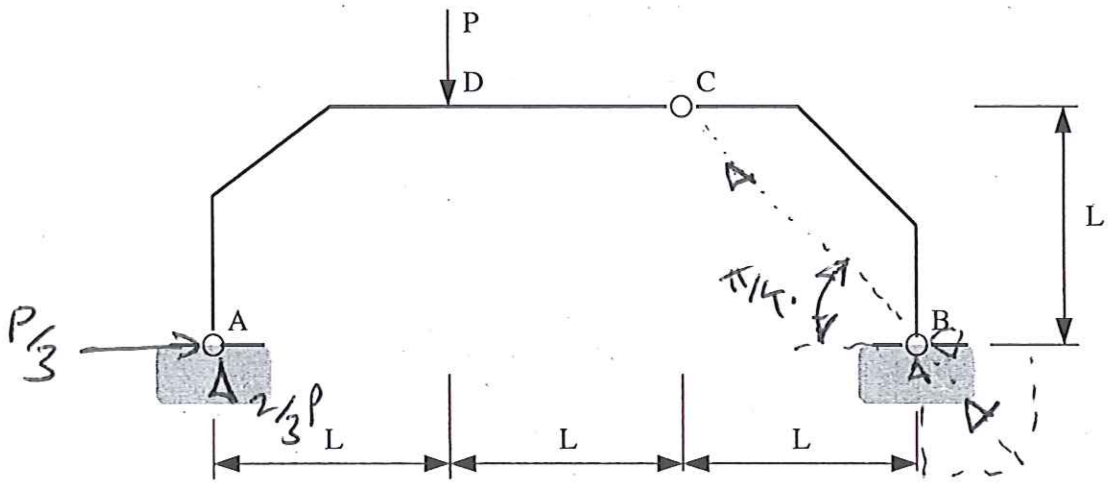


Figure 3: Front elevation view of a simple three-pinned arch.

[3a] (5 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of L and P.

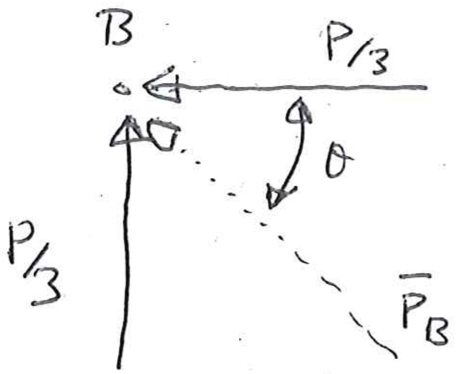
$\sum M_A = 0 \Rightarrow PL = V_B 3L \Rightarrow V_B = P/3$

$\sum V = 0 \Rightarrow V_A + V_B = P \Rightarrow V_A = \frac{2}{3}P.$

$\sum' M_C = 0 \Rightarrow V_B \cdot L + H_B L = 0 \Rightarrow H_B = -\frac{P}{3}$

$\sum' H = 0 \Rightarrow H_A + H_B = 0 \Rightarrow H_A = P/3.$

[3b] (3 pts) Compute the magnitude and orientation of the **total reaction force vector** at support B. Show that it passes through the hinge at C. You can annotate Figure 3 if you think it will help to explain your solution.



Magnitude $P_B = \frac{\sqrt{2}}{3} P$

$\tan(\theta) = \left(\frac{P}{3} / \frac{P}{3}\right) = 1 \Rightarrow \theta = \pi/4$.

[3c] (2 pts) Suppose that your calculations indicated that the "total reaction force at support B" **did not** pass through the hinge at C. What would that mean?

- Structure not in equilibrium.
- Calculations wrong...