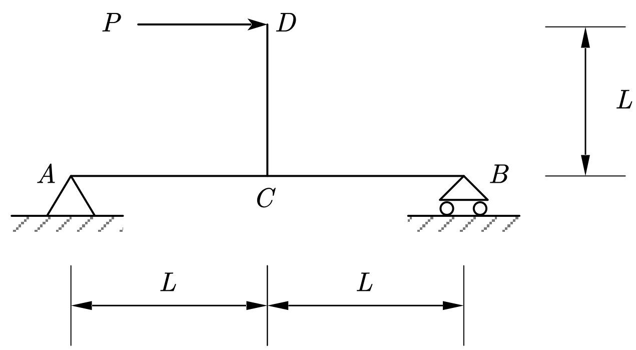
University of Maryland Fall 2018

ENCE353: Introduction to Structural Analysis

In-Class Problems #5 Solution

Consider a T shape structure shown below:

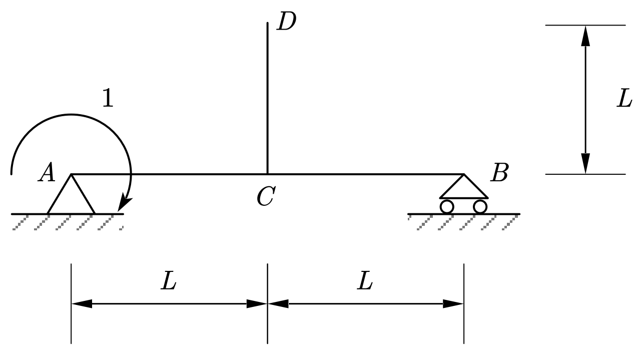


Assuming EI is constant for the structure and EA is sufficient large.

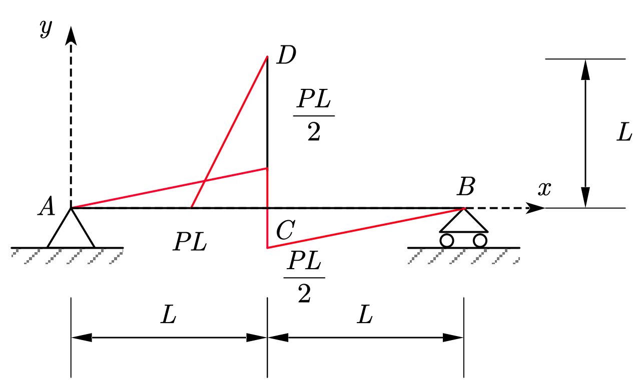
(1) Use principle of virtual work to calculate the rotation at A;

Solution:

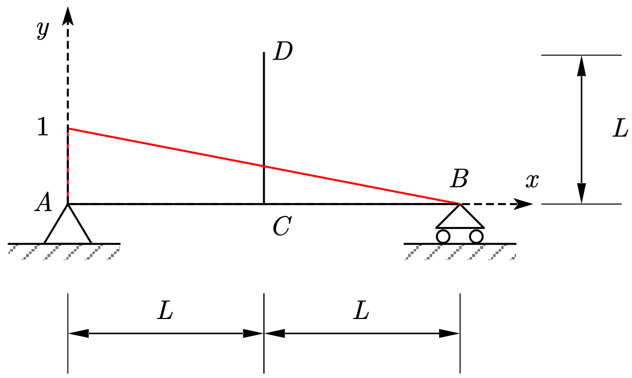
Apply a virtual unit moment at A and draw the moment diagram for real load and virtual load respectively:



Bending Moment Diagram (real load):



Bending Moment Diagram (virtual load):



Assume the origin of the coordinate system is at A:

$$m(x)=\frac{-1}{2L}x+1,\ 0\leqslant x\leqslant 2L$$

$$M(x)=\begin{cases}\frac{P}{2}x,\ 0\leqslant x\leqslant L\\ \frac{-P}{2}x+PL,\ L\leqslant x\leqslant 2L\end{cases}$$

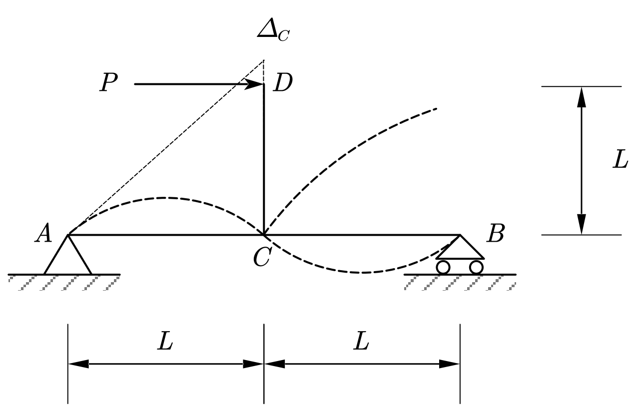
$$\theta_A=\int_0^{2L}\frac{m(x)M(x)}{EI}dx$$

$$=\frac{1}{EI}\left(\int_0^L\left(\frac{-1}{2L}x+1\right)\cdot\left(\frac{P}{2}x\right)dx-\int_L^{2L}\left(\frac{-1}{2L}x+1\right)\cdot\left(\frac{-P}{2}x+PL\right)dx\right)$$

$$=\frac{1}{EI}\left(\frac{PL^2}{6}-\frac{PL^2}{12}\right)=\frac{PL^2}{12EI}$$

(2) Use method of moment area to calculate the rotation at A.

Solution:



$$\Delta_C=\frac{\frac{1}{2}\cdot\frac{PL}{2}\cdot L\cdot\frac{L}{3}}{EI}=\frac{PL^3}{12EI}$$

$$\theta_A=\frac{\Delta_C}{L}=\frac{PL^2}{12EI}$$