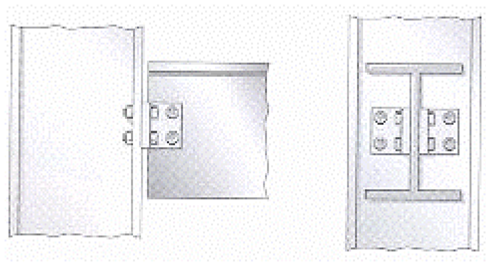
Analysis of Statically Determinate Structures

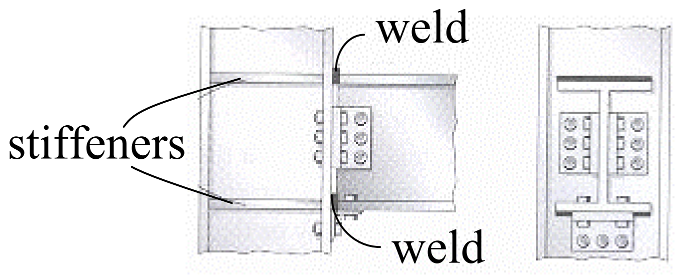
- **Idealized Structure**
- **Principle of Superposition**
- **Equations of Equilibrium**
- **Determinacy and Stability**
  - **Beams**
  - **Frames**
  - **Gable Frames**
- **Application of the Equations of Equilibrium**
- **Analysis of Simple Diaphragm and Shear Wall Systems Problems**

Classification of Structures

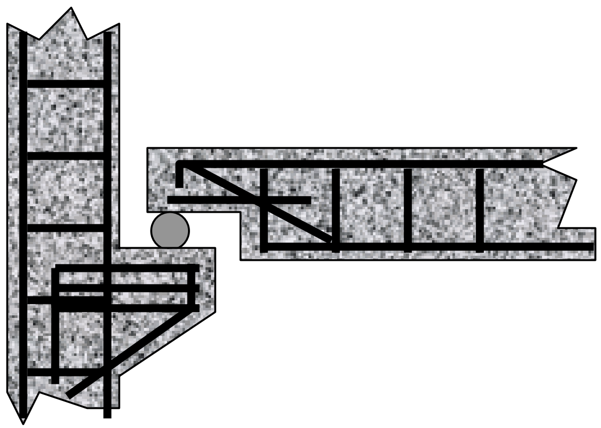
- **Support Connections**



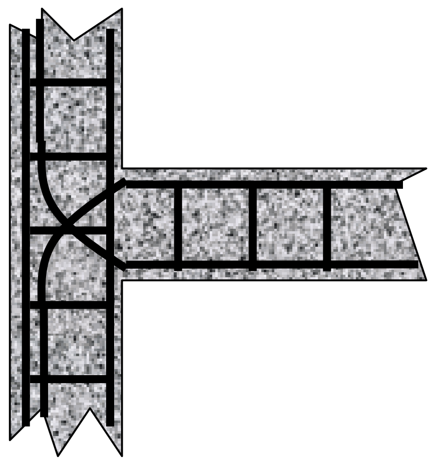
typical "pin-supported" connection (metal)



typical "fixed-supported" connection (metal)

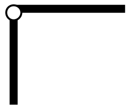


typical "roller-supported" connection (concrete)

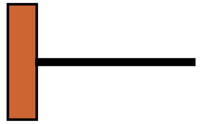


typical "fixed-supported" connection (concrete)

pin support



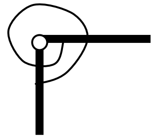
pin-connected joint



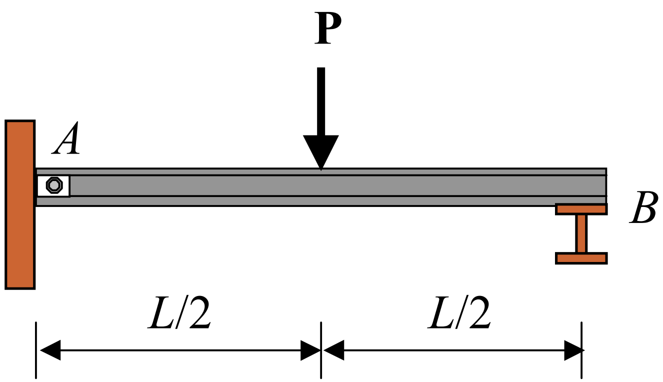
fixed support

fixed-connected joint

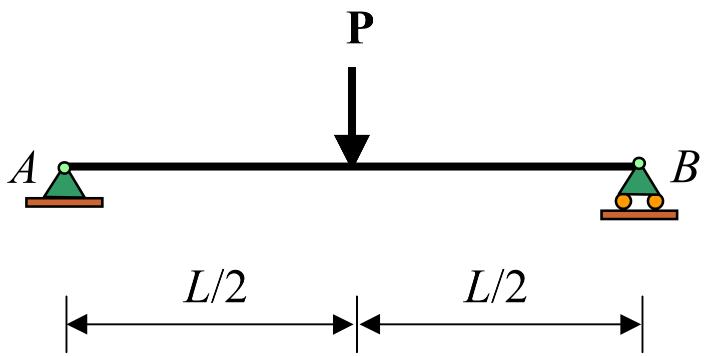
torsional spring support



torsional spring joint

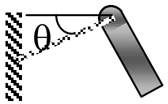
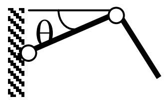
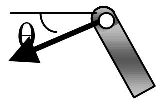
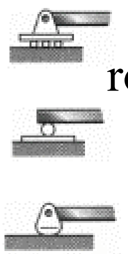
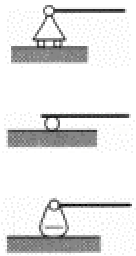
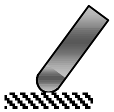
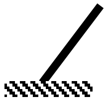
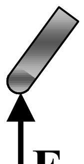
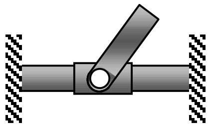
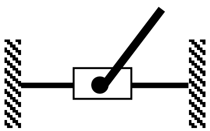
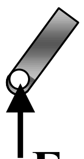


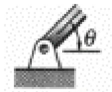
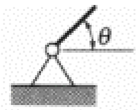
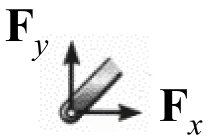
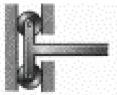
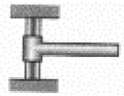
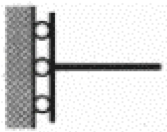
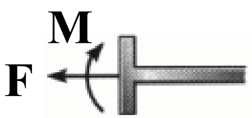
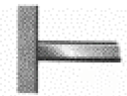
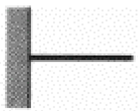
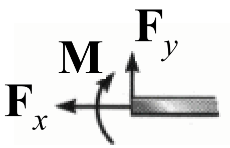
actual beam



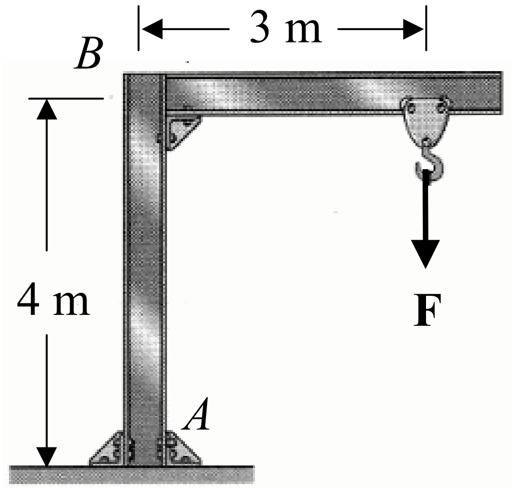
idealized beam

Table 2-1 Supports for Coplanar Structures

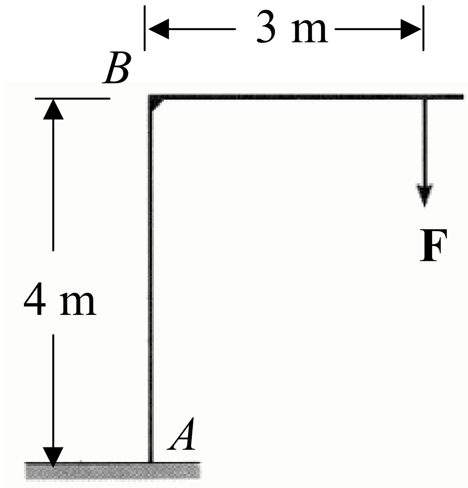
Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  Light cable			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers rockers		 **F**	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3) 		 **F**	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4) 		 **F**	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(5)  Smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider  fixed-connected collar			Two unknowns. The reactions are a force and moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.

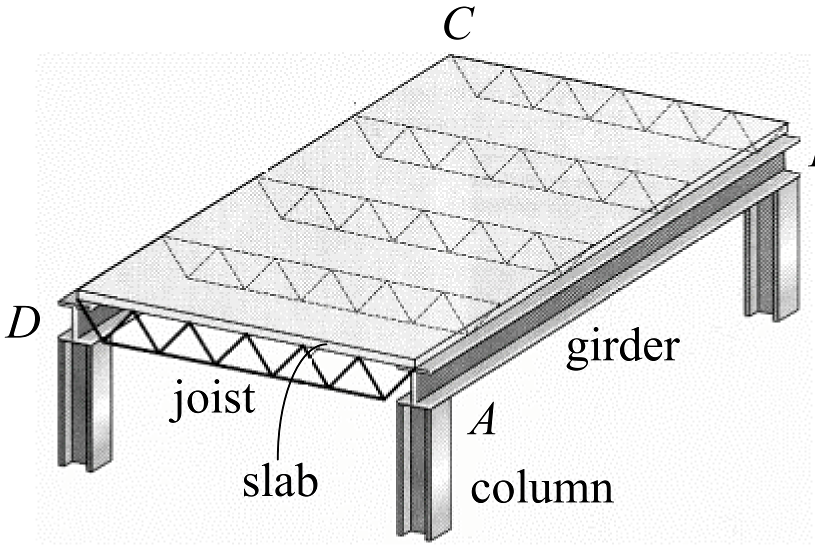
• **Idealized Structure.**

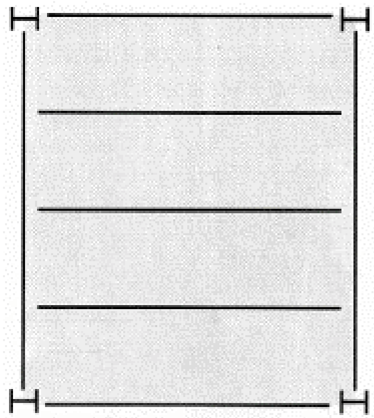


actual structure

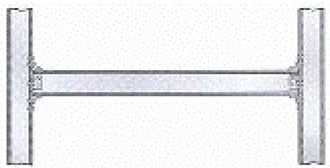


idealized structure

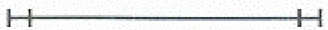




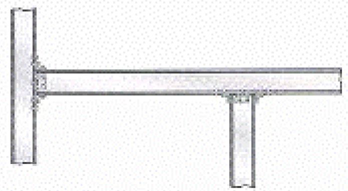
idealized framing plan



fixed-connected beam

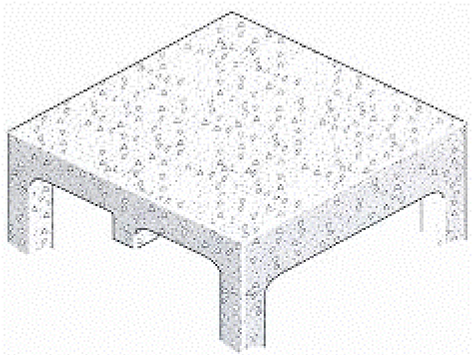


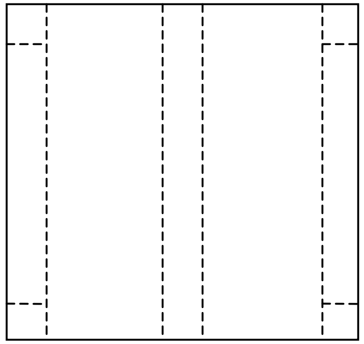
idealize beam



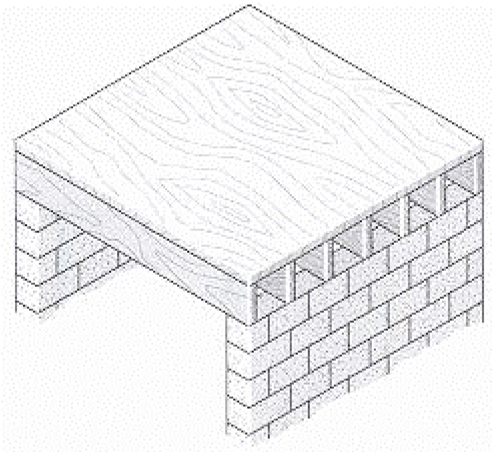
fixed-connected overhanging beam

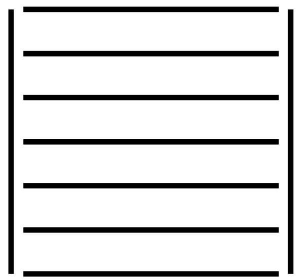
Idealized beam





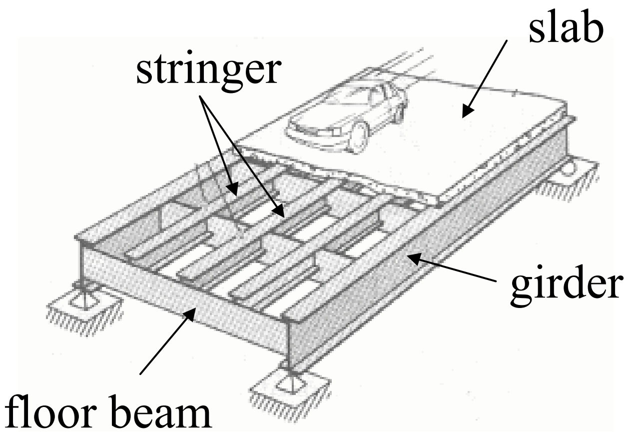
idealized framing plan

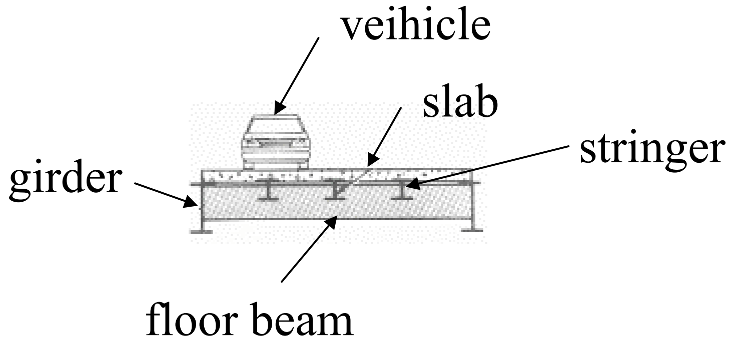


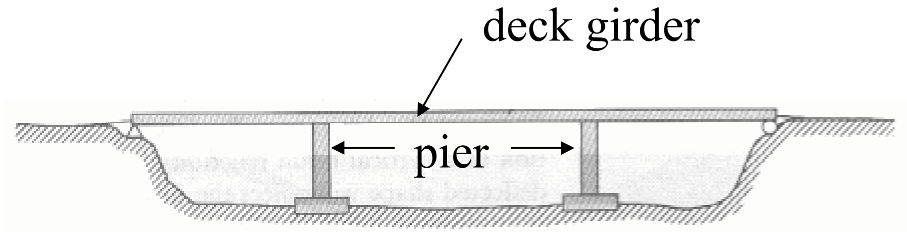


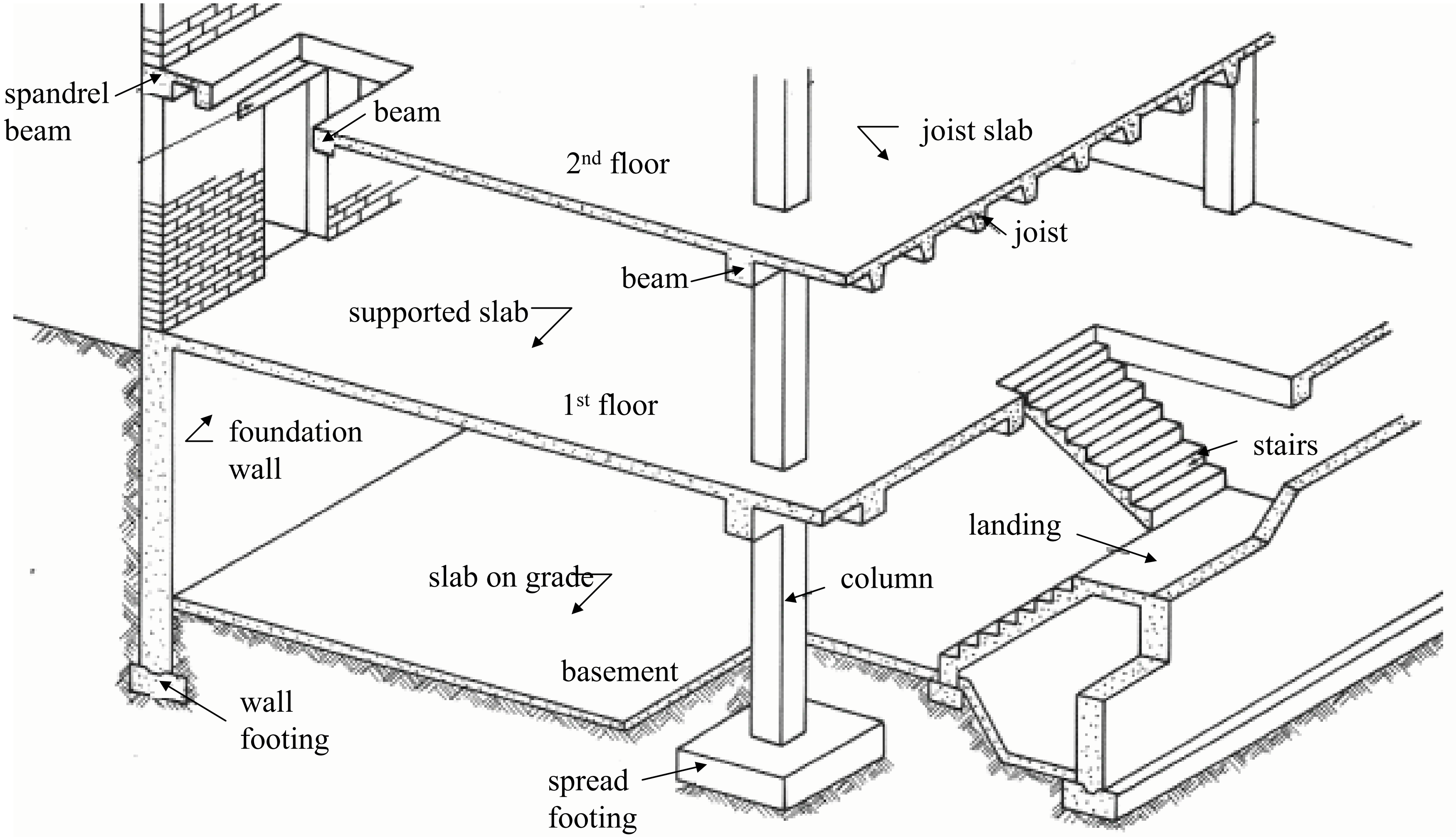
idealized framing plan

• **Tributary Loadings.**

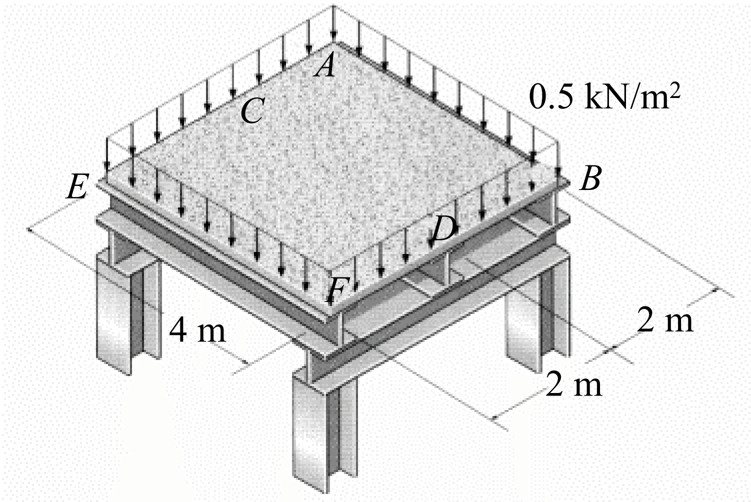


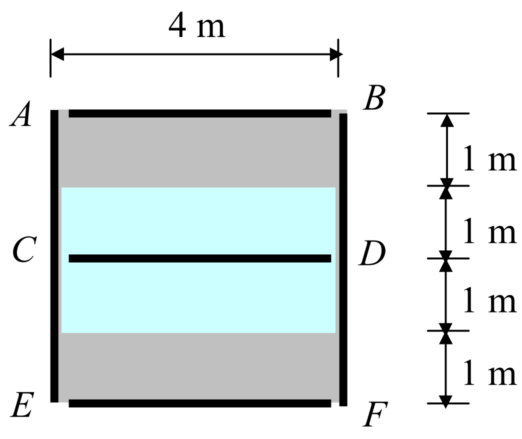




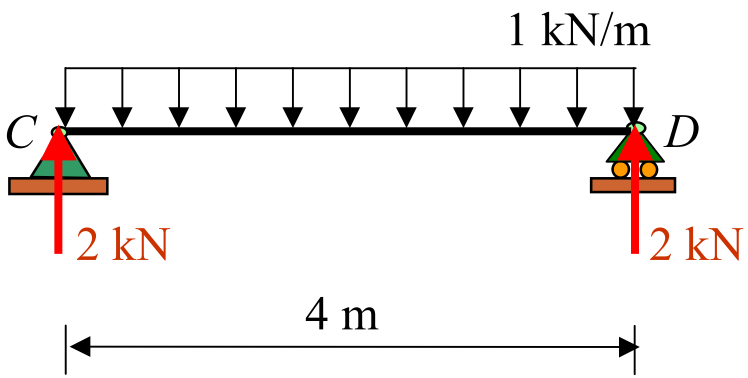

spandrel beam
beam
2nd floor
joist slab
joist
beam
supported slab
1st floor
foundation wall
stairs
landing
slab on grade
column
basement
wall footing
spread footing

One-Way System.

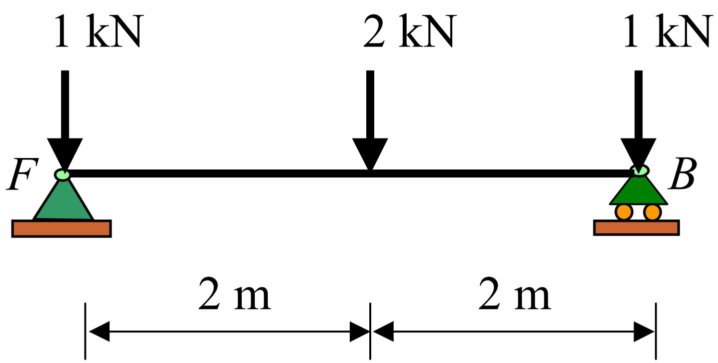




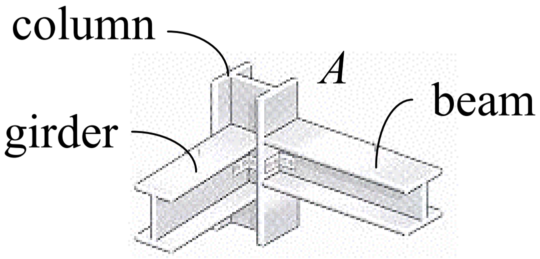
idealized framing plan



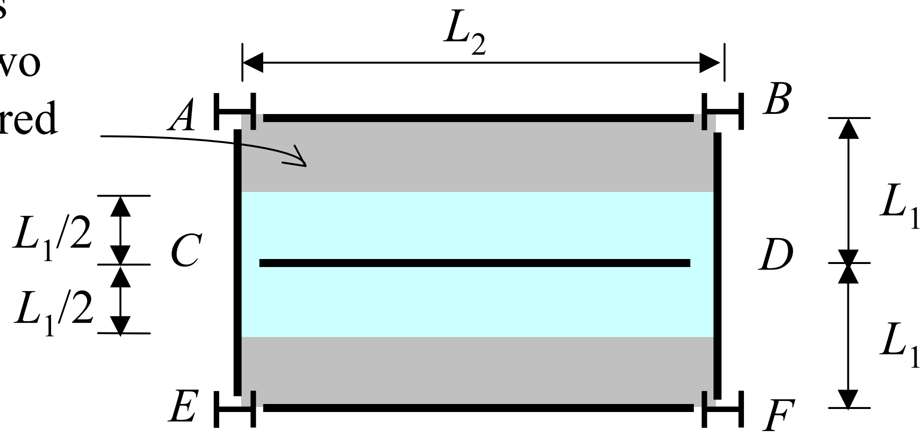
idealized beam



idealized girder

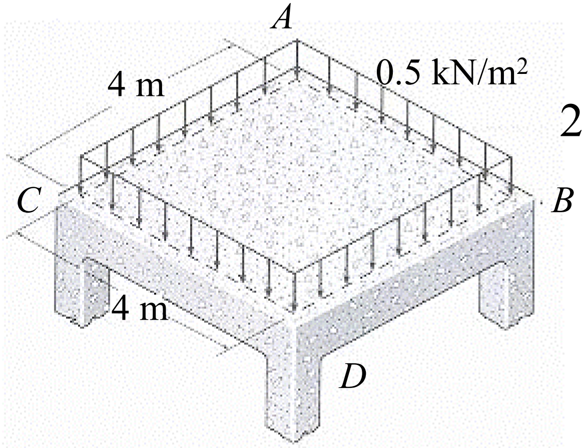


concrete slab is reinforced in two directions, poured on plane forms

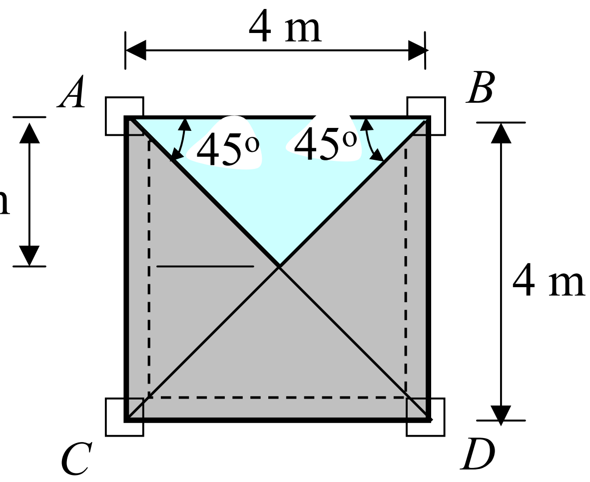


Idealized framing plan for one-way slab action requires $L_2 / L_1 \geq 2$

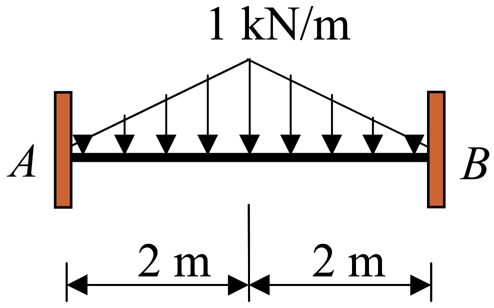
Two-Way System.



$L_2/L_1 = 1$

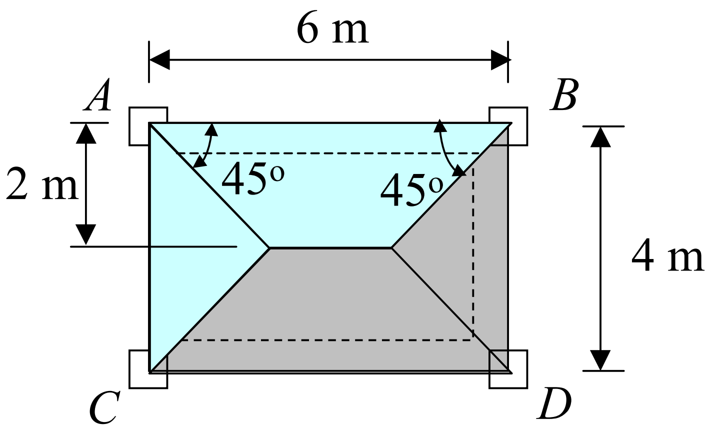


idealized framing plan

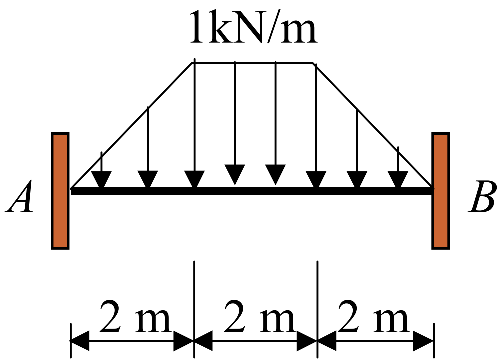


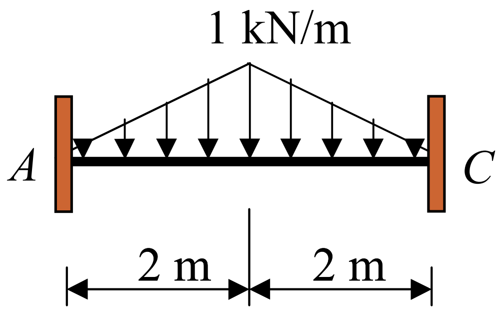
idealized beam, all

$L_2/L_1 = 1.0 < 2$



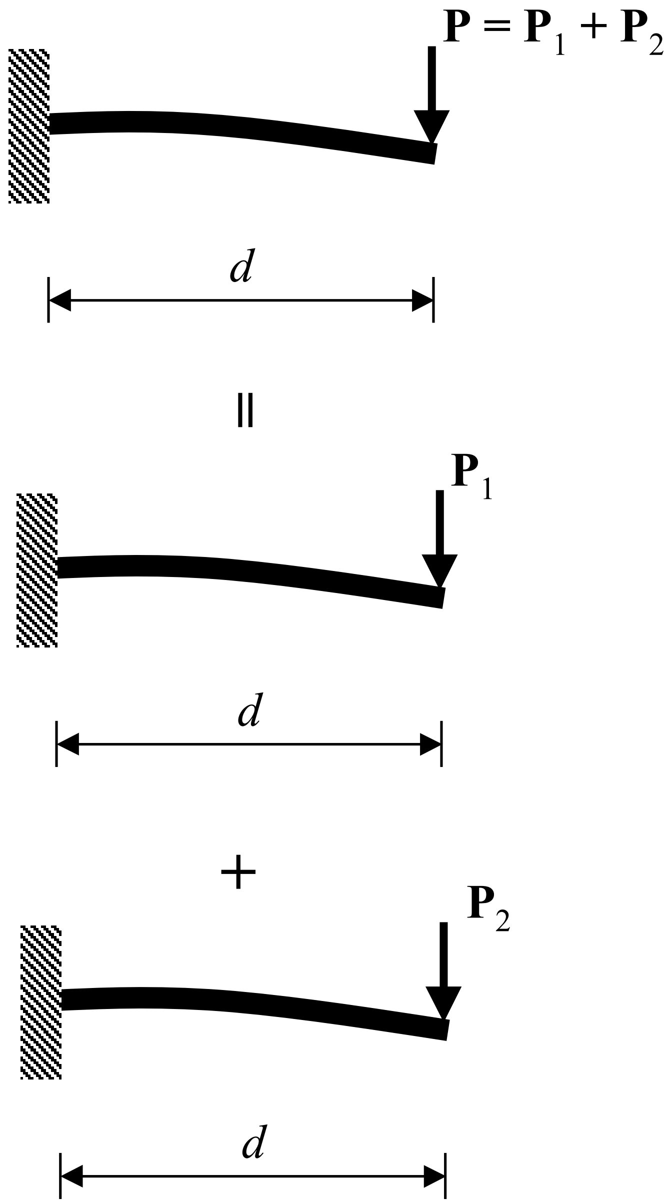
idealized framing plan





idealized beam

Principle of Superposition



Two requirements must be imposed for the principle of superposition to apply :

1. The material must behave in a linear-elastic manner, so that Hooke's law is valid, and therefore the load will be proportional to displacement.

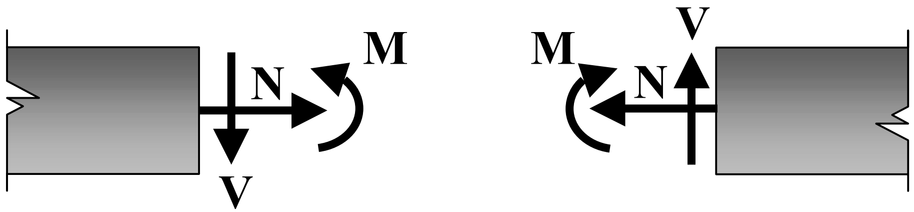
$$\sigma = P/A$$
$$\delta = PL/AE$$

2. The geometry of the structure must not undergo significant change when the loads are applied, i.e., small displacement theory applies. Large displacements will significantly change and orientation of the loads. An example would be a cantilevered thin rod subjected to a force at its end.

Equations of Equilibrium

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$



internal loadings

Determinacy and Stability

- **Determinacy**

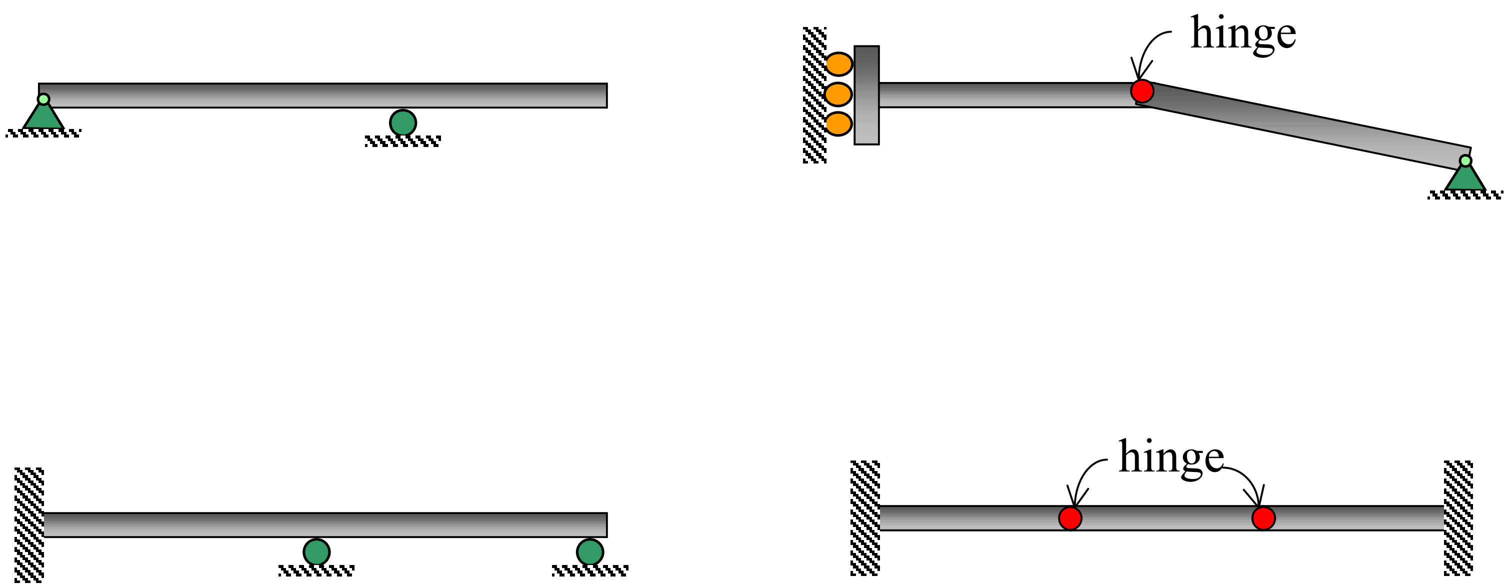
$r = 3n$, statically determinate

$r > 3n$, statically indeterminate

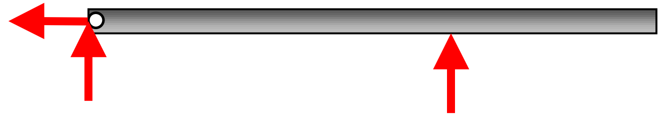
n = the total parts of structure members.
r = the total number of unknown reactive force and moment components

Example 2-1

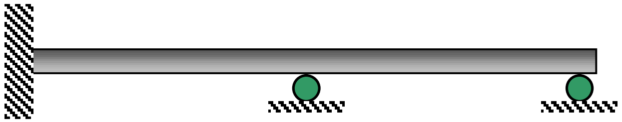
Classify each of the beams shown below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.

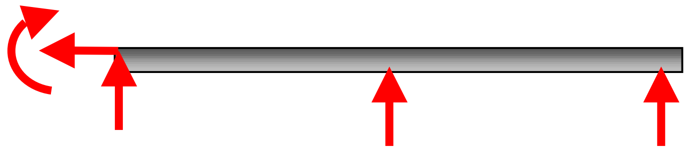


SOLUTION

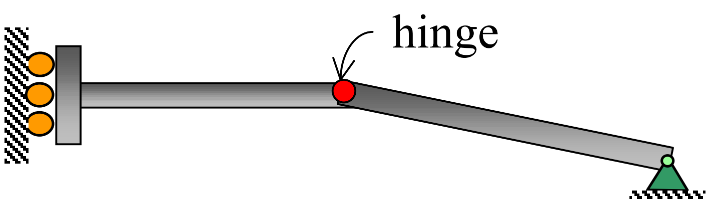


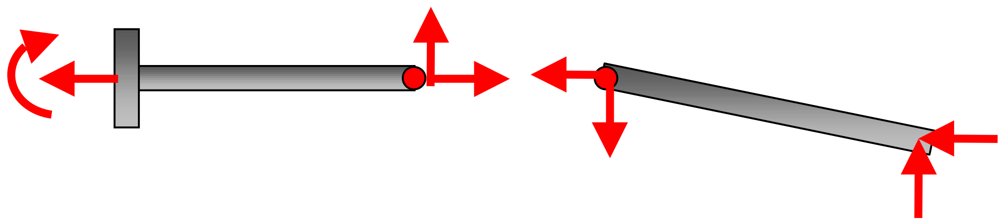
$r = 3$, $n = 1$, $3 = 3(1)$ Statically **determinate**



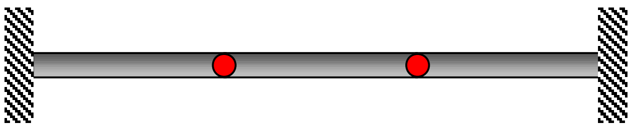


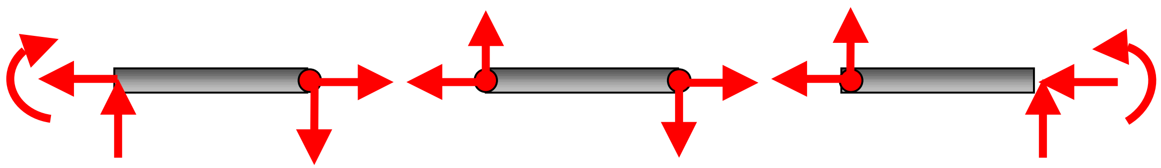
$r = 5$, $n = 1$, $5 - 3(1) = 2$ Statically **indeterminate** to the second degree





$r = 6$, $n = 2$, $6 = 3(2)$ Statically **determinate**

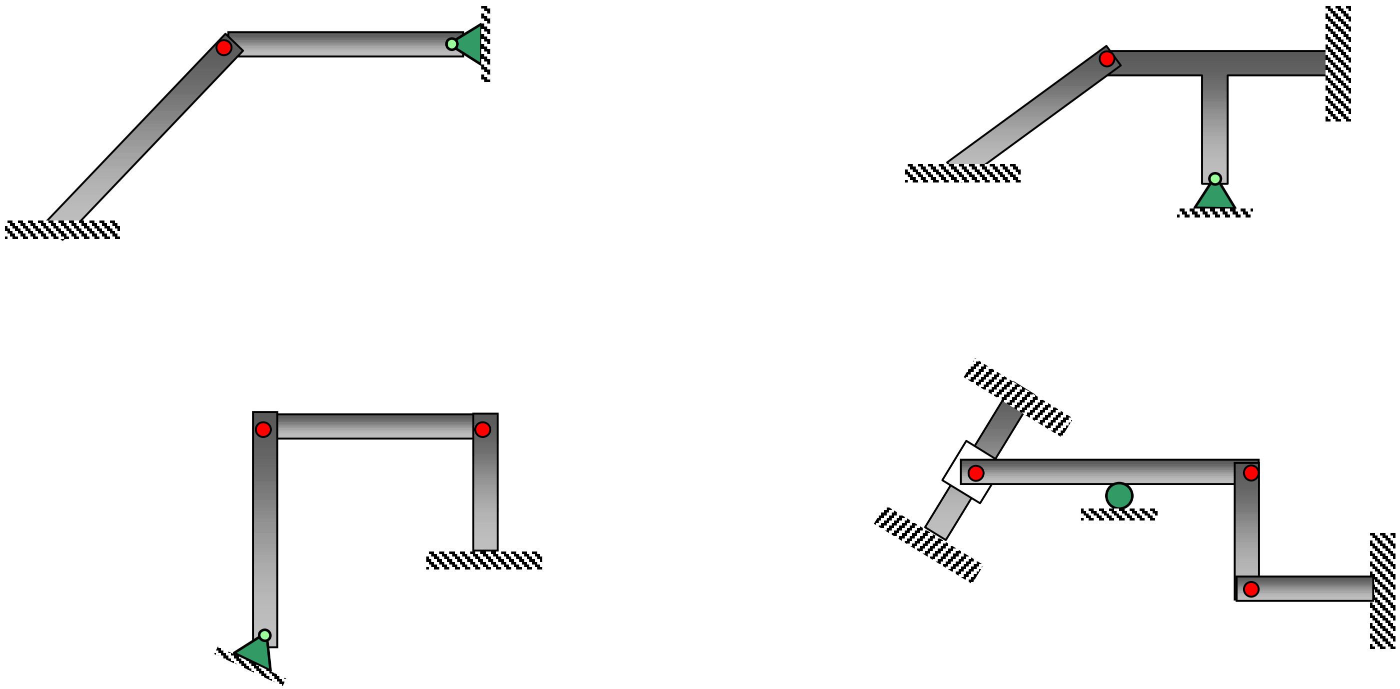




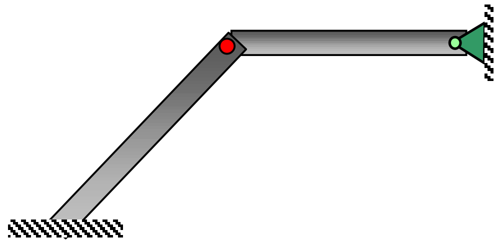
$r = 10$, $n = 3$, $10 - 3(3) = 1$ Statically **indeterminate** to the first degree

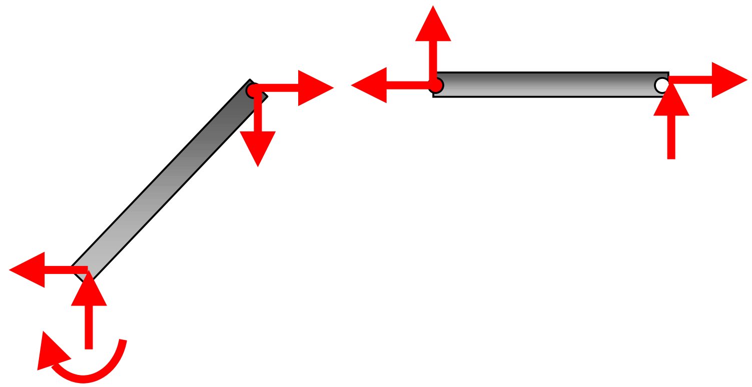
Example 2-2

Classify each of the pin-connected structures shown in figure below as statically determinate or statically indeterminate. If statically are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.



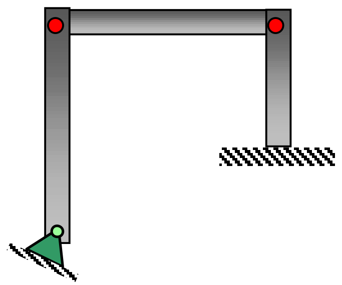
SOLUTION

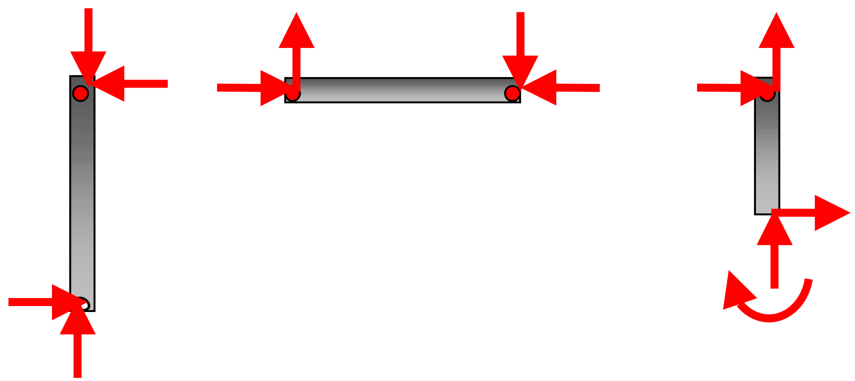




$r = 7$, $n = 2$, $7 - 3(2) = 1$

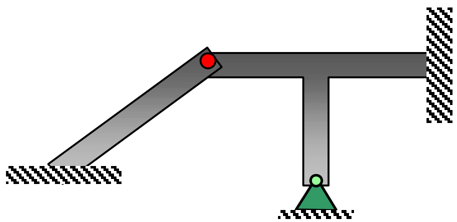
Statically **indeterminate** to the first degree

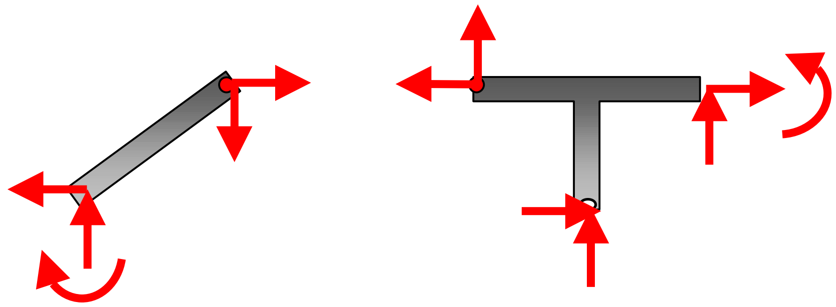




$r = 9$, $n = 3$, $9 = 3(3)$

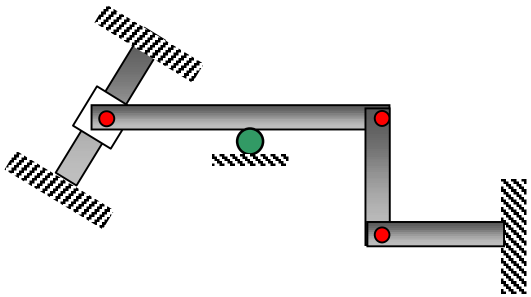
Statically **determinate**

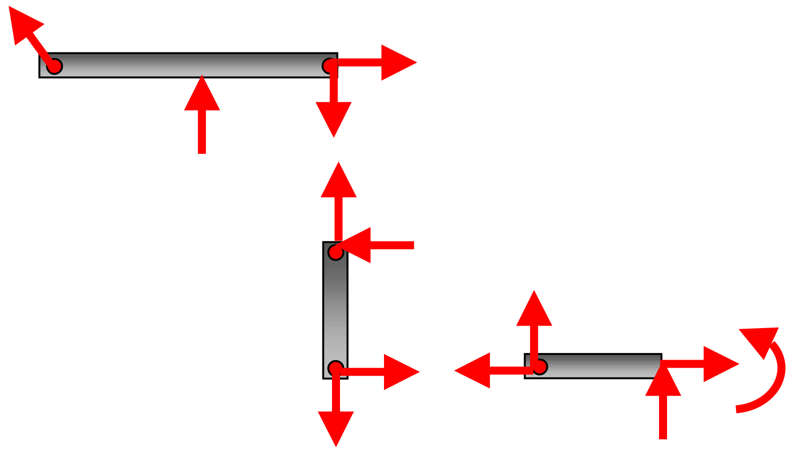




$r = 10$, $n = 2$, $10 - 6 = 4$

Statically **indeterminate** to the fourth degree



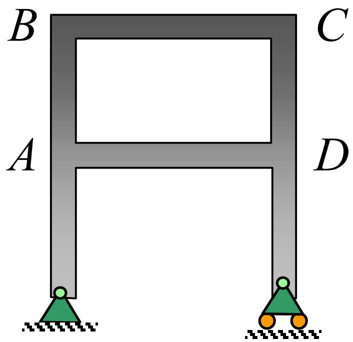


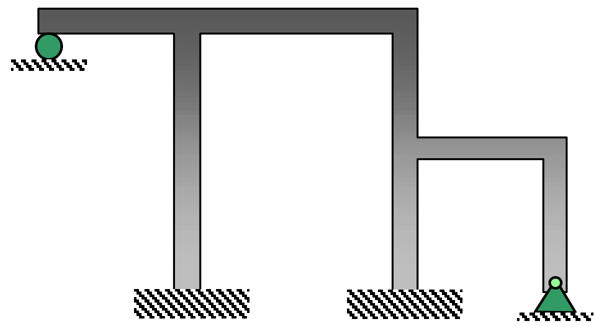
$r = 9$, $n = 3$, $9 = 3(3)$

Statically **determinate**

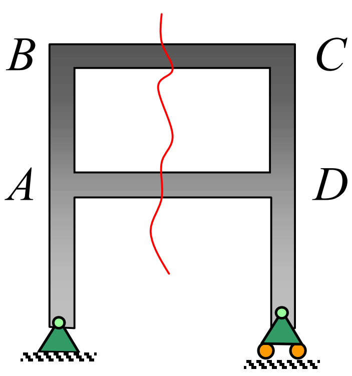
Example 2-3

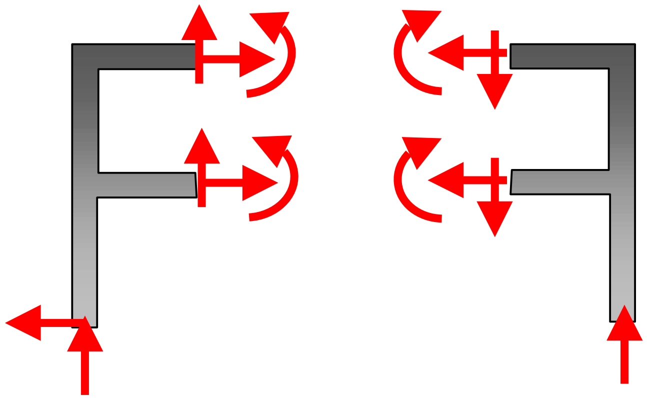
Classify each of the frames shown in figure below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The frames are subjected to external loadings that are assumed to be known and can act anywhere on the frames.





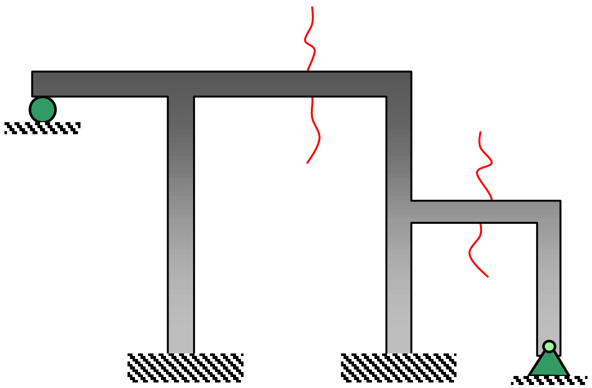
SOLUTION

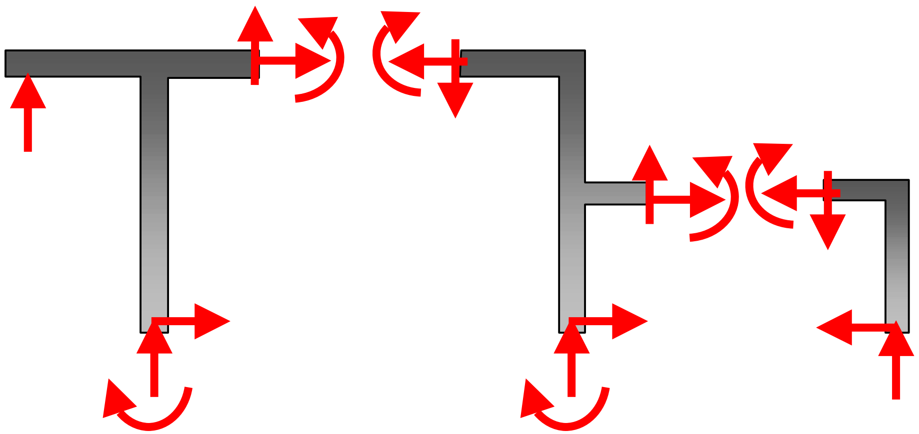




$r = 9$, $n = 2$, $9 - 6 = 3$

Statically **indeterminate** to the third degree





$r = 15$, $n = 3$, $15 - 9 = 6$

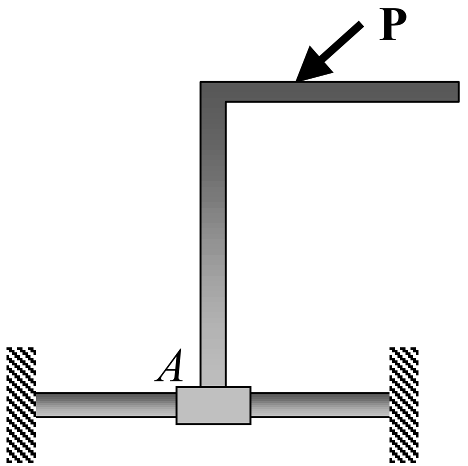
Statically **indeterminate** to the sixth degree

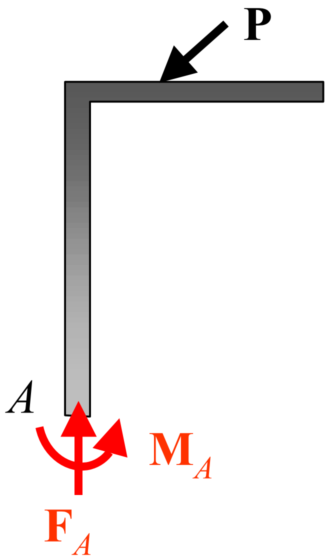
• **Stability**

$r < 3n$, unstable

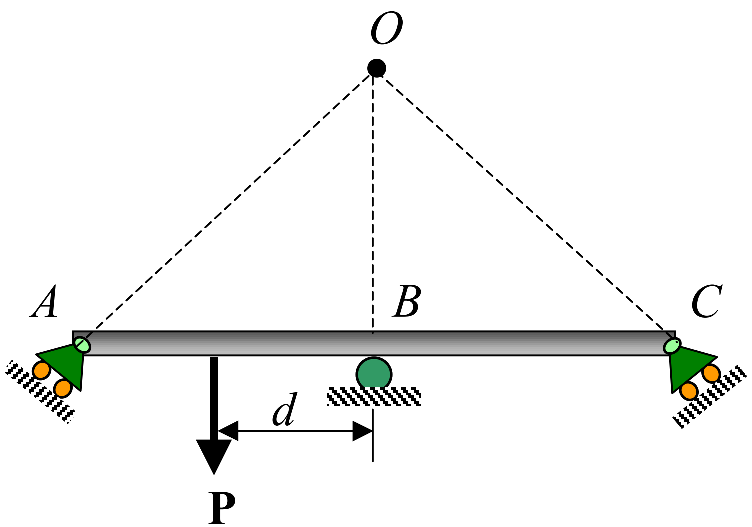
$r \geqslant 3n$, unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism

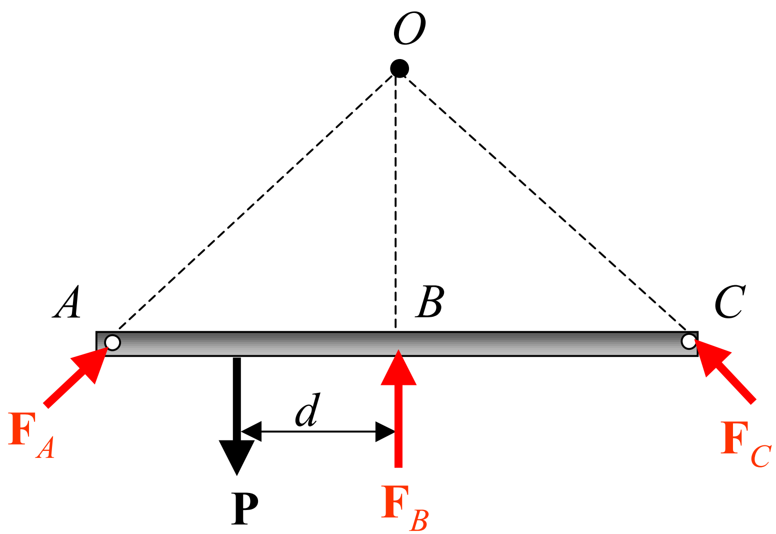
Partial Constrains

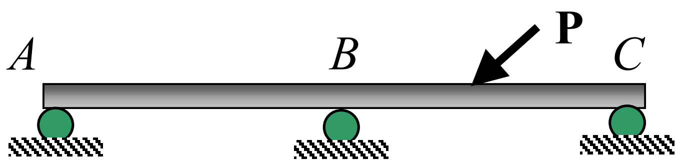


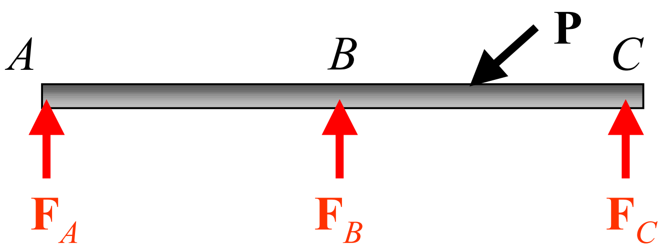


Improper Constraints



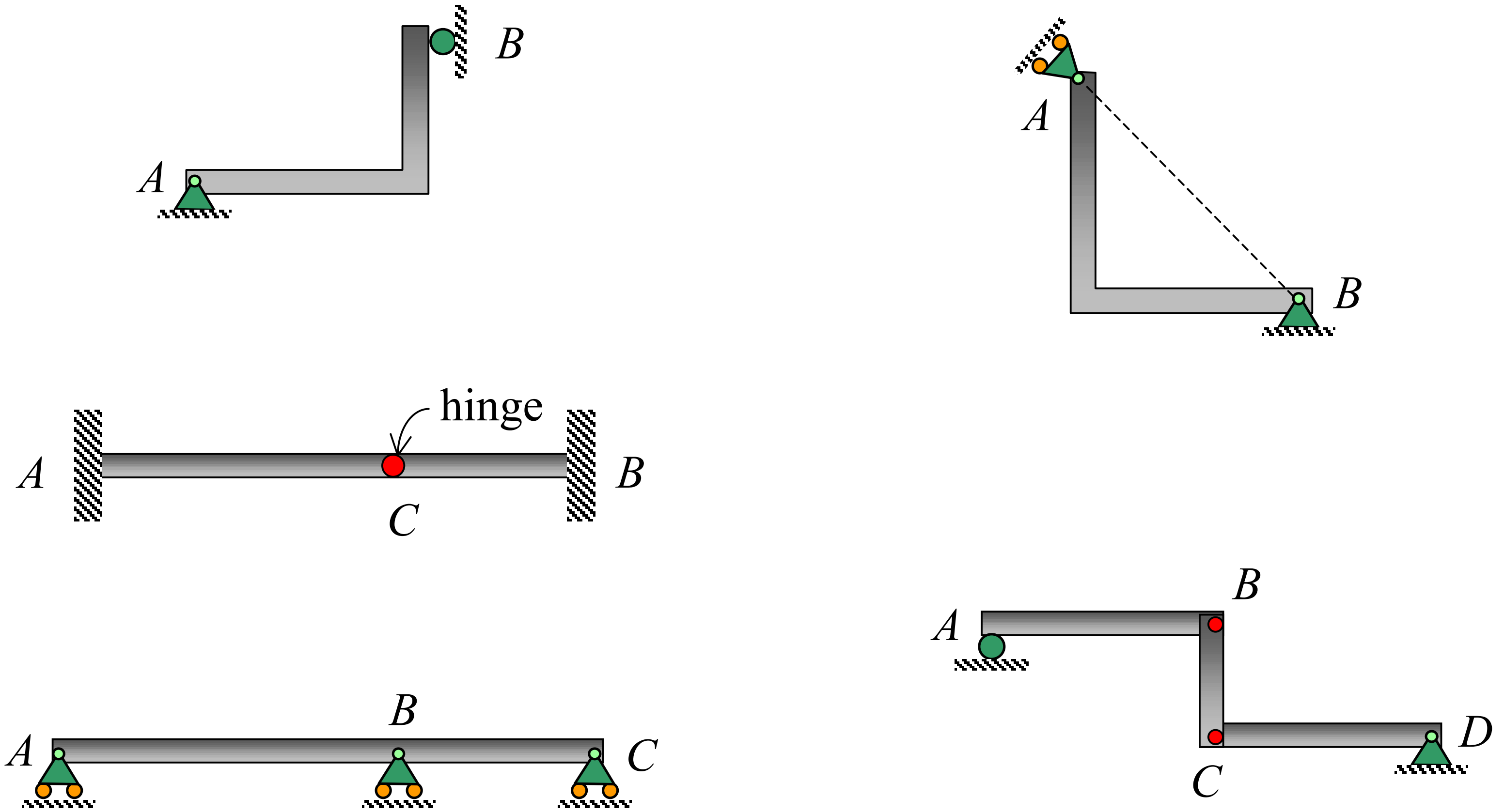




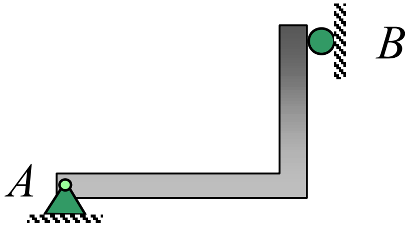


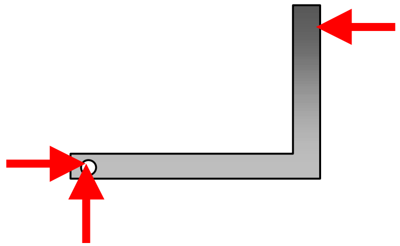
Example 2-4

Classify each of the structures in the figure below as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.

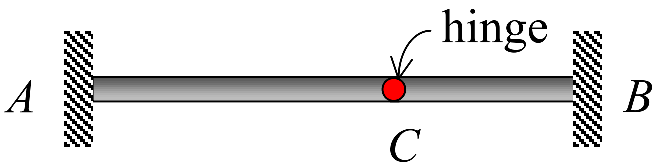


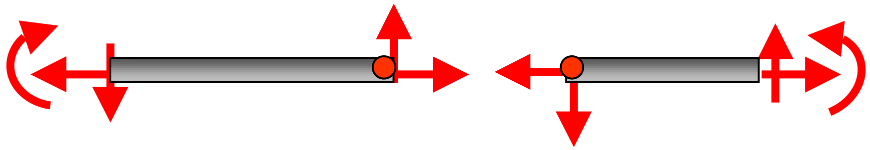
SOLUTION



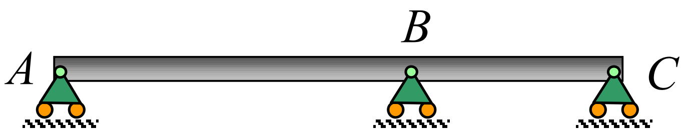


The member is ***stable*** since the reactions are non-concurrent and nonparallel. It is also statically **determinate**.

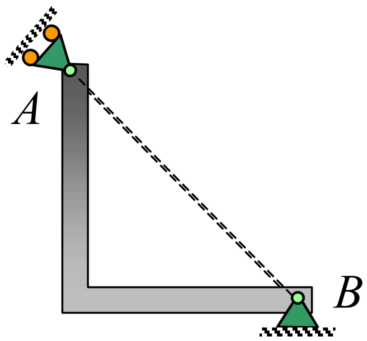


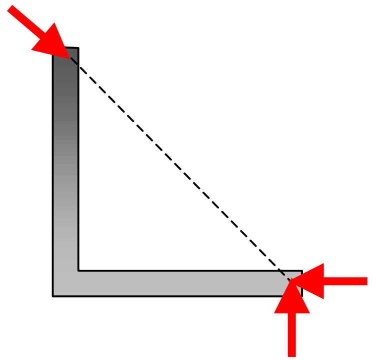


The compound beam is ***stable***. It is also **indeterminate** to the second degree.

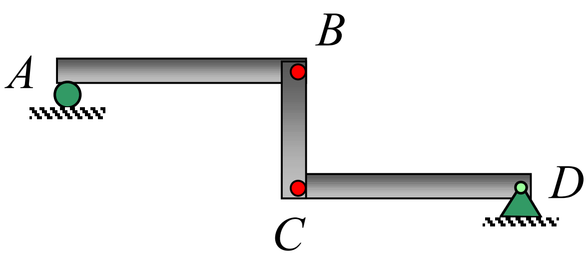


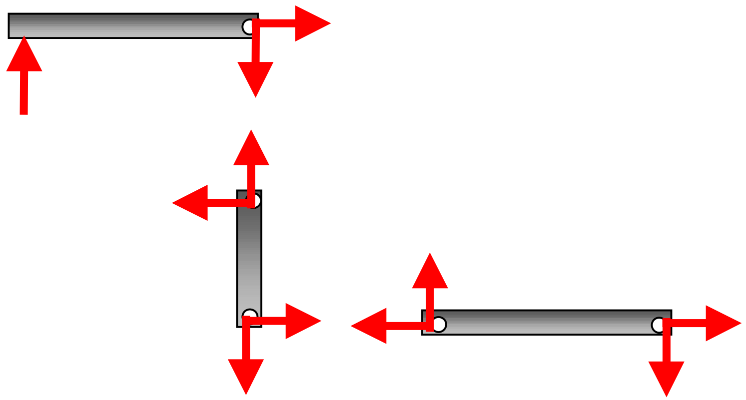
The compound beam is ***unstable*** since the three reactions are all parallel.





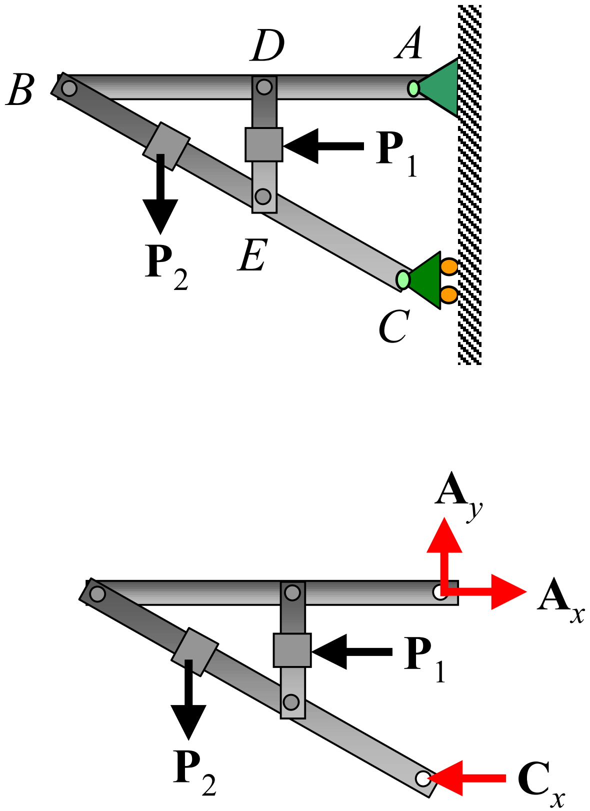
The member is ***unstable*** since the three reactions are concurrent at *B*.

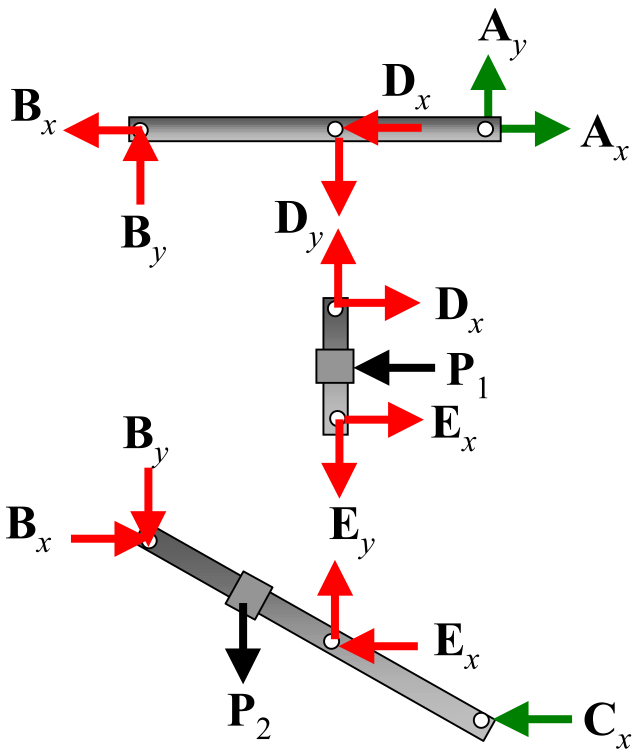




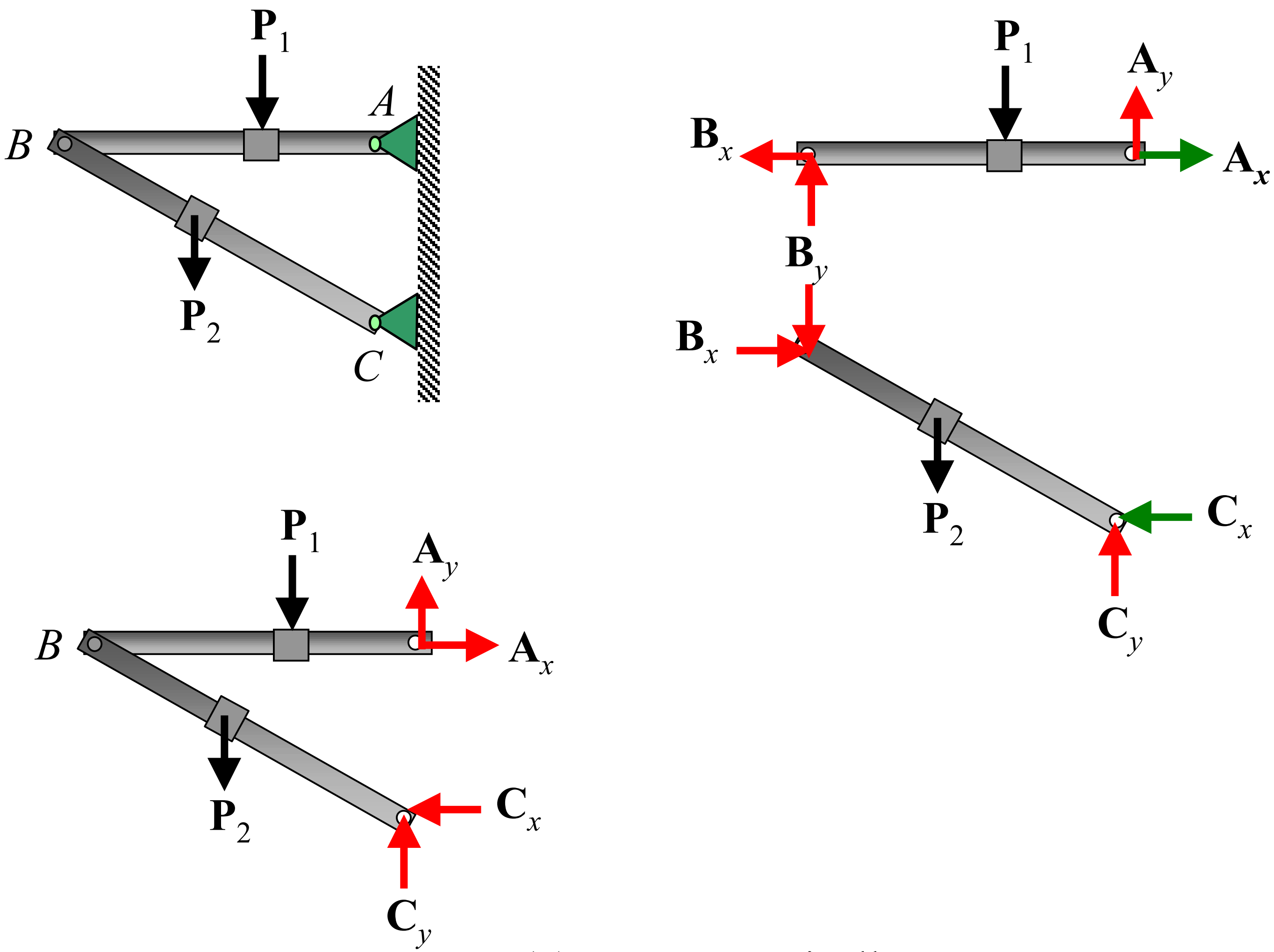
The structure is ***unstable*** since $r = 7$, $n = 3$, so that, $r < 3n$, $7 < 9$. Also, this can be seen by inspection, since *AB* can move horizontally without restraint.

Application of the Equations of Equilibrium





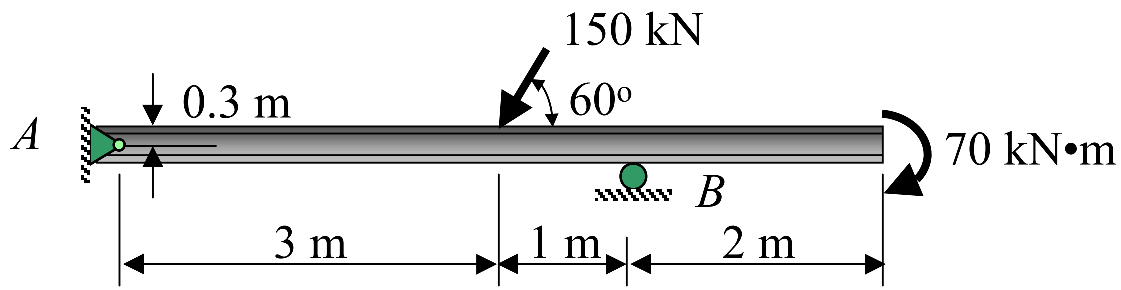
$r = 9$, $n = 3$, $9 = 3(3)$; statically **determinate**



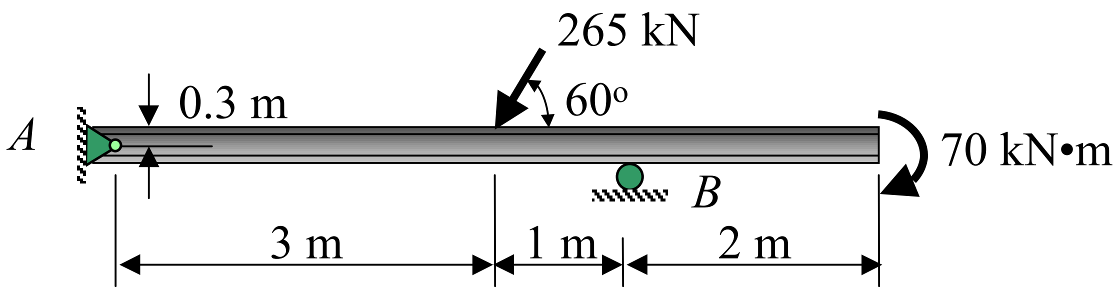
$r = 6$, $n = 2$, $6 = 3(2)$; statically **determinate**

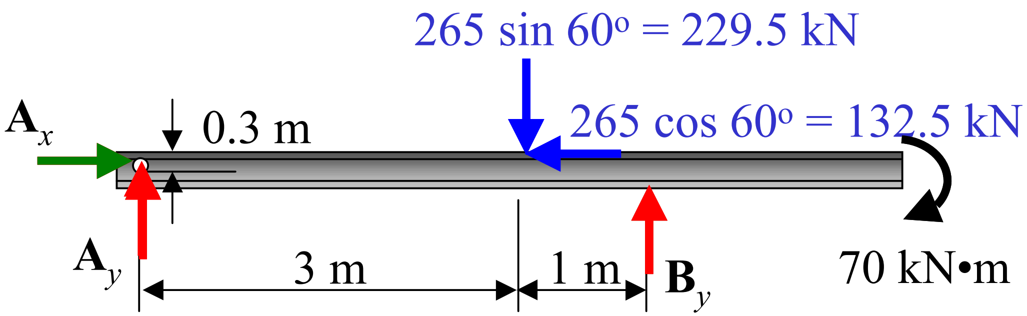
Example 2-5

Determine the reactions on the beam shown.



SOLUTION





$\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x - 132.5 = 0$: $A_x = 132.5$ kN , $\rightarrow$

$\overset{+}{\curvearrowright} \Sigma M_A = 0$: $\quad B_y(4) - (229.5)(3) + (132.5)(0.3) - 70 = 0$

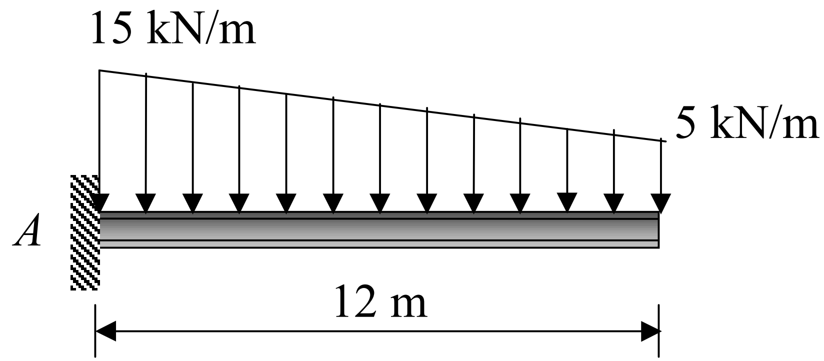
$B_y = 179.69$ kN, $\uparrow$

$+\uparrow \Sigma F_y = 0$: $\quad A_y - 229.5 + 179.69 = 0$

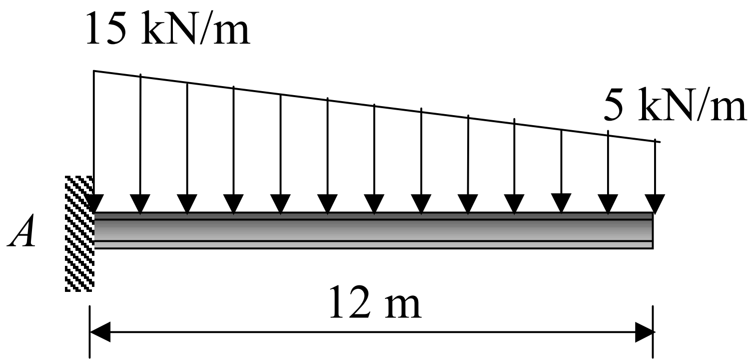
$A_y = 49.81$ kN , $\uparrow$

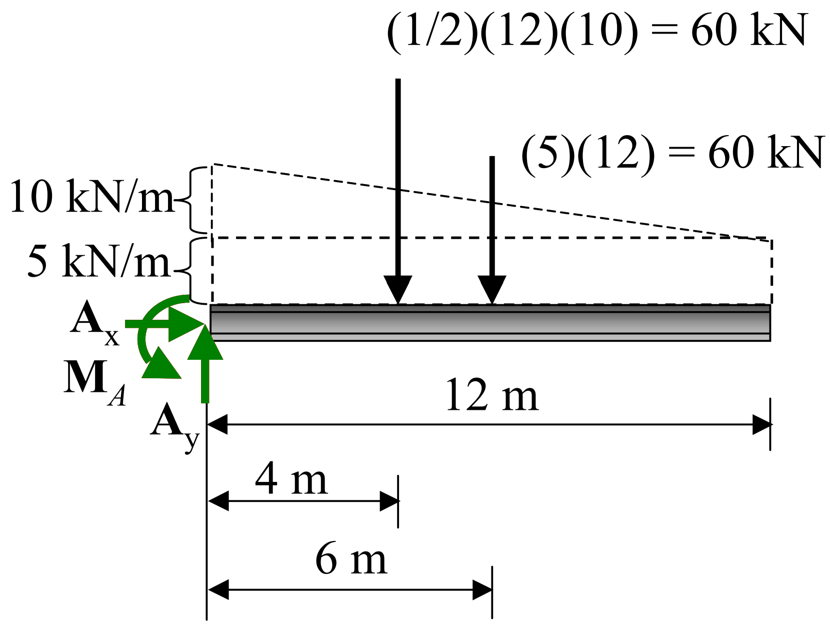
Example 2-6

Determine the reactions on the beam shown.



SOLUTION





$\overset{+}{\rightarrow} \Sigma F_x = 0$: $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0$: $\quad A_y - 60 - 60 = 0$

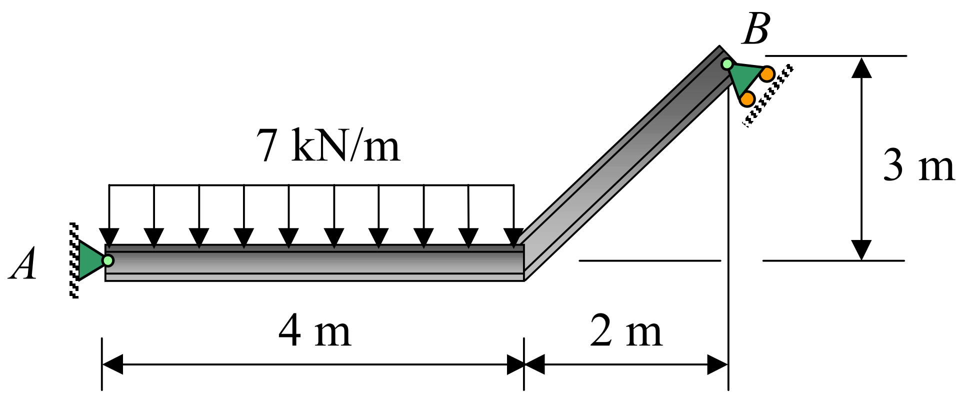
$$A_y = 120 \text{ kN}, \uparrow$$

$+\circlearrowright \Sigma M_A = 0$: $\quad M_A - (60)(4) - (60)(6) = 0$

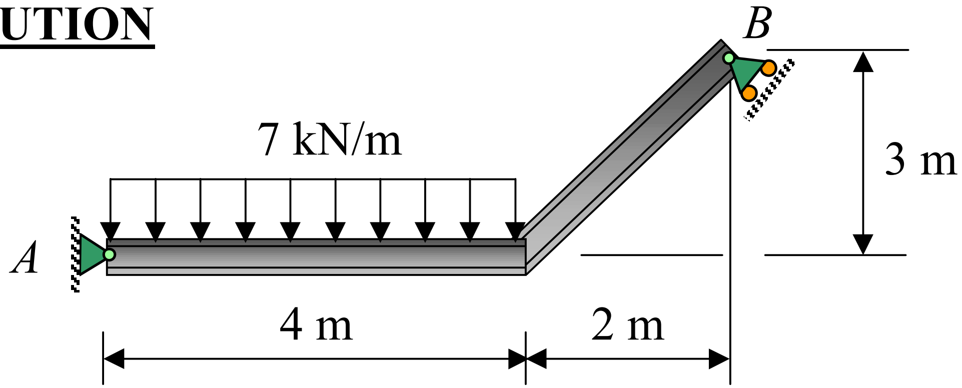
$$M_A = 600 \text{ kN} \bullet \text{m}$$

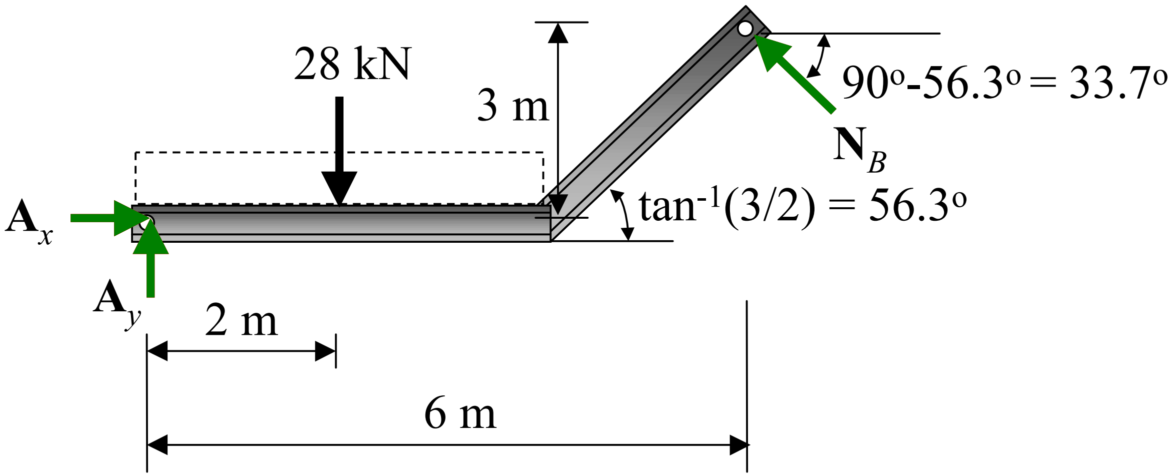
Example 2-7

Determine the reactions on the beam shown. Assume *A* is a pin and the support at *B* is a roller (smooth surface).



SOLUTION





$\circlearrowright + \; \Sigma M_A = 0$: $\quad -28(2) + N_B \sin 33.7(6) + N_B \cos 33.7(3) = 0$

$N_B = 9.61$ kN

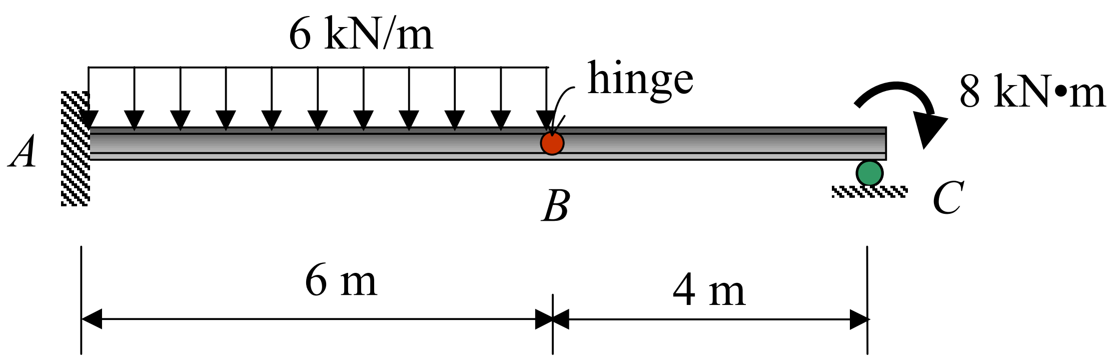
$\xrightarrow{+} \; \Sigma F_x = 0$: $\quad A_x - N_B \cos 33.7 = 0;\; A_x = 9.61 \cos 33.7 = 8$ kN , $\rightarrow$

$+\uparrow \; \Sigma F_y = 0$: $\quad A_y - 28 + 9.61 \cos 33.7 = 0$

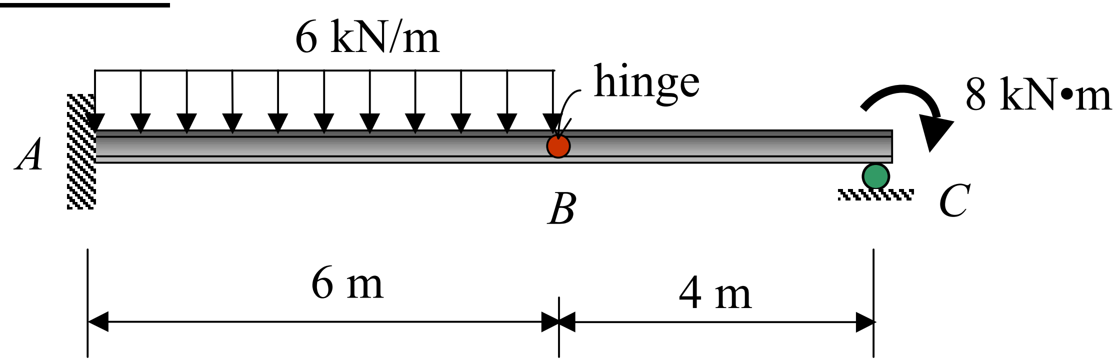
$A_y = 22.67$ kN , $\uparrow$

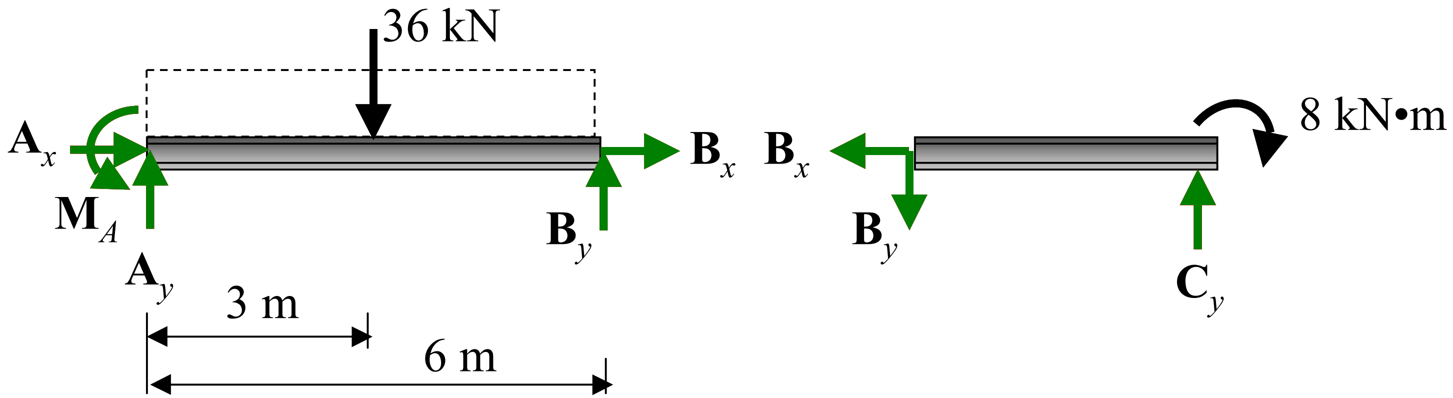
Example 2-8

The compound beam in figure below is fixed at *A*. Determine the reactions at *A*, *B*, and *C*. Assume that the connection at pin and *C* is a rooler.



SOLUTION





Member *BC*

$+\circlearrowright \Sigma M_B = 0$: $\quad C_y(4) - 8 = 0$

$C_y = 2 \text{ kN}, \uparrow$

$\xrightarrow{+} \Sigma F_x = 0$: $\quad B_x = 0$

$+\uparrow \Sigma F_y = 0$: $\quad C_y - B_y = 0;$

$B_y = C_y = 2 \text{ kN}, \uparrow$

Member *AB*

$+\circlearrowright \Sigma M_A = 0$: $\quad M_A - 36(3) + 2(6) = 0$

$M_A = 96 \text{ kN•m}$

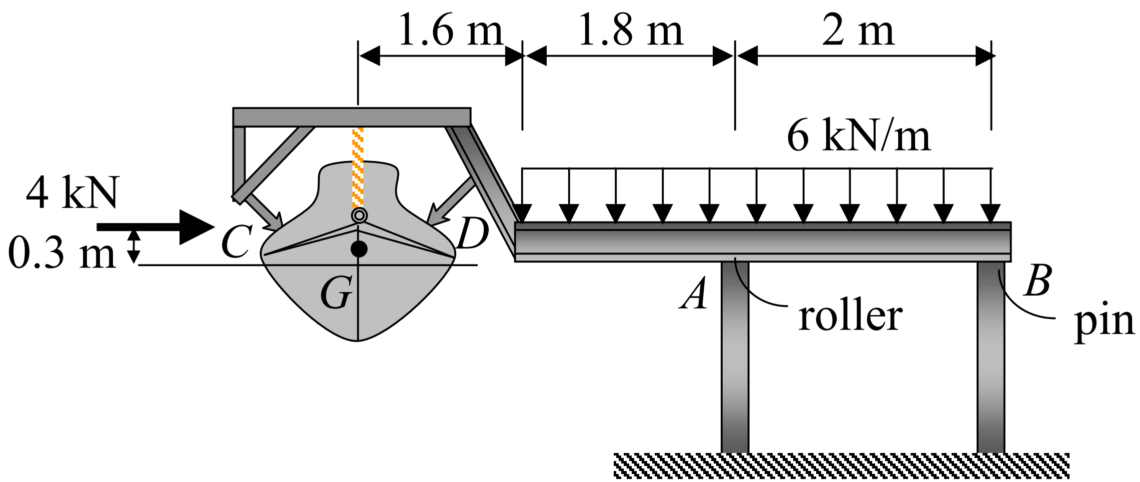
$\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x - B = 0 ; A_x = B_x = 0$

$+\uparrow \Sigma F_y = 0$: $\quad A_y - 36 + 2 = 0$

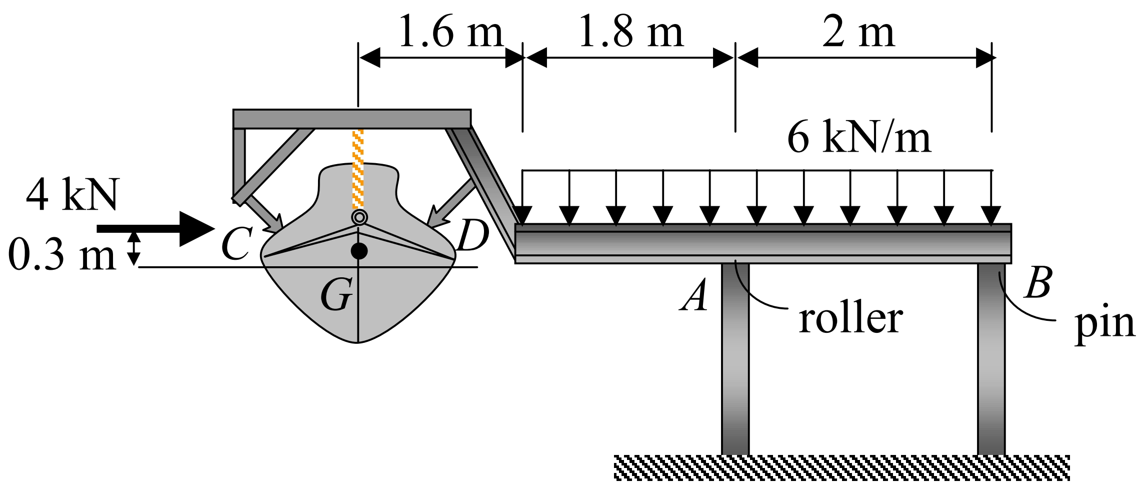
$A_y = 34 \text{ kN}, \uparrow$

Example 2-9

The side girder shown in the photo supports the boat and deck. An idealized model of this girder is shown in the figure below, where it can be assumed *A* is a roller and *B* is a pin. Using a local code the anticipated deck loading transmitted to the girder is 6 kN/m. Wind exerts a *resultant* horizontal force of 4 kN as shown, and the mass of the boat that is supported by the girder is 23 Mg. The boat's mass center is at *G*. Determine the reactions at the supports.



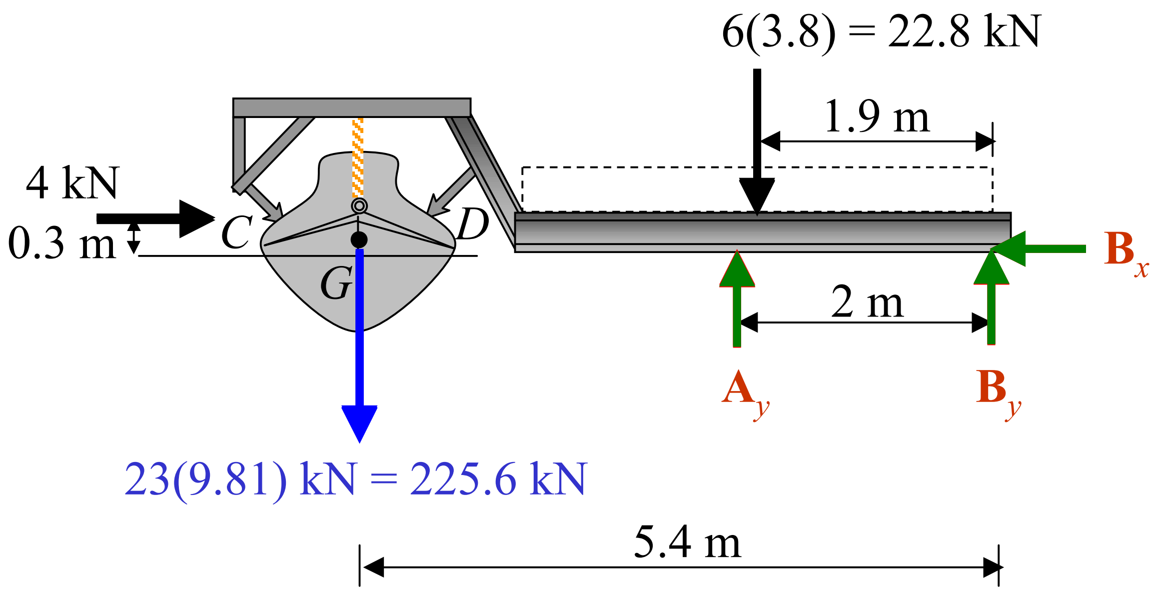
SOLUTION



$\xrightarrow{+} \Sigma F_x = 0$:

$4 - B_x = 0$

$B_x = 4$ kN , $\leftarrow$



$+\circlearrowright \Sigma M_B = 0$:

$22.8(1.9) - A_y(2) + 225.6(5.4) - 4(0.3) = 0$

$A_y = 630.2$ kN , $\uparrow$

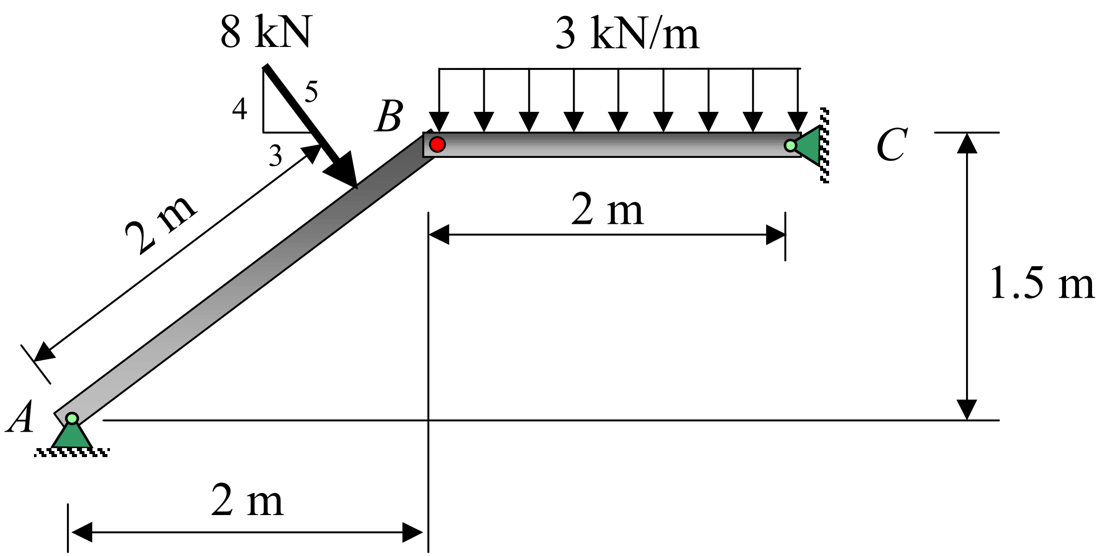
$+\uparrow \Sigma F_y = 0$:

$-225.6 + 630.2 - 22.8 + B_y = 0$

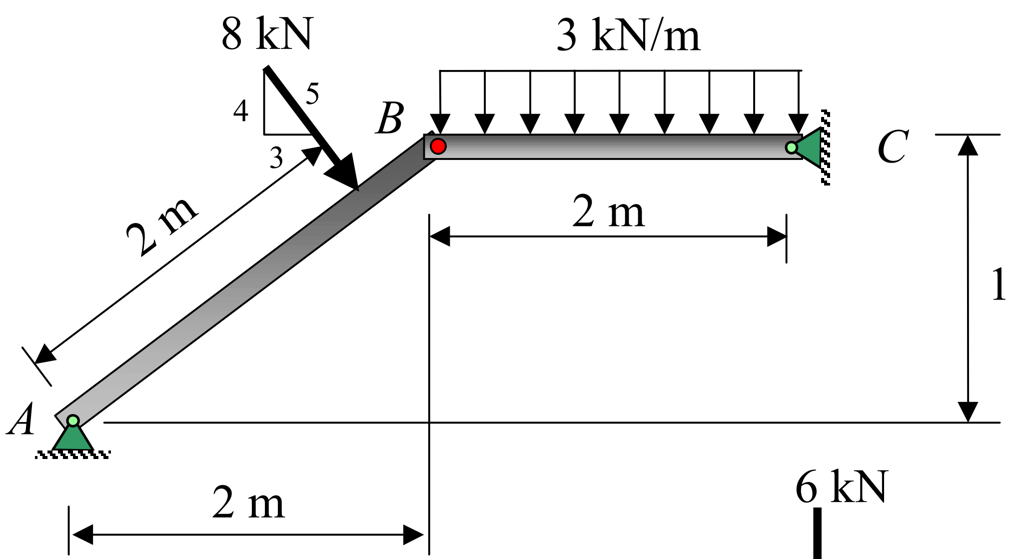
$B_y = 382$ kN , $\uparrow$

Example 2-10

Determine the horizontal and vertical components of reaction at the pins *A*, *B*, and *C* of the two-member frame shown in the figure below.



<u>SOLUTION</u>



Member *BC*

$\circlearrowright + \; \Sigma M_C = 0:$

$-B_y(2) + 6(1) = 0$

$B_y = 3 \text{ kN}, \uparrow$

Member *AB*

$\circlearrowright + \; \Sigma M_A = 0:$

$-8(2) - 3(2) + B_x(1.5) = 0$

$B_x = 14.7 \text{ kN}, \leftarrow$

$\xrightarrow{+} \; \Sigma F_x = 0:$

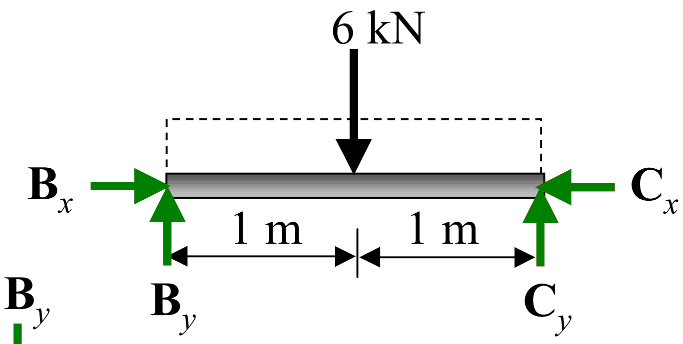
$A_x + (3/5)8 - 14.7 = 0$

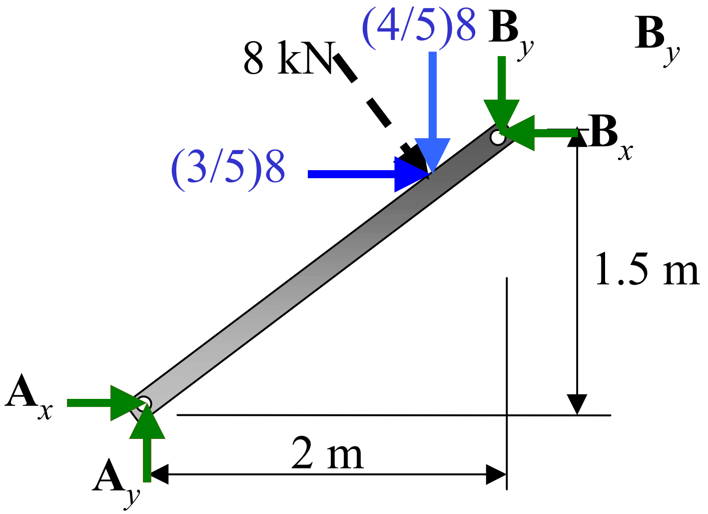
$A_x = 9.87 \text{ kN}, \rightarrow$

$+\uparrow \; \Sigma F_y = 0:$

$A_y - (4/5)8 - 3 = 0$

$A_y = 9.4 \text{ kN}, \uparrow$





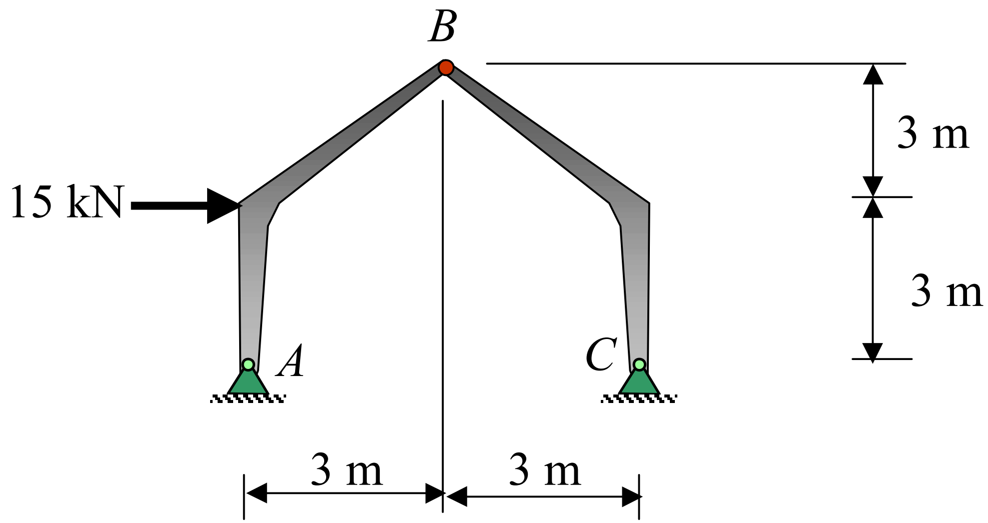
Member *BC*

$\xrightarrow{+} \; \Sigma F_x = 0: \quad C_x - B_x = 0; \; C_x = B_x = 14.7 \text{ kN}, \leftarrow$

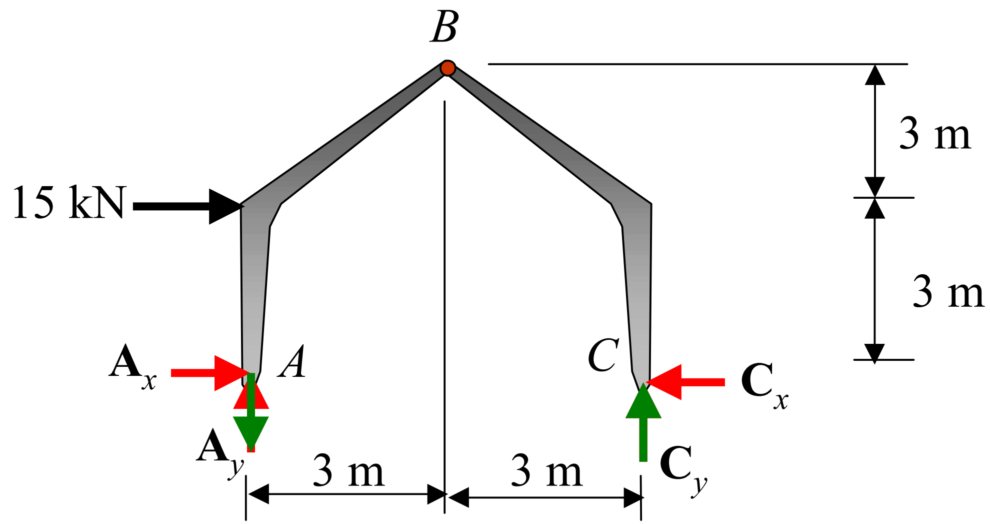
$+\uparrow \; \Sigma F_y = 0: \quad 3 - 6 + C_x = 0 \; ; \quad C_y = 3 \text{ kN}, \uparrow$

Example 2-11-1

From the figure below, determine the horizontal and vertical components of reaction at the pin connections *A*, *B*, and *C* of the supporting gable arch.



SOLUTION



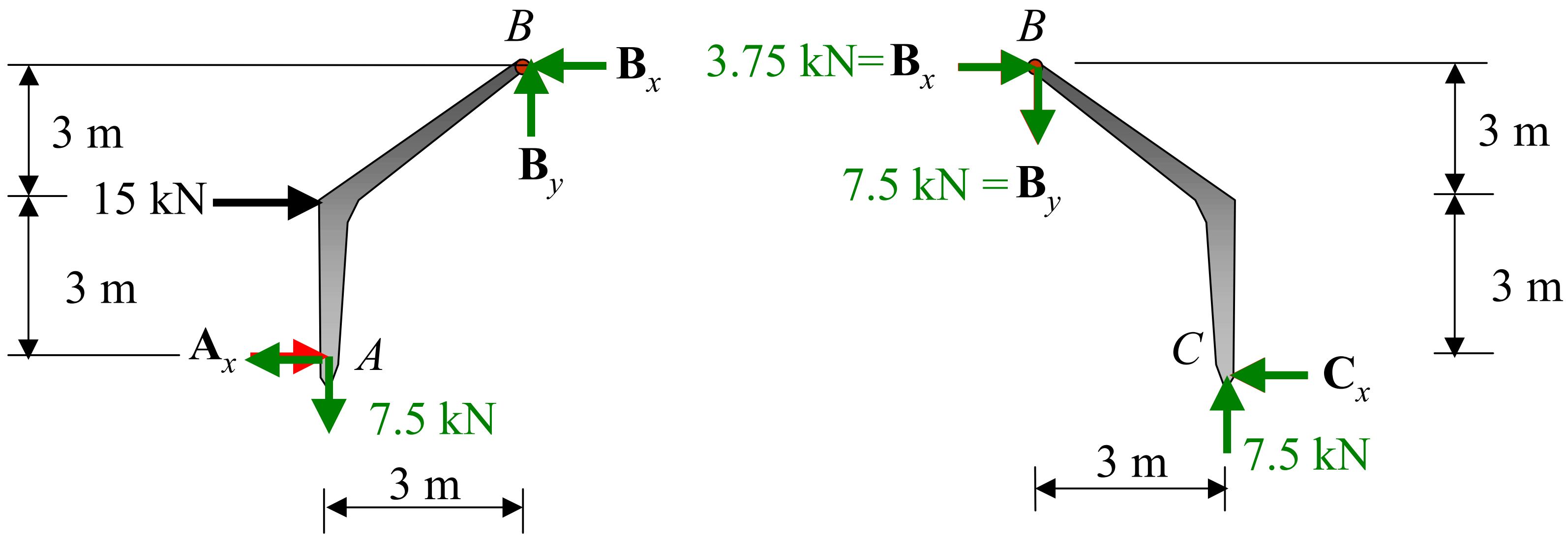
Entire Frame

$$\circlearrowright + \ \Sigma M_A = 0: \qquad C_y(6) - 15(3) = 0$$

$$C_y = 7.5 \text{ kN} \,, \uparrow$$

$$+\uparrow \ \Sigma F_y = 0: \qquad A_y + 7.5 = 0$$

$$A_y = \text{-}7.5 \text{ kN} \,, \downarrow$$



Member *AB*

$$\overset{+}{\curvearrowright} \Sigma M_B = 0: \quad 15(3) + A_x(6) + 7.5(3) = 0$$

$$A_x = \text{-}11.25 \text{ kN}, \leftarrow$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -11.25 + 15 - B_x = 0$$

$$B_x = 3.75 \text{ kN}, \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad -7.5 + By = 0$$

$$B_y = 7.5 \text{ kN}$$

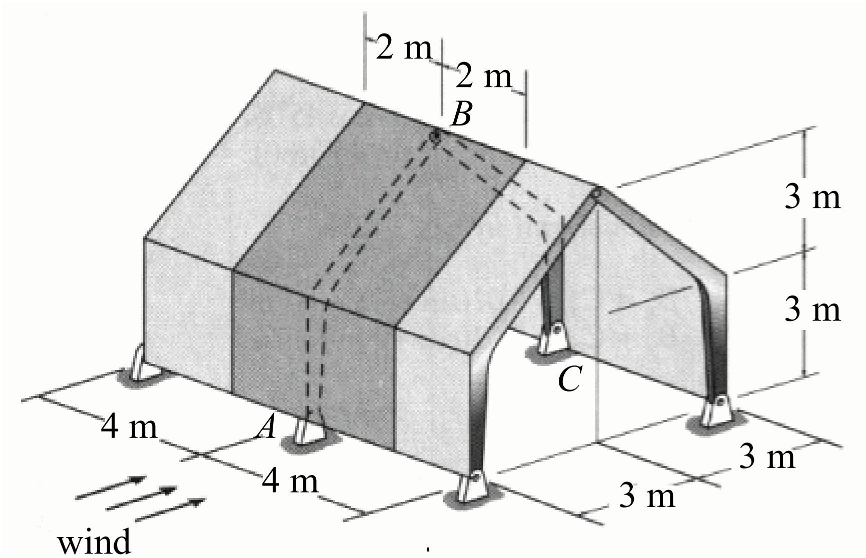
Member *BC*

$$\overset{+}{\rightarrow} \Sigma \text{F}_x = 0: \quad 3.75 - C_x = 0$$

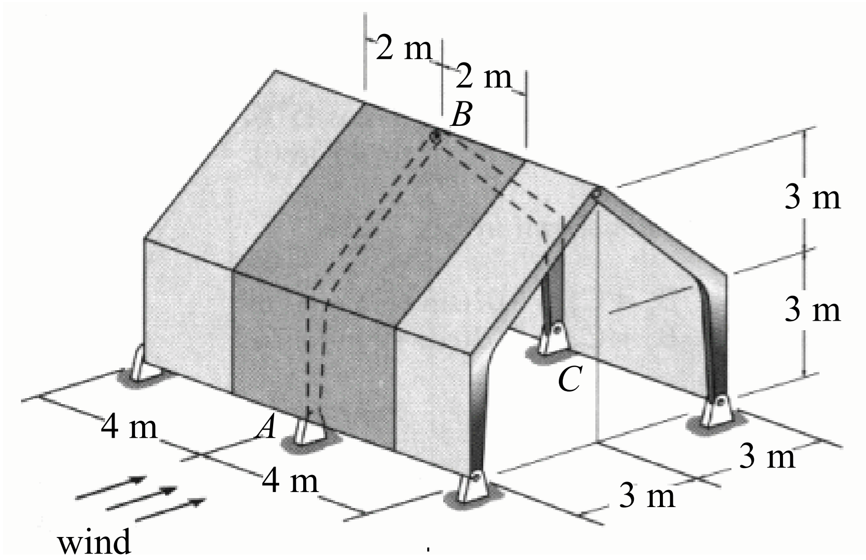
$$C_x = 3.75 \text{ kN}$$

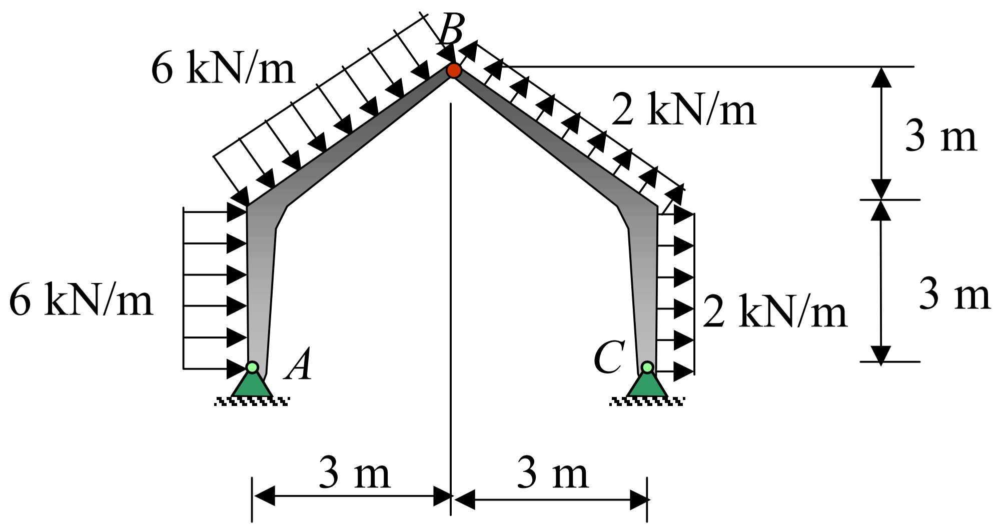
Example 2-11-2

The side of the building in the figure below is subjected to a wind loading that creates a uniform *normal* pressure of 1.5 kPa on the windward side and a suction pressure of 0.5 kPa on the leeward side. Determine the horizontal and vertical components of reaction at the pin connections *A*, *B*, and *C* of the supporting gable arch.



SOLUTION



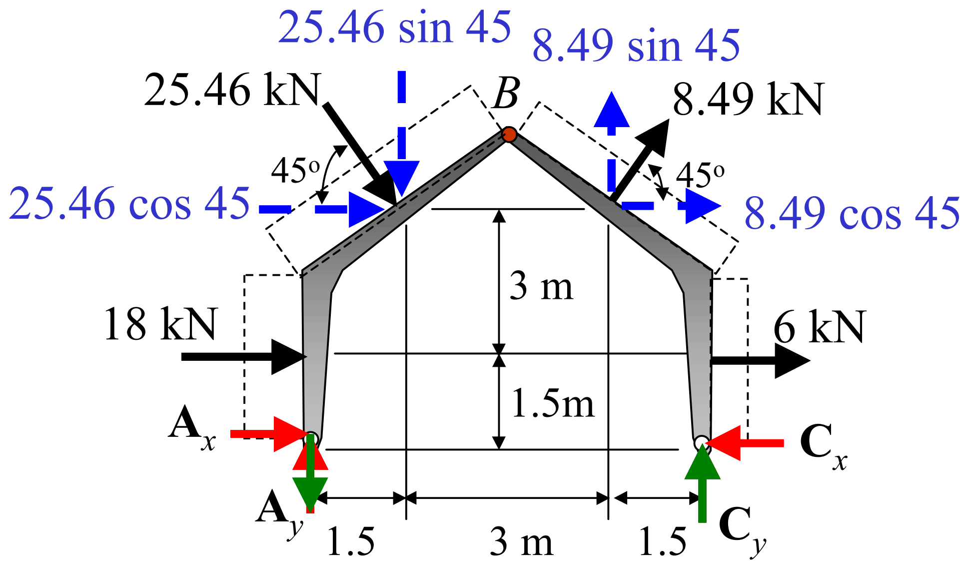


A uniform distributed load on the *windward* side is

$$(1.5\ \text{kN/m}^2)(4\ \text{m}) = 6\ \text{kN/m}$$

A uniform distributed load on the *leeward* side is

$$(0.5\ \text{kN/m}^2)(4\ \text{m}) = 2\ \text{kN/m}$$



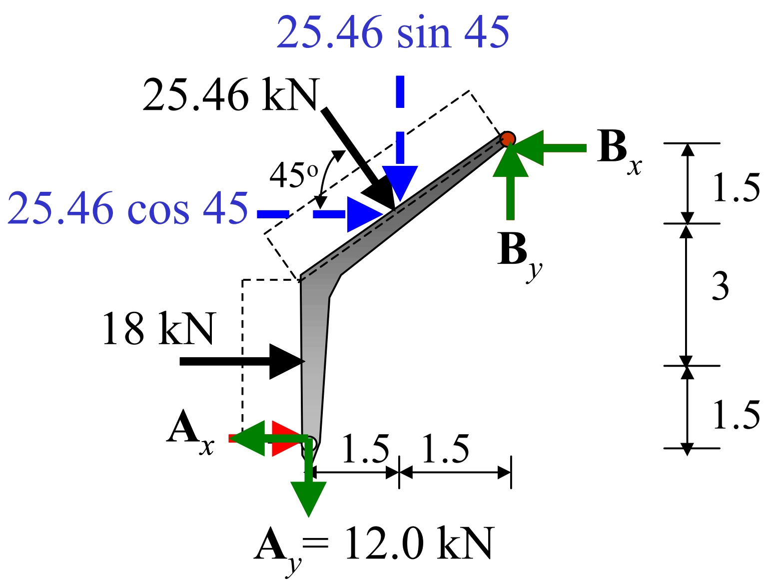
Entire Frame

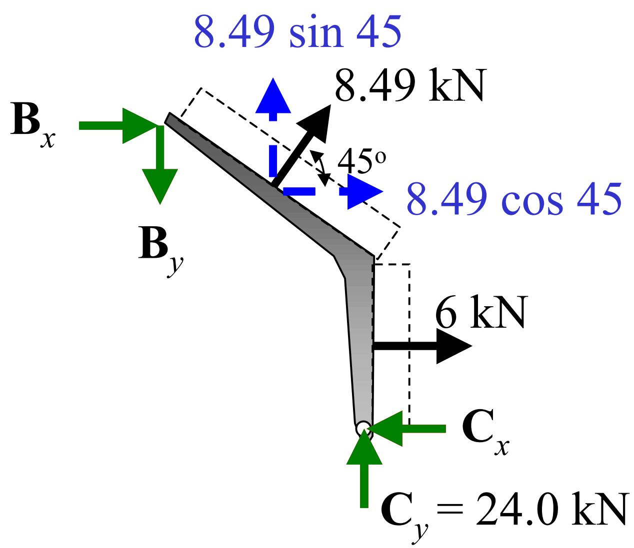
$$+\circlearrowright \Sigma M_A = 0: \quad -(18+6)(1.5) - (25.46+8.49)\cos 45^{o}(4.5) - (25.46 \sin 45^{o})(1.5) + (8.49 \sin 45^{o})(4.5) + C_y(6) = 0$$

$$C_y = 24.0 \text{ kN}, \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 25.46 \sin 45^{o} + 8.49 \sin 45^{o}\ 3 + 24 = 0$$

$$A_y = -12.0 \text{ kN} \quad \downarrow$$





Member *AB*

$$\overset{+}{\curvearrowright} \Sigma M_B = 0: \quad (25.46 \sin 45^{\circ})(1.5) + (25.46 \cos 45^{\circ})(1.5) + (18)(4.5) + A_x(6) + 12(3) = 0$$

$$A_x = -28.5 \text{ kN} \quad \longleftarrow$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad -28.5 + 18 + 25.46 \cos 45^{\circ} - B_x = 0$$

$$B_x = 7.5 \text{ kN}, \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad -12 - 25.46 \sin 45^{\circ} + B_y = 0$$

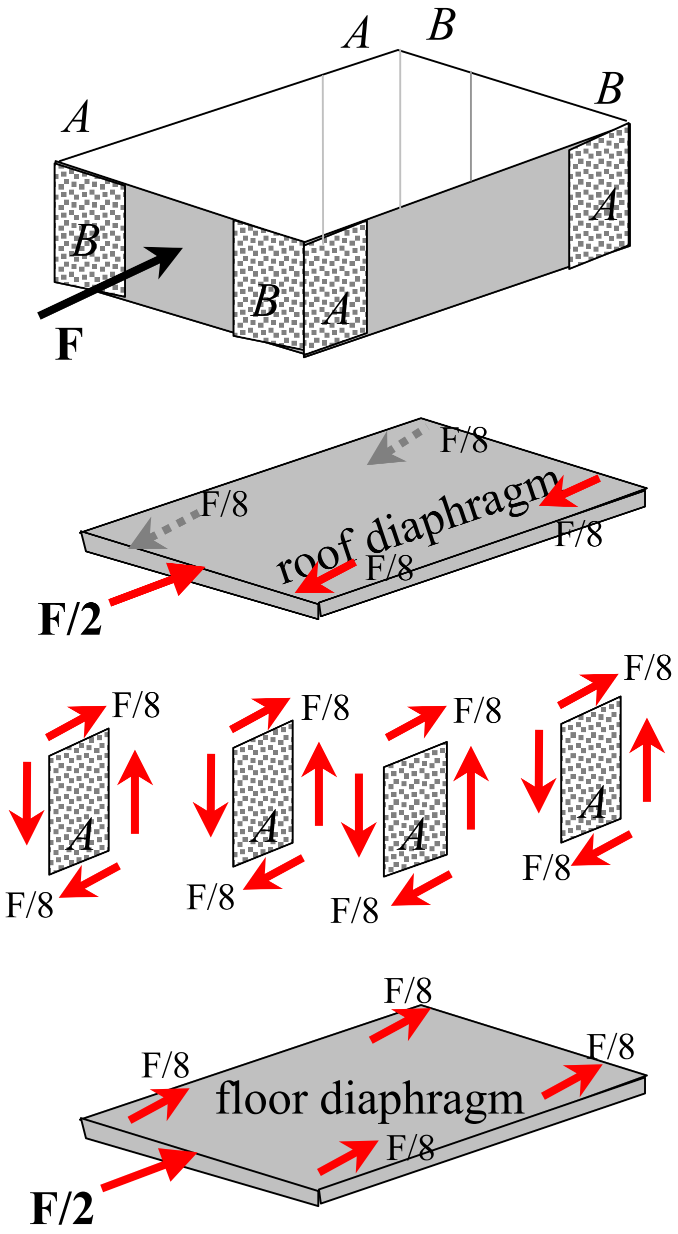
$$B_y = 30.0 \text{ kN}, \uparrow$$

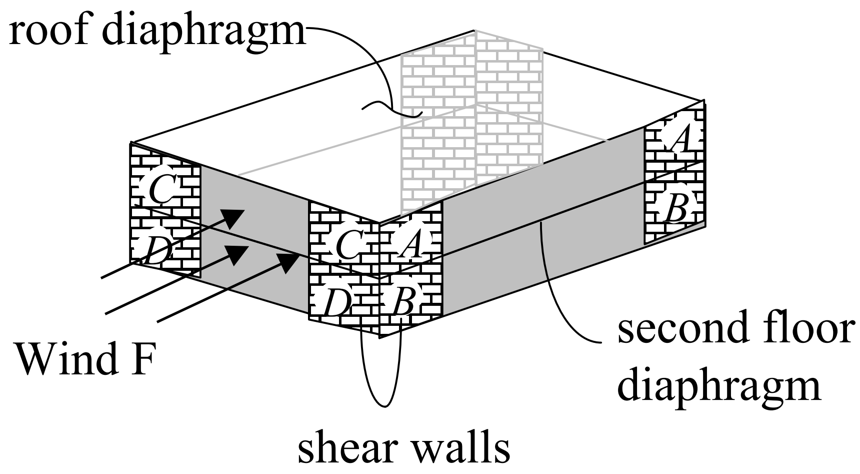
Member *CB*

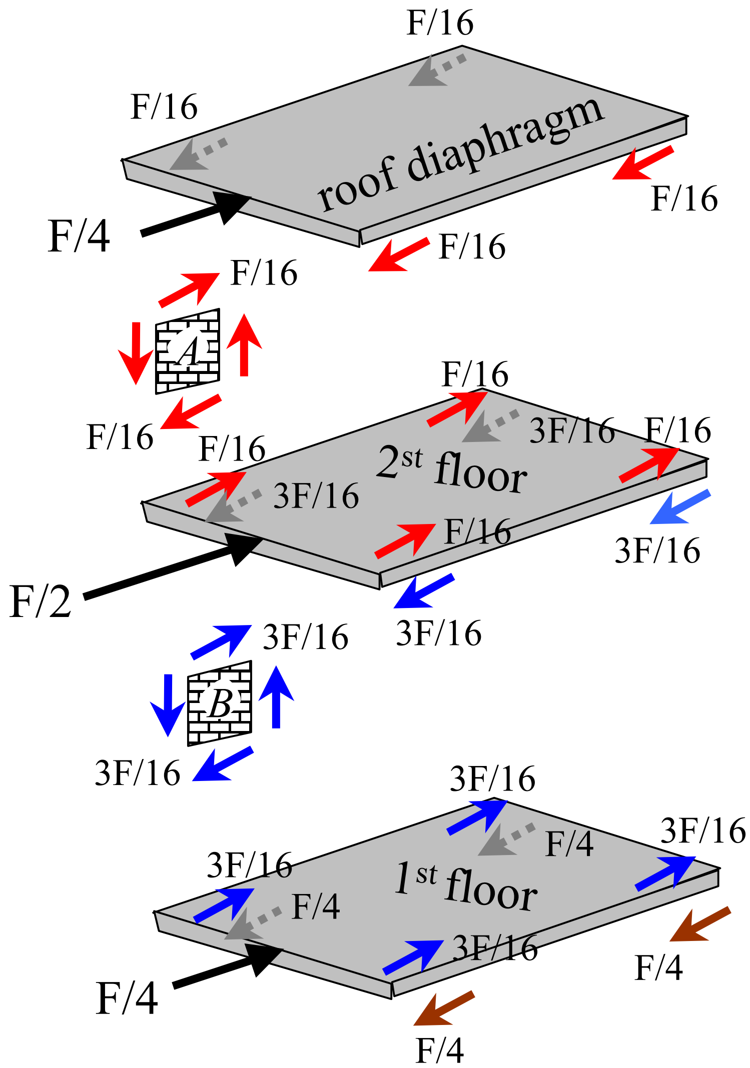
$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad 7.5 + 8.49 \cos 45^{\circ} + 6 - C_x = 0$$

$$C_x = 19.50 \text{ kN}, \leftarrow$$

Analysis of Simple Diaphragm and shear Wall Systems

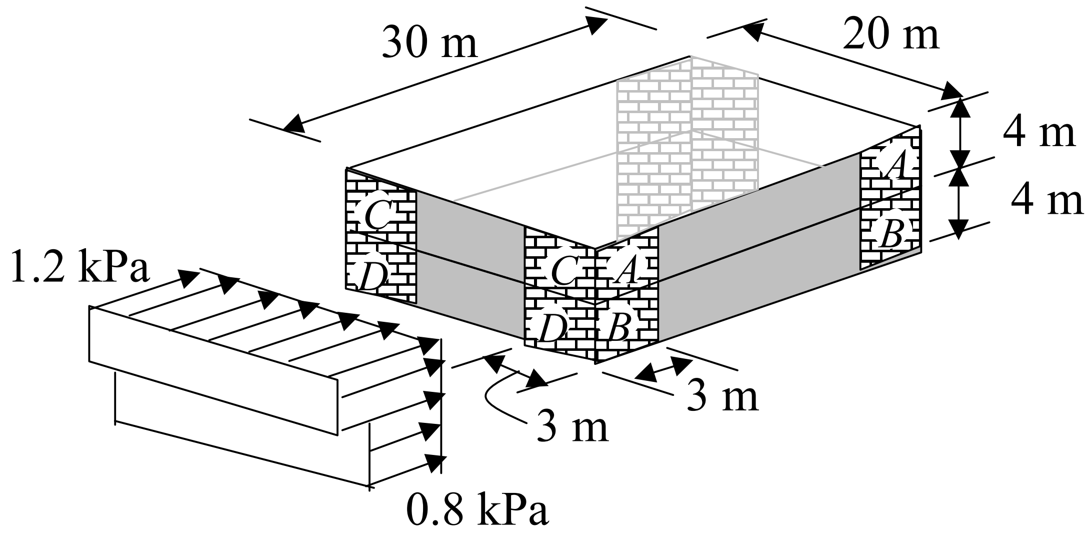



roof diaphragm
A
B
C
D
C
A
D
B
Wind F
second floor
diaphragm
shear walls

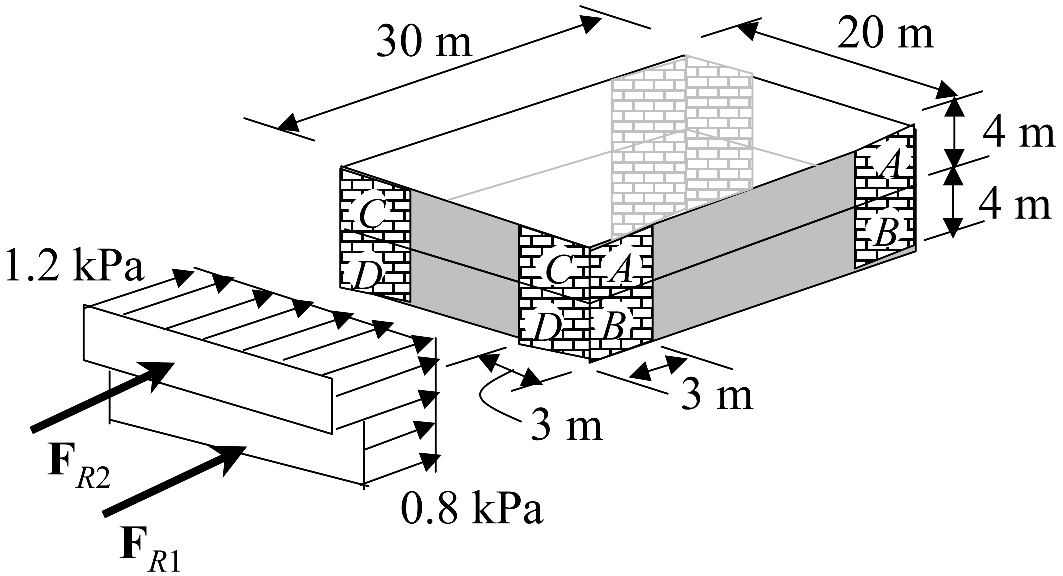

F/16
F/16
roof diaphragm
F/16
F/4
F/16
F/16
A
F/16
F/16
F/16
3F/16
F/16
3F/16
2st floor
F/16
3F/16
F/2
3F/16
3F/16
B
3F/16
3F/16
3F/16
F/4
3F/16
1st floor
F/4
3F/16
F/4
F/4
F/4

Example 2-12

Assume the wind loading acting on one side of a two-story building is as shown in the figure below. If shear walls are located at each of the corners as shown and flanked by columns, determine the shear in each panel located between the floors and the shear along the columns.



SOLUTION

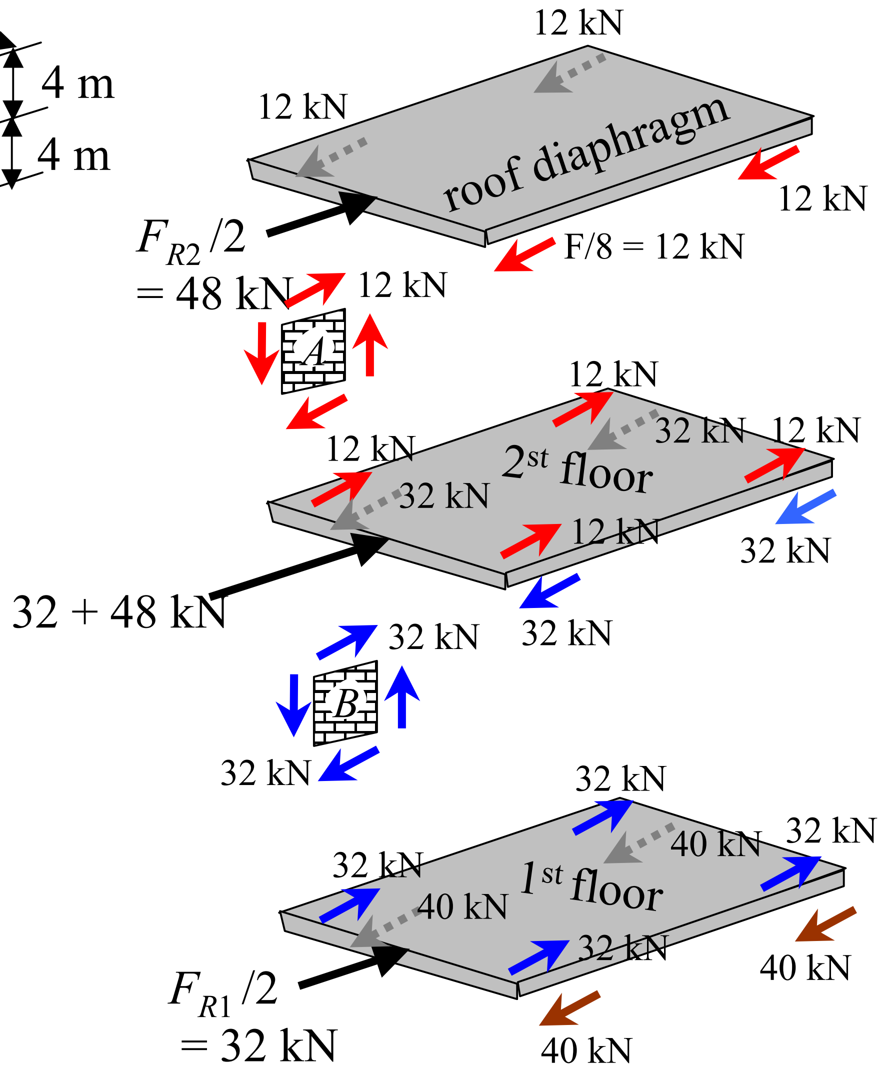


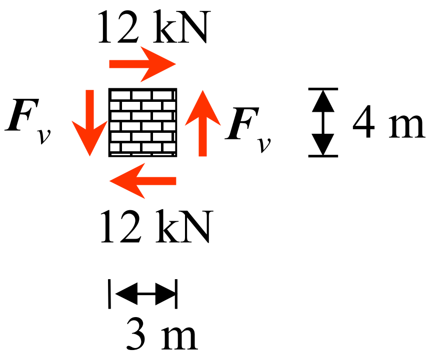
$$F_{R1} = 0.8(10^3)\ \text{N/m}^2\,(20\ \text{m})(4\ \text{m}) = 64\ \text{kN}$$

$$F_{R1}/2 = 32$$

$$F_{R2} = 1.2(10^3)\ \text{N/m}^2\,(20\ \text{m})(4\ \text{m}) = 96\ \text{kN}$$

$$F_{R2}/2 = 48$$

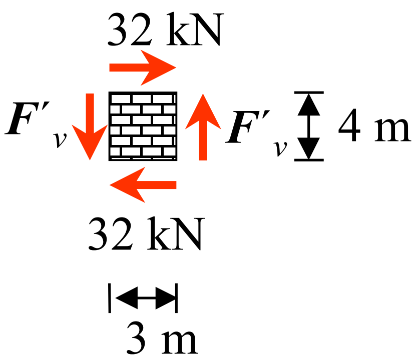




$+\circlearrowright \Sigma M = 0$:

$F_v(3) - 12(4) = 0$

$F_v = 16$ kN



$+\circlearrowright \Sigma M = 0$:

$F'_v(3) - 32(4) = 0$

$F'_v = 42.7$ kN