## ENCE 353 Midterm 2, Open Notes and Open Book

Name:

Exam Format and Grading. Answer question 1. Then answer either question two or question three. Cross out the question you do not want graded in the table below. Partial credit will be given for partially correct answers, so please show all your working.

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 25 |  |
| 2 | 15 |  |
| 3 | 15 |  |
| Total | 40 |  |

## Question 1: 25 points

COMPULSORY: Moment-Area, Virtual Work. Figure 1 is a front elevation view of a cantilever beam carrying two external loads P. EI is constant along the cantilever beam.


Figure 1: Cantilever beam carrying two applied loads P (kN).
[1a] (5 pts) Use the method of moment area to show that the end rotation at B (measured clockwise) is:

$$
\begin{equation*}
\theta_{A}=\frac{3}{2} \frac{P L^{2}}{E I} \tag{1}
\end{equation*}
$$

[1b] (5 pts) Use the method of moment area to show that the vertical beam deflection at B is:

$$
\begin{equation*}
\triangle_{B}=\frac{11}{6} \frac{P L^{3}}{E I} \tag{2}
\end{equation*}
$$

[1c] (10 pts) Use the method of virtual forces to compute the vertical displacement at B.
[1d] (5 pts) Indicate how you would use the method of virtual forces to compute the clockwise rotation of the beam at Point B.

Note: I am looking for the method - so just set up the appropriate bending moment functions and integrals, but don't take the time to actually evaluate them.

## Question 2: 15 points

OPTIONAL: Virtual Work. Consider the truss structure shown in Figure 2.


Figure 2: Nine-bar truss structure.

A vertical load of P kN is applied at nodes D and F . AE is constant for all truss members.
[2a] ( 5 pts ) Use the method of virtual forces to show that the horizontal displacement at node D is:

$$
\begin{equation*}
\triangle_{x}=(2+\sqrt{2}) \frac{P L}{A E} . \tag{3}
\end{equation*}
$$

[2b] (5 pts) Use the method of virtual forces to show that the vertical displacement at node D is:

$$
\begin{equation*}
\triangle_{y}=(2+3 \sqrt{2}) \frac{P L}{A E} . \tag{4}
\end{equation*}
$$

[2c] (5 pts) Hence, show that:

$$
\begin{equation*}
\text { Total displacement at node } \mathrm{D}=2 \sqrt{(7+4 \sqrt{2})} \frac{P L}{A E} . \tag{5}
\end{equation*}
$$

## Question 3: 15 points

OPTIONAL: Virtual Displacements. Consider the T-shaped structure shown in Figure 3.


Figure 3: T-shaped structure.
[3a] (5 pts) By taking moments about the supports (or otherwise) compute symbolic expressions for vertical support reactions at A and B .
[3b] (5 pts) Use the principle of virtual displacements to compute a symbolic expression for the vertical reaction at B.
[3c] (5 pts) Use the principle of virtual displacements to compute a symbolic expression for the vertical reaction at $A$.

