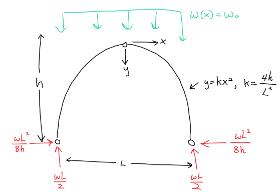
10/07/2019

Monday, October 7, 2019 8:51 AM

Last time:



The horizontal reaction increases when the arch become shallow

Can show M(x) = 0 => V(x) = 0

- Loads are transferred to foundation by axial compression alone
- Position of hinge doesn't matter!

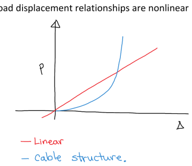
Cable structures

Strategy - flip arch upside down, and explore mechanism for carrying load in tension!

Applications: suspension bridges, gondolas, hot air balloons, tug boats and log rafts.

Key ideas

1. Cable is flexible. Can only carry loads in tension
   - No bending and no shear forces.
2. Cable needs to be in equilibrium.
   - It will change its shape to achieve this objective.
3. Load displacement relationships are nonlinear

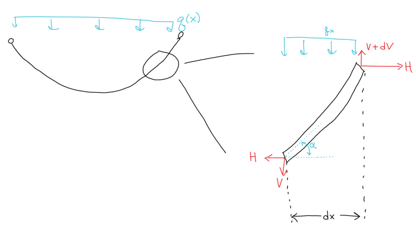


→ Linear function

$- f(x+y) = f(x) + f(y)$

$- f(kx) = k f(x)$

Equation of Equilibrium



$\Sigma F_y = 0$

$V + dV - V - q_x dx = 0$

$\Rightarrow dV - q_x dx = 0$

$$\boxed{\frac{dV}{dx} = q_x} \text{ — A}$$

Need to relate forces to cable geometry:

- Cable can only carry loads in tension.

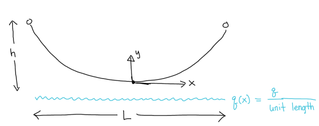
  => $\tan(\alpha) = \frac{V}{H} = \frac{dy}{dx}$ => $\boxed{V = H\frac{dy}{dx}}$ — B

Combine A and B

$\frac{dV}{dx} = H\frac{d^2y}{dx^2} = q_x$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{q_x}{H}} \text{ — C}$$

Example



Sag/span ratio $= \frac{f}{L}$

$\frac{d^2y}{dx^2} = \frac{q}{H} \xrightarrow{\text{integrate}} y(x) = \frac{qx^2}{2H} + C_1 x + C_2$

Apply boundary conditions

$y(0) = 0 \Rightarrow C_2 = 0$

$\frac{dy}{dx} = 0$ @ $x = 0 \Rightarrow C_1 = 0$

$\Rightarrow y(x) = \frac{qx^2}{2H}$

But!!

$y\left(\frac{L}{2}\right) = h$

$\Rightarrow h = \frac{qL^2}{8H}$

Therefore

$$\boxed{H = \frac{qL^2}{8h}} \text{ — D}$$

↳ verticle force

Notice: D is equal to equal and opposite to the horizontal reaction force of for the arch!