Department of Civil and Environmental Engineering,   Fall Semester, 2017

ENCE 353 Final Exam, Open Notes and Open Book

Name: Austin.

Exam Format and Grading. The exam will be 2 hrs plus five minutes to read the questions.

Answer question 1. Then answer **three of the four** remaining questions.

Only the first four questions that you answer will be graded, so please cross out the question you do not want graded in the table below. Partial credit will be given for partially correct answers, so please show all your working.

After you have finished working on the exam, look at the bonus problem for additional credit. No partial credit for this part of the exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Bonus	4	
Total	40	

Question 1: 10 points

Mtdhc of Virtl Forces.

COMPULSORY: Moment-Area, Virtual Work. Figure 1 is a front elevation view of a cantilever beam carrying two external loads P. EI is constant along the cantilever beam.
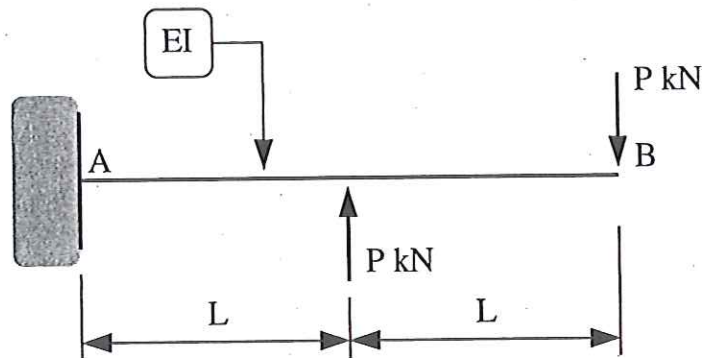

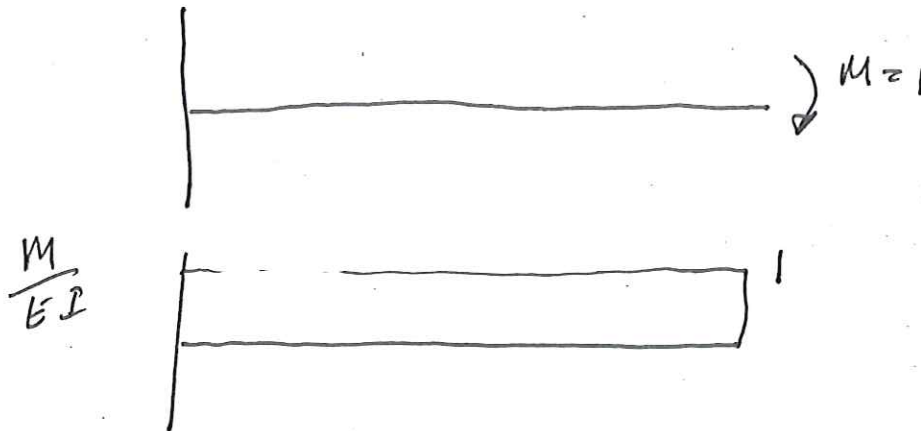Figure 1: Cantilever beam carrying two applied loads P (kN).

[1a] (5 pts) Use the method of VIRTUAL FORCES to compute the vertical displacement at B.

$M(x) = PL.$

M/EI

$PL.$

$M(x) = Px.$

$M(x) = x$

$\Delta_B = \int_0^{2L} \frac{M(x) m(x)}{EI} dx = \int_0^L \frac{P(x) \cdot x}{EI} dx + \int_L^{2L} \frac{PL \cdot x}{EI} dx$

$$= \frac{11}{6} \frac{PL^3}{EI}$$

2

[1b] (5 pts) Use the method of **VIRTUAL FORCES** to to compute clockwise rotation of the beam at Point B. .

$$M = 1$$

$$\frac{M}{EI}$$

$$\theta_B = \int_0^{2L} \frac{M(x) \, M(x)}{EI} \, dx = \int_0^{L} \frac{1 \cdot Px}{EI} \, dx + \int_L^{2L} \frac{1 \cdot PL}{EI} \, dx$$

$$\boxed{\theta_B = \frac{3 \, PL^2}{2 \, EI}}$$

3

Question 2: 10 points

OPTIONAL: Structural Analysis of a T-shaped Beam Structure. Consider the t-shaped beam structure shown in Figure 2.
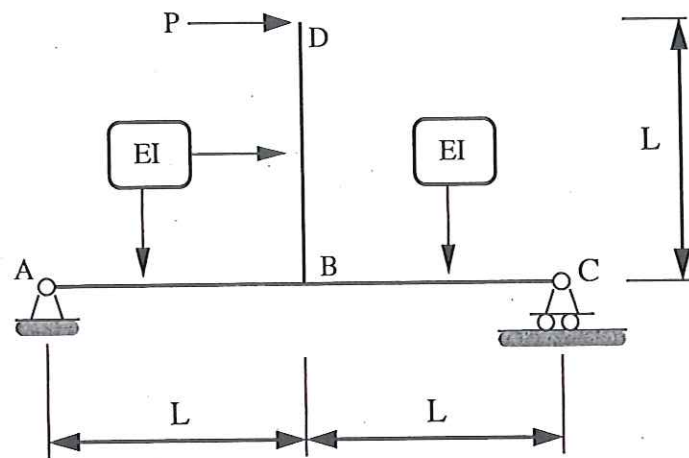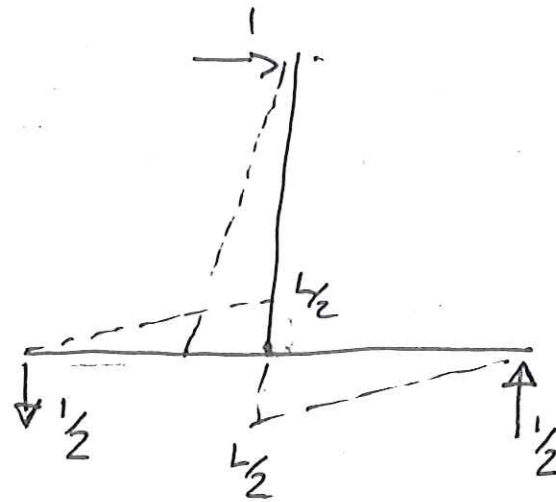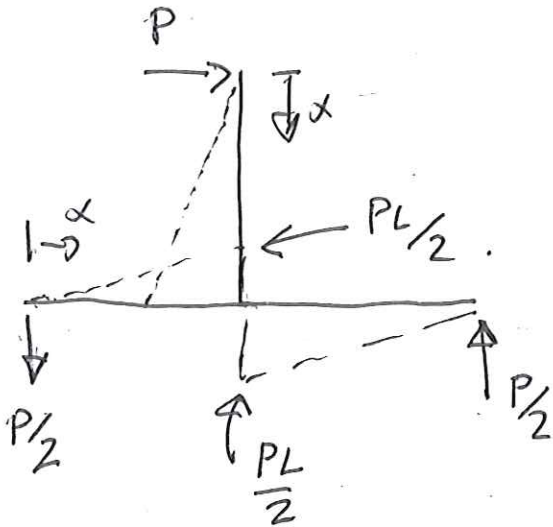

Figure 2: Front elevation view of a simple T-shaped beam structure.

[2a] (3 pts) Use the method of **VIRTUAL DISPLACEMENTS** to compute the virtical reaction at C.

$$\sum EWD = 0$$

$$\Rightarrow V_C \, \Delta_1^{**} + \Delta_D^{**}(-P) = 0$$

from geometry

$$\phi = \frac{\Delta_1^{**}}{2L} \qquad \Delta_D^{**} = \phi \cdot L$$

$$\Rightarrow \Delta_D^{**} = \frac{\Delta_1^{**}}{2}$$

$$\Rightarrow \boxed{V_C = \frac{P}{2}}$$

4

[2b] (7 pts) Use the method of **VIRTUAL FORCES** to compute a formula for the horizontal displacement at B.

P

$\downarrow x$

$1 \to \alpha$

$\leftarrow PL/2$

$P/2$

$\uparrow \frac{PL}{2}$

$P/2$

$1 \to$

$L/2$

$\downarrow \frac{1}{2}$

$L/2$

$\uparrow \frac{1}{2}$

$M_c(x) = Px$

$\&\ M_b(x) = \frac{P}{2}x$

$M_c(x) = x$

$M_b(x) = \frac{x}{2}$

$$\Delta_D = \int_0^L \frac{M_c(x)\, M_c(x)\, dx}{EI} + 2\int_0^L \frac{M_b(x)\, M_b(x)\, dx}{EI}$$

$$= \frac{P}{EI}\int_0^L x^2\, dx + 2\frac{P}{EI}\cdot\frac{1}{4}\int_0^L x^2\, dx.$$

$$= \frac{P}{EI}\left[\frac{1}{3}x^3\right]_0^L + \frac{P}{2EI}\left[\frac{1}{3}x^3\right]_0^L$$

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$$\boxed{\Delta_D = \frac{PL^3}{2EI}.}$$

Question 3: 10 points

OPTIONAL: Analysis of a Supported Cantilever Beam Structure. Consider the supported cantilever beam structure shown in Figure 3.
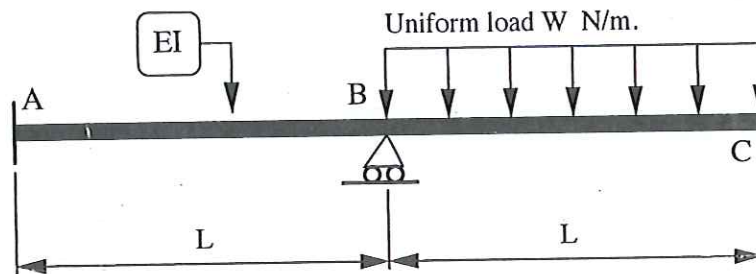

Figure 3: Front elevation view of a supported cantilever beam structure.

The cantilever is fully fixed (no rotation) at support A and is restrained against vertical displacements at B. It carries a uniform load W (N/m) along the segment length B-C.

[3a] (5 pts) Use the methods of **MOMENT AREA** and **COMPATIBILITY OF DISPLACEMENTS** to show that the support reaction at B is:

$$V_b = \frac{7}{4}WL. \tag{1}$$

$$A_1 = A_2 = \frac{WL^3}{2}.$$
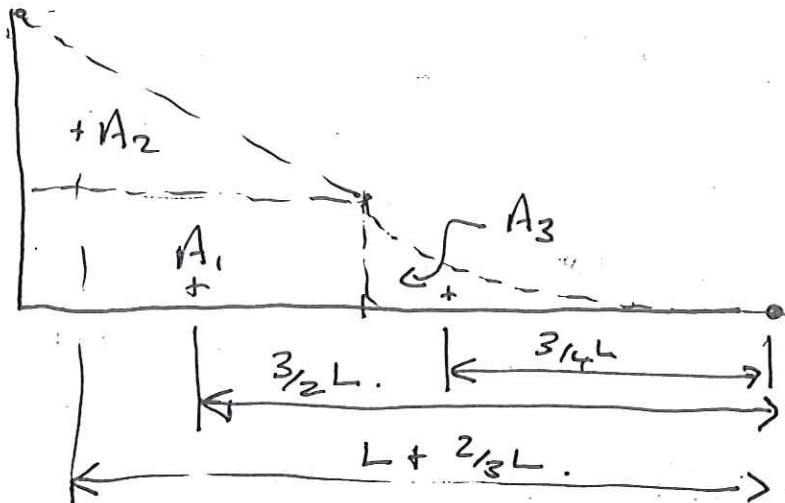
$$A_3 = \frac{A_1}{3} = \frac{WL^3}{6}$$

$$\Delta_B = A_1 \bar{x}_1 + A_2 \bar{x}_2 = \frac{WL^3}{2EI}\left[\frac{L}{2} + \frac{2}{3}L\right] = \frac{7}{12}\frac{WL^4}{EI.}$$
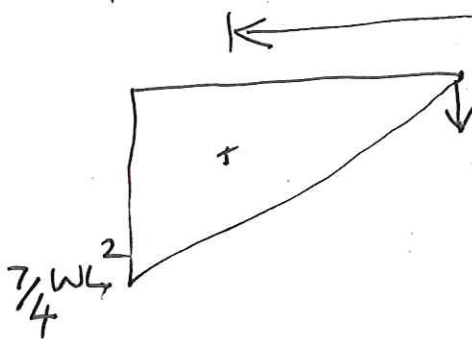
$$\frac{V_B L^3}{3EI} = \Delta_B = \frac{7}{12}\frac{WL^4}{EI} \Rightarrow \boxed{V_B = \frac{7}{4}WL}$$

[3b] (5 pts) Use the method of **MOMENT AREA** to compute the vertical displacement at C.

A_C, no support at B.

$$\Delta_{C_1} = A_1 \bar{\alpha_1} + A_2 \bar{x_2} + A_3 \bar{\alpha_3} = \frac{41 wL^4}{24 EI}$$

$$\Delta_{C_2} = \left(-\frac{7}{4} wL^2\right)\left(\frac{L}{2}\right)\left(L + \frac{2}{3}L\right)$$

$$= -\frac{35}{24} \frac{wL^4}{EI}$$

Net displacement.

$$\Delta_C = \Delta_{C_1} + \Delta_{C_2} = \frac{41 wL^4}{24 EI} - \frac{35 wL^4}{24 EI}$$

$$\boxed{\Delta_C = \frac{7 wL^4}{4 EI} \downarrow}$$

Question 4: 10 points

OPTIONAL: Principle of Virtual Work. Figure 4 is a front elevation view of a simple truss that supports a horizontal load P at node C. All three truss members have cross section properties AE.
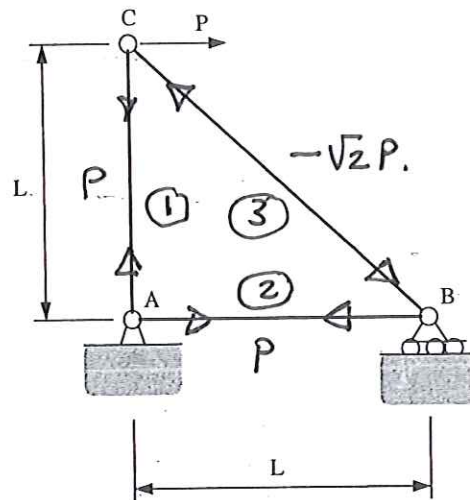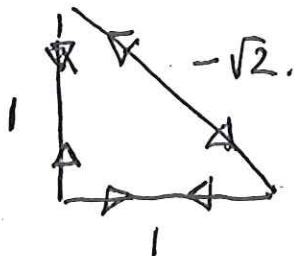

Figure 4: Front elevation view of a simple truss.

[4a] (10 pts). Use the method of **VIRTUAL FORCES** to show that the horizontal deflection at node C is:

$$\triangle = \frac{2PL}{AE}\left[1 + \sqrt{2}\right]. \tag{2}$$

$$\triangle = \sum_{1}^{3} \frac{f_i \, F_i \, L_i}{EA} = \frac{1}{AE}\left[\frac{-\sqrt{2}\,P \cdot -\sqrt{2} \cdot \sqrt{2}L}{\textcircled{3}} + \frac{P \cdot 1 \cdot L}{\textcircled{1}} + \frac{P \cdot 1 \cdot L}{\textcircled{2}}\right]$$

$$= \frac{2PL}{AE}\left[\sqrt{2} + 1\right].$$

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OPTIONAL: Deriving equations of beam deflection. Figure 5 shows a beam structure that carries a uniform load W (N/m) along its length.
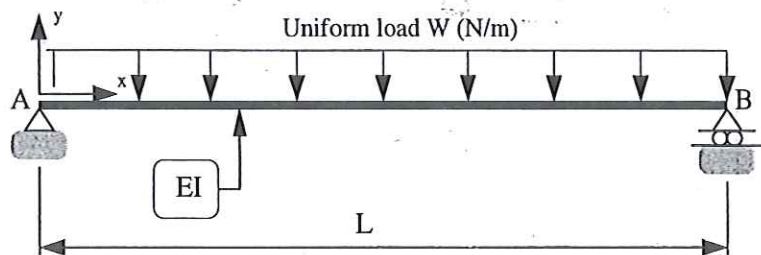

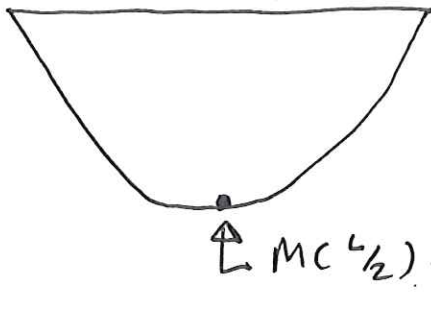
Figure 5: Front elevation view of a simple beam.

[5a] (10 pts). Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{-M(x)}{EI}\right], \qquad (3)$$

(notice the minus sign on M(x)) and appropriate boundary conditions, show that that vertical beam deflection is given by:

$$y(x) = -\frac{Wx}{24EI}\left(x^3 - 2Lx^2 + L^3\right). \qquad (4)$$

BMD

Let $M(x) = A \times (L-x)$

$M(^L/_2) = \frac{\omega L^2}{8} \implies A = \frac{\omega}{2}$.

$\uparrow M(^L/_2) = \frac{\omega L^2}{8}$

$\implies EI \frac{d^2y}{dx^2} = \frac{-\omega}{2}(x)(L-x)$.

$\implies EI \frac{dy}{dx} = \frac{-\omega}{2}\int_0^L Lx\,dx + \frac{\omega}{2}\int_0^L x^2\,dx + A$

$\implies EI y(x) = \frac{-\omega L}{2}\cdot\frac{1}{2}x^2 + \frac{\omega}{2\cdot3}x^3 + A$

$= \frac{-\omega L}{2}\frac{1}{2}x^2 + \frac{\omega}{2}\frac{1}{3}x^3 + A$.

$$\Rightarrow EI \; y(x) = -\frac{WL}{2} \cdot \frac{1}{3} \cdot x^3 + \frac{W}{2} \cdot \frac{1}{12} x^4 + Ax + B,$$

BC. $y(0) = 0 \quad \Rightarrow B = 0$

$\qquad y(L) = 0 \quad \Rightarrow EI \; y(L) = -\frac{WL^4}{12} + \frac{WL^4}{24} + AL = 0$

$$\Rightarrow A = \frac{WL^3}{24}.$$

$$\Rightarrow EI \; y(x) = -\frac{WL x^3}{12} + \frac{W x^4}{24} + \frac{WL^3 x}{24}$$

$$y(x) = -\frac{Wx}{24EI} \left[x^3 - 2Lx^2 + L^3 \right] \quad \cancel{\;}$$

BONUS PROBLEM: 4 points

As I mentioned in class, the problem below comes from the recent Grade 12 calculus exam in New Zealand. It has nothing to do with structural analysis, but it's fun, and it has a short and elegant solution.

Problem: Find **all integer** solutions to the problem:

$$x^4 - y^2 = 71. \qquad (5)$$

Justify your solution with the appropriate equations and rationale.

$$(x^4 - y^2) = (x^2 + y)(x^2 - y) = 71 = 1 \times 71 \text{ or } 71 \times 1$$

$$71 \text{ prime}$$

Case 71×1.

$$x^2 + y = 71$$
$$x^2 - y = 1$$

$$\Rightarrow x = \pm 6, \ y = 35$$

$$\underline{1 \times 71}$$
$$x^2 - y = 71$$
$$x^2 + y = 1$$

$$x = \pm 6, \ y = -35.$$