Department of Civil and Environmental Engineering, Fall Semester, 2016

ENCE 353 Final Exam, Open Notes and Open Book

Name: Austin

Exam Format and Grading. The exam will be 2 hrs plus five minutes to read the questions.

Answer question 1. Then answer **three of the four** remaining questions.

Only the first four questions that you answer will be graded, so please cross out the question you do not want graded in the table below.

Partial credit will be given for partially correct answers, so please show all your working.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	40	

Question 1: 10 points

COMPULSORY: Deriving Formulae for the Deflection of a Cantilever Beam. Consider the cantilever beam structure shown in Figure 1.

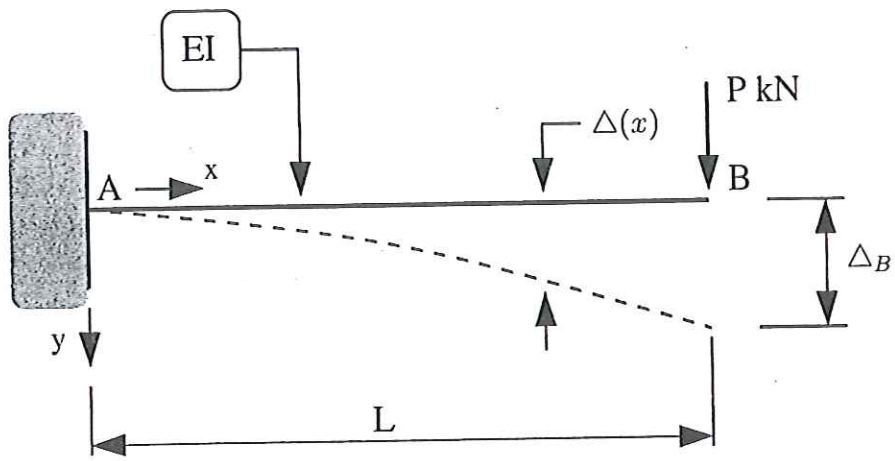


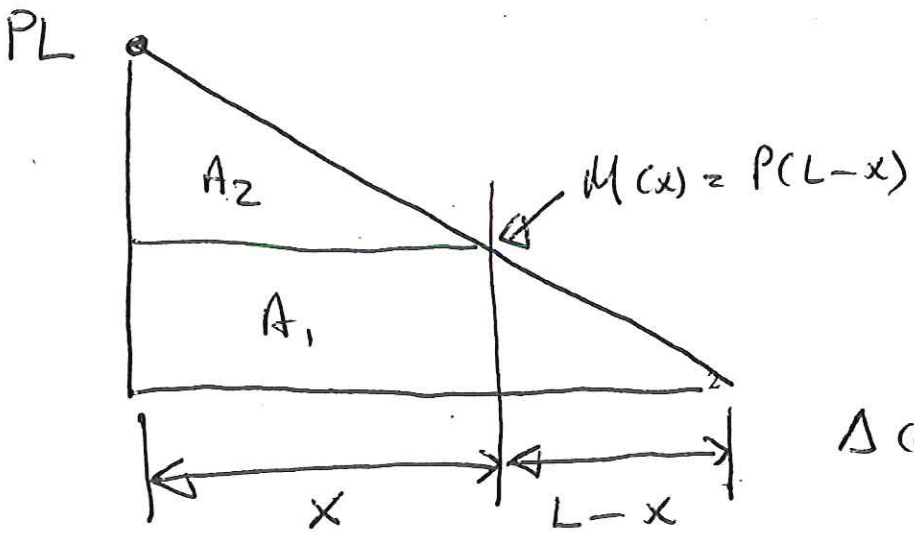
Figure 1: Front elevation view of a cantilever.

In this problem setup: (1) the beam is fully fixed at point A and the flexural stiffness EI is constant along the beam, (2) there is a vertical load P kN acting downwards at point B, and (1) the coordinate system is positioned at point A.

[1a] (3 pts) Use the method of **moment area** to show that:

$$\Delta(x) = \left(\frac{P}{6EI}\right) x^2 \left(3L - x\right). \tag{1}$$

Notice that when x = L, equation 1 gives the formula we have used in class.



$A_1 = P(L-x)x$

$A_2 = \frac{Px^2}{2}$

$\bar{x}_1 = x/2$

$\bar{x}_2 = 2/3\, x$

$\Delta(x) = A_1 \bar{x}_1 + A_2 \bar{x}_2$

$= \frac{Px^2}{6EI}(3L - x).$

[1b] (4 pts) Use the method of **virtual forces** to show that:

$$\Delta(x) = \left(\frac{P}{6EI}\right) x^2 (3L - x). \tag{2}$$

Hint: This part is a bit tricky. Remember that x is the boundary of the integration, and not the variable over which the integration needs to occur. Otherwise, the math is straight forward.

$$\Delta(x) = \int_0^{x} \frac{M(y)\, m(y)}{EI}\, dy.$$

$M(y) = P(L-y)$

$m(y) = (x-y) \quad y < x$

$= 0 \quad y \geq x$

$$= \int_0^{x} \frac{P(L-y)}{EI} \cdot (x-y)\, dy + \int_x^{L} \frac{P(L-y)}{EI} \cdot 0\, dy$$

$$= \frac{Px^2}{6EI}\left[3L - x\right]$$

Using equation (1) as a starting point,

[1c] (3 pts) Derive a formula for the slope of the beam as a function of x. This is a one line solution. Use your formula to verify that the beam rotation at B is:

$$\theta_B = \left(\frac{PL^2}{2EI}\right). \tag{3}$$

$$\theta_x = \frac{d\Delta}{dx} = \frac{P}{6EI} \frac{d}{dx}\left[x^2(3L - x)\right]$$

$$= \frac{P}{2EI}\left[2Lx - x^2\right]$$

$$\theta_x(L) = \frac{PL^2}{2EI} \star$$

Question 2: 10 points

OPTIONAL: Member forces in a Propped Cantilever. Figure 2 is an elevation view of a propped cantilever structure that carries external point loads P at points B and C. The cantilever support is fully fixed at point A.

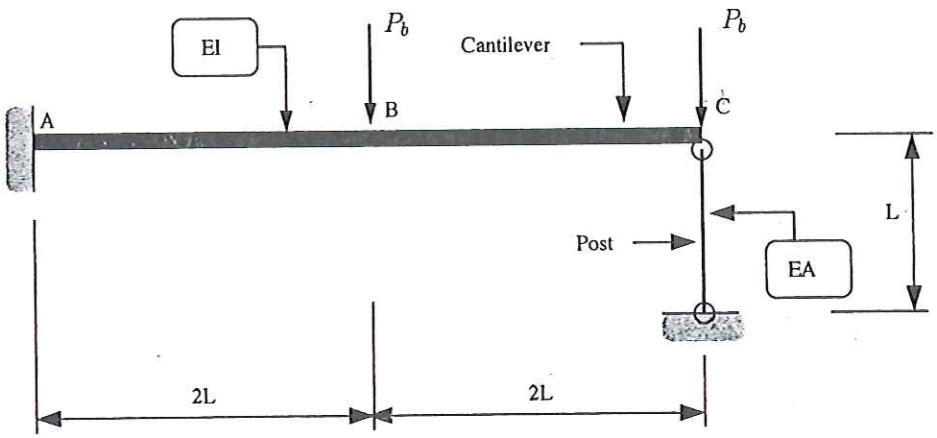


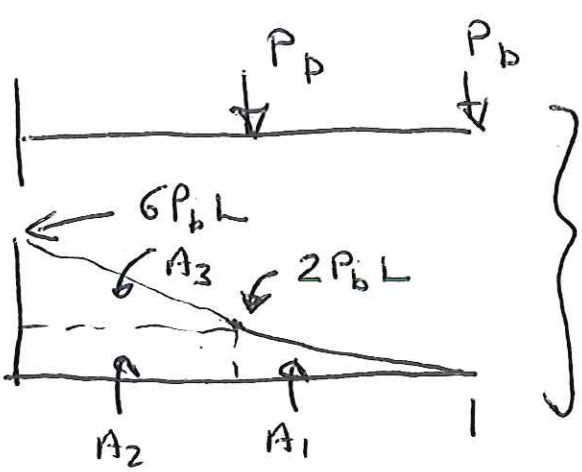
Figure 2: Elevation view of a propped cantilevel beam (not to scale).

The structural system has constant section properties EI along the beam, and is supported by a post element having section properties EA.

[2a] (1 pt) Compute the degree of indeterminacy for the propped cantilever beam.

[2b] (5 pt) Using the method of moment-area, or otherwise, show that the compressive force in the post element, P_p, is related to the externally applied loads P_b by the equation:

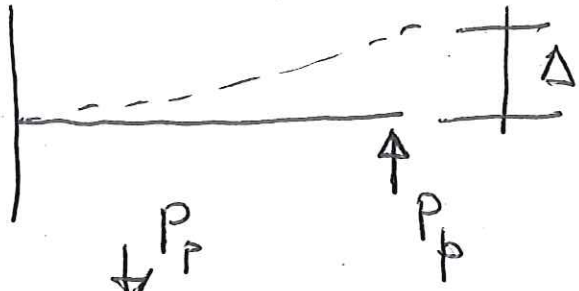
$$\frac{28P_bL^3}{EI} = P_p\left[\frac{64L^3}{3EI} + \frac{L}{EA}\right]. \tag{4}$$



$$\Delta_B = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$$

$$= \frac{28P_bL^3}{EI}$$

Question 2b continued ...



$$\Delta_c = \frac{P_p (4L)^3}{3EI} = \frac{64 P_p L^3}{3EI}$$

$$\Delta_{bar} = \frac{P_p L}{EA}.$$

Displacement Compatibility

$$\Delta_{bar} + \Delta_{c_1} = \Delta_{c_2}$$

$$\Rightarrow \frac{P_b L}{AE} + \frac{64 P_p L^3}{3EI} = \frac{28 P_b L^3}{EI}$$

[2c] (4 pt) Explain how the value of bending moment at the cantilever support (i.e., at point A) will change as a function of the problem parameters (i.e., P_b, P_p, L, E, I and A).

$$M_A = 2P_b L + 4P_b L - P_p 4L$$

$$= 6P_b L - 4P_p L$$

$$= (6P_b - 4P_b) L.$$

If A $\to \infty$, bar will support most of P_b.

~~[illegible]~~ etc $\frac{P_p L}{AE} \to 0 \Rightarrow \frac{64}{3} P_p = 28 P_b$

$$\Rightarrow \boxed{P_p = \frac{84}{64} P_b}$$

Question 3: 10 points

OPTIONAL: Bending Moment Diagram for a Propped Cantilever Beam. This problem involves computation of the bending moment diagram for the propped cantilever shown in Figure 3.

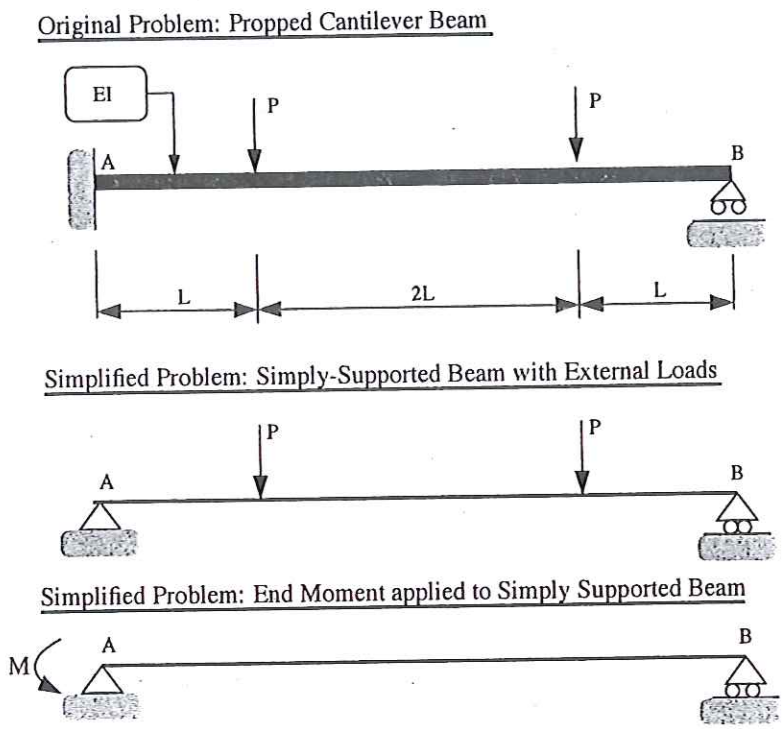


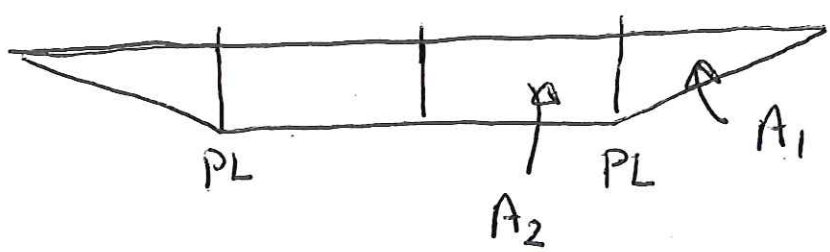
Figure 3: Elevation view of a propped cantilevel beam.

The cantilever is fully-fixed at point A and is propped up at point B. As such, it is statically indeterminate to degree 1. The middle and bottom figures show the same beam with: (1) the rotational degree of freedom released at the wall support, and (2) the external loads removed, and replaced by a moment M applied to the left-hand support.

[3a] (2 pt) Compute the rotation at A due to the applied loads P.

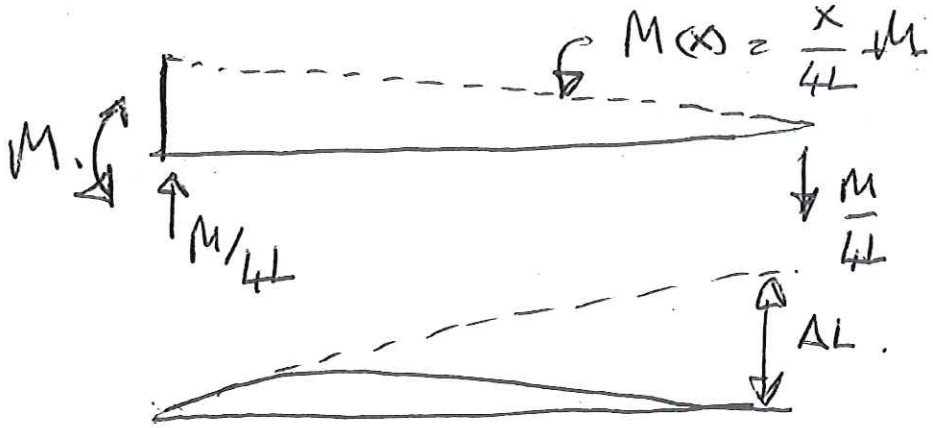
$$A_1 = \frac{1}{2}\frac{PL^2}{EI} \quad x_1 =$$

$$A_2 = \frac{PL^2}{EI}$$



$$\theta_A = A_1 + A_2 = \frac{3}{2}\frac{PL^2}{EI} \Rightarrow \theta_{A_1} = \frac{3}{2}\frac{PL^2}{EI}.$$

[3b] (3 pt) Compute the rotation at A due to the applied moment M.



$$\theta_M = \frac{\Delta L}{4L}$$

$$\Delta L = \frac{1}{2} M \cdot 4L \cdot \frac{2}{3} 4L$$

$$= \frac{16 M L^2}{3EI}$$

$$\theta_A = \frac{4ML}{3EI}.$$

[3c] (3 pt) Use compatbility of (rotational) displacements to determine the bending moment at the wall support that will result in zero rotations at the wall support.

$$\theta_{A1} - \theta_{A2} = 0 \Rightarrow \frac{3}{2}\frac{PL^2}{EI} = \frac{4ML}{3EI}$$

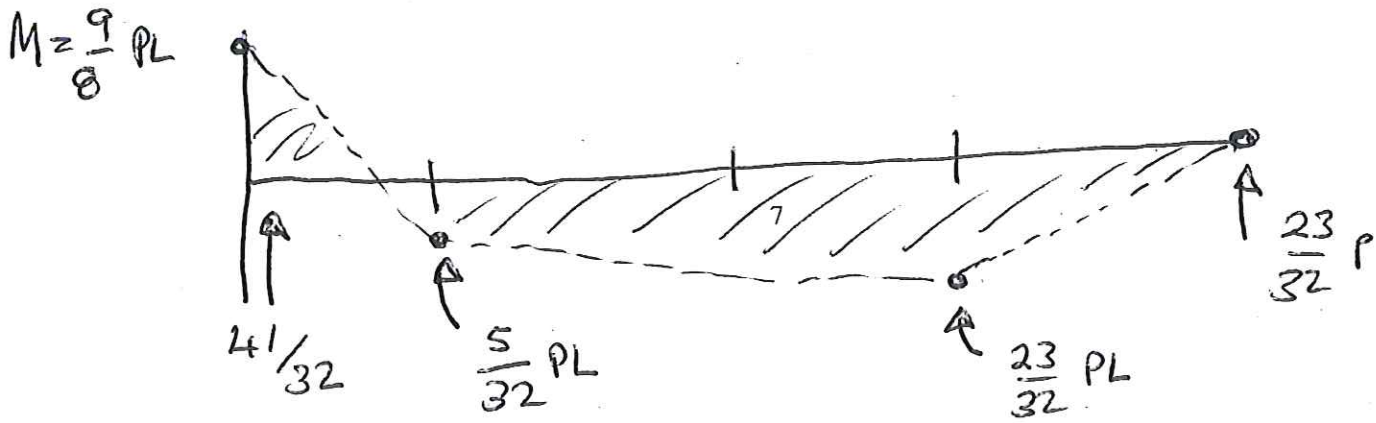
$$\Rightarrow M = \frac{9}{8} PL.$$

$$\text{Reaction Force} = \frac{M}{4L} = \frac{9}{32} P$$

[3d] (2 pt) Draw and label the bending moment diagram for the propped cantilever.

$$\text{Reaction at } A = P + \frac{9}{32} P = \frac{41}{32} P$$

$$\text{BMD} \quad \text{" at } B = P - \frac{9}{32} P = \frac{23}{32} P.$$



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Question 4: 10 points

OPTIONAL: Reaction Forces in a Three-Pinned Arch Structure. Consider the three-pinned arch structure shown in Figure 4.

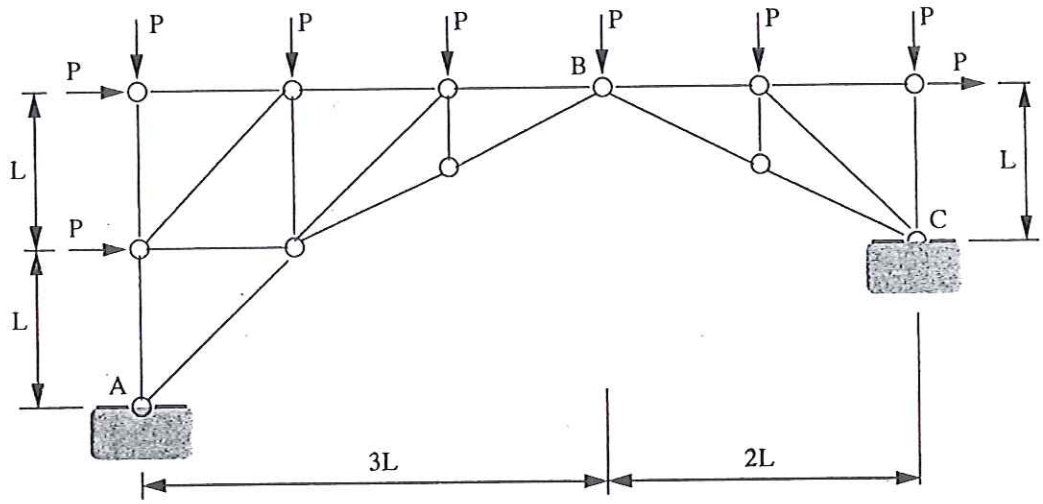
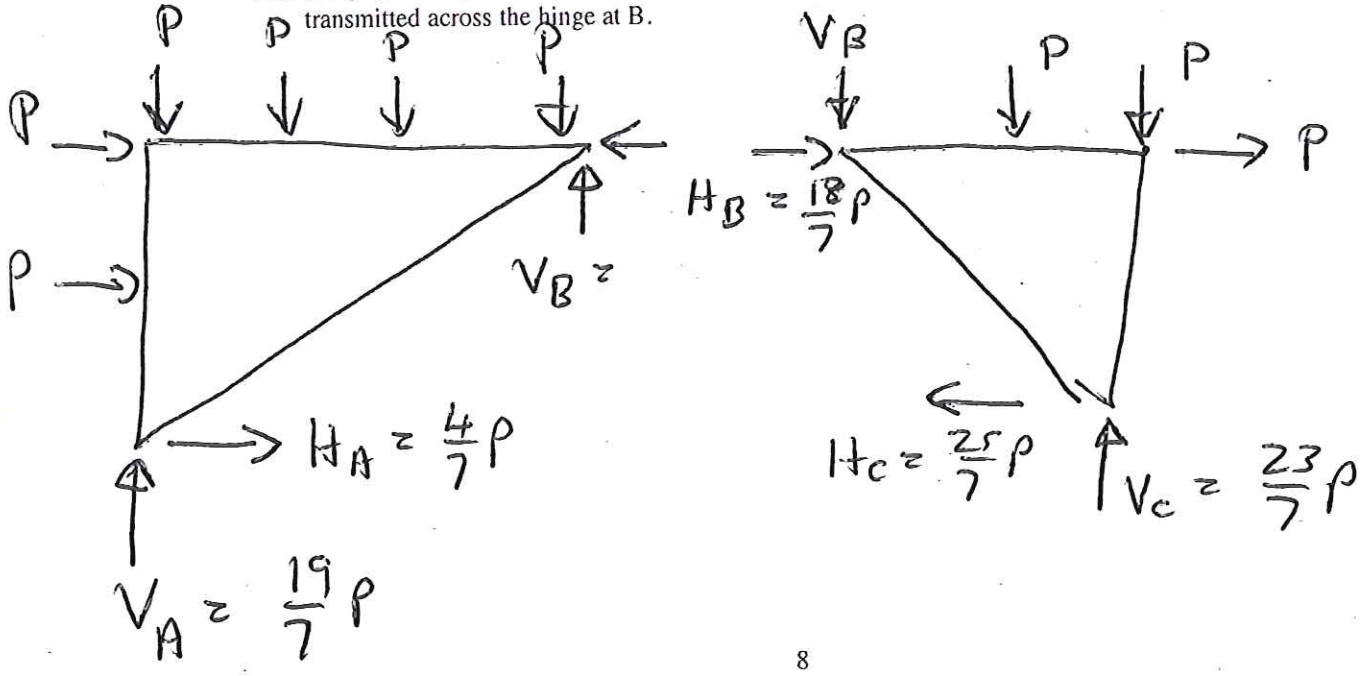


Figure 4: Front elevation view of a three-pinned arch structure.

Note: Nodes A, B and C are pin joints that can transmit vertical and horizontal forces, but not moments.

[4a] (10 pts). Compute the horizontal and vertical reaction forces at supports A and C, and the force transmitted across the hinge at B.



Question 4 continued ...

Full Structure

$\sum F_y = 0 \Rightarrow V_A + V_C = 6P$ — (A)

$\sum F_x = 0 \Rightarrow H_C - H_A = 3P$ — (B)

$\sum M_A = 0 \Rightarrow H_C L + 5V_C L = 20PL$ — (C)

Sub-structures

RHS $\sum M_B = 0 \Rightarrow 3PL + H_C L = V_C 2L$

$\Rightarrow 2V_C - H_C = 3P.$ — (D)

LHS $\sum M_B = 0 \Rightarrow 7PL + 2H_A L = 3V_A L$

$\Rightarrow 3V_A - 2H_A = 7P.$ — (E)

Solve Equations

Add (C) + (D) $\left.\begin{array}{l} 5V_C + H_C = 20P \\ 2V_C - H_C = 3P \end{array}\right\}$ $\boxed{V_C = \frac{23}{7}P.}$

From (A) $\boxed{V_A = \frac{42-23}{7}P = \frac{19}{7}P}$

From (C) $H_C = 20P - 5V_C \Rightarrow \boxed{H_C = \frac{25}{7}P}$

From (B) $H_A = H_C - 3P \Rightarrow \boxed{H_A = \frac{4}{7}P}$

$\Rightarrow \boxed{V_B = -\frac{4}{7}P}$ $\boxed{H_B = \frac{18}{7}P}$

Question 5: 10 points

OPTIONAL: Compute Cable Profile and Tension in a Suspension Bridge. Figure 5 is a front elevation view of a cable span in a suspension bridge.

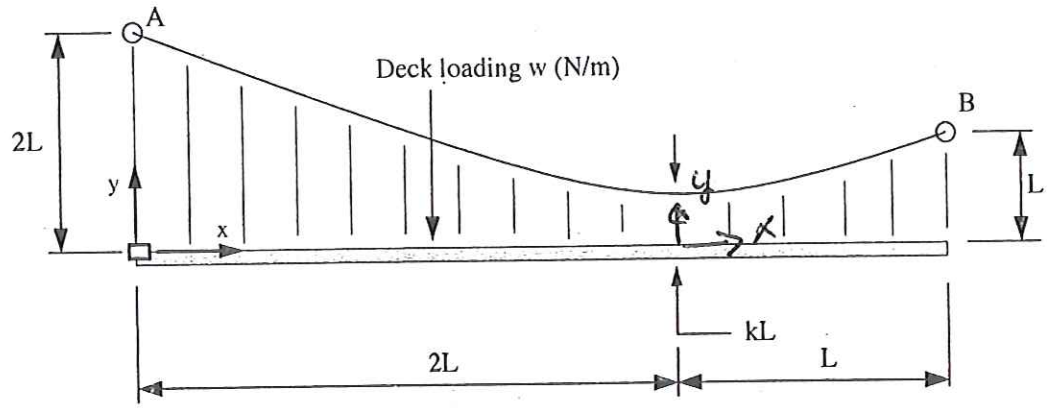


Figure 5: Elevation view of a single cable span in a suspension bridge (not to scale).

The span has length 3L and carries a uniform load w (N/m). The lowest point of the cable profile is kL above the bridge deck.

[5a] (4 pts) If the lowest point in the cable profile is at distance 2L from the left-hand side, show that:

$$k = \left[\frac{2}{3}\right]. \tag{5}$$

$$\frac{d^2y}{dx^2} = \frac{w}{H} \Rightarrow y(x) = ax^2 \text{ \& } a = \frac{w}{2H}.$$

From geometry.

RHS: $L(1-k) = A\,L^2$

LHS: $L(2-k) = A\,4L^2$

$\Rightarrow 4 - 4k = 2 - k$

$\Rightarrow 3k = 2$

$\Rightarrow k = 2/3.$

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[5b] (3 pts) Hence, show that the horizontal component of cable force is:

$$H = \left[\frac{3}{2}\right] wL. \tag{6}$$

At $x = L$.

$$1 - k = \frac{WL}{2H} \Rightarrow H = \frac{WL}{2(1-k)} = \frac{3}{2} WL$$

[5c] (3 pts) Compute the vertical components of cable force at supports A and B.

$$y(x) = \frac{W}{2H} \cdot x^2 \Rightarrow \frac{dy}{dx} = \frac{Wx}{H} = \frac{V}{H}.$$

~~but~~ $\Rightarrow \boxed{V(x) = Wx.}$

$$\left.\begin{aligned} V_A &= 2WL \\ V_B &= WL \end{aligned}\right\} V_A + V_B = 3WL \checkmark$$