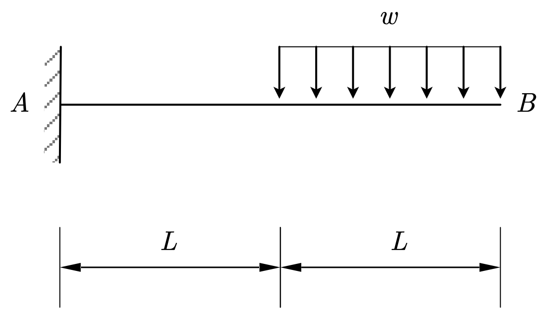
University of Maryland Fall 2018

ENCE353: Introduction to Structural Analysis

In-Class Problems #4 Solution

A cantilever beam is shown below, assuming the EI is constant along the beam:



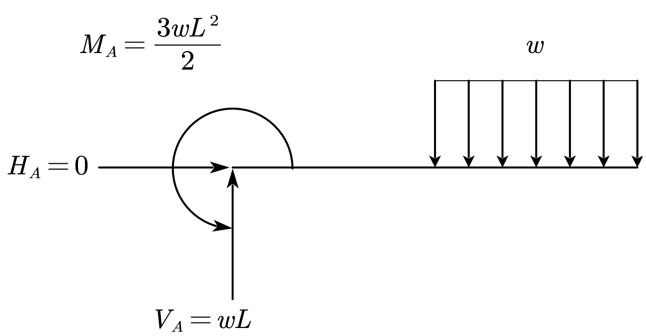
(1) Derive the deflection curve y(x) based on *elastic beam theory* (i.e. $d^2y/dx^2=M(x)/EI$);

(2) Based on the results of (1), calculate the deflection y(x=2L) and the angle of tangent (or slope) at point B (θ_B);

(3) Use *the method of moment-area* to verify your results from (2).

Solution:

(1-2) Draw the free body diagram of the beam and replace the boundary conditions with reactions:



For interval 0<=x<=L,

$$M(x) = \frac{3wL^2}{2} - wLx$$

$$\Rightarrow \frac{dy}{dx} = \int \frac{M(x)}{EI} dx = \frac{3wL^2}{2} x - \frac{wL}{2} x^2 + A$$

$$\because \frac{dy}{dx}|_{x=0} = 0$$

$$\therefore A = 0$$

$$\Rightarrow y(x) = \int \int \frac{M(x)}{EI} dx\, dx = \frac{3wL^2}{4} x^2 - \frac{wL}{6} x^3 + B$$

$$\because y(0) = 0$$

$$\therefore B = 0$$

$$\Rightarrow y(x) = \frac{3wL^2}{4}x^2 - \frac{wL}{6}x^3$$

$$y(L) = \frac{7}{12}wL^4,\ \frac{dy}{dx}|_{x=L} = wL^3$$

For interval L<=x<=2L

$$M(x) = \frac{3wL^2}{2} - wLx + \frac{1}{2}w(x-L)^2 = 2wL^2 - 2wLx + \frac{1}{2}wx^2$$

$$\Rightarrow \frac{dy}{dx} = \int \frac{M(x)}{EI}dx = 2wL^2x - wLx^2 + \frac{1}{6}wx^3 + C$$

$$\because \frac{dy}{dx}|_{x=L} = wL^3$$

$$\therefore 2wL^3 - wL^3 + \frac{1}{6}wL^3 + C = wL^3 \Rightarrow C = -\frac{1}{6}wL^3$$

$$\Rightarrow y(x) = \int\int \frac{M(x)}{EI}dx\,dx = wL^2x^2 - \frac{wL}{3}x^3 + \frac{1}{24}wx^4 - \frac{1}{6}wL^3x + D$$

$$\because y(L) = \frac{7}{12}wL^4$$

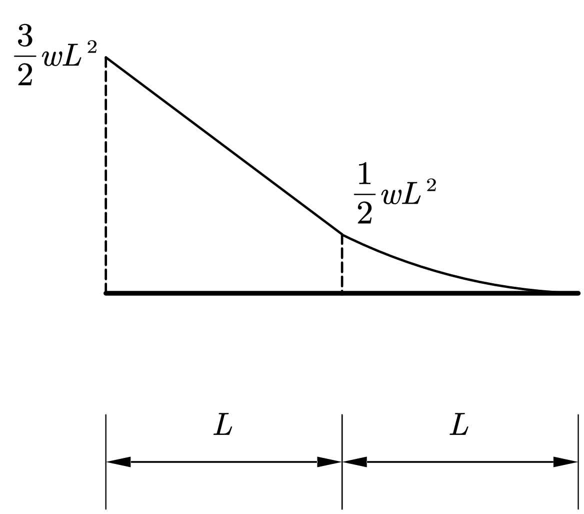
$$\therefore wL^4 - \frac{wL^4}{3} + \frac{1}{24}wL^4 - \frac{1}{6}wL^4 + D = \frac{7}{12}wL^4 \Rightarrow D = \frac{1}{24}wL^4$$

$$\Rightarrow y(x) = wL^2x^2 - \frac{wL}{3}x^3 + \frac{1}{24}wx^4 - \frac{1}{6}wL^3x + \frac{1}{24}wL^4$$

$$\Rightarrow \Delta_B = y(2L) = 4wL^4 - \frac{8wL^4}{3} + \frac{16}{24}wL^4 - \frac{2}{6}wL^4 + \frac{1}{24}wL^4 = \frac{41}{24}wL^4$$

$$\Rightarrow \theta_B = \frac{dy}{dx}\bigg|_{x=2L} = 4wL^3 - 4wL^3 + \frac{8}{6}wL^3 - \frac{1}{6}wL^3 = \frac{7}{6}wL^3$$

(3) Draw the moment diagram:



Calculate the deflection at point B:

$$\Delta_B = \frac{\frac{1}{2} \cdot L \cdot wL^2}{EI} \cdot \left(\frac{2}{3}L + L\right) + \frac{L \cdot \frac{1}{2}wL^2}{EI} \cdot \left(\frac{1}{2}L + L\right) + \frac{\frac{1}{3} \cdot \frac{1}{2}wL^2 \cdot L}{EI} \cdot \frac{3}{4}L = \frac{41}{24}\frac{wL^4}{EI}$$

Calculate the slope at point B:

$$\theta_B = \left(\frac{3}{2}wL^2 + \frac{1}{2}wL^2\right)\frac{L}{2} + \frac{1}{3}L\frac{1}{2}wL^2 = \frac{7}{6}wL^3$$

Thus, the results are confirmed.