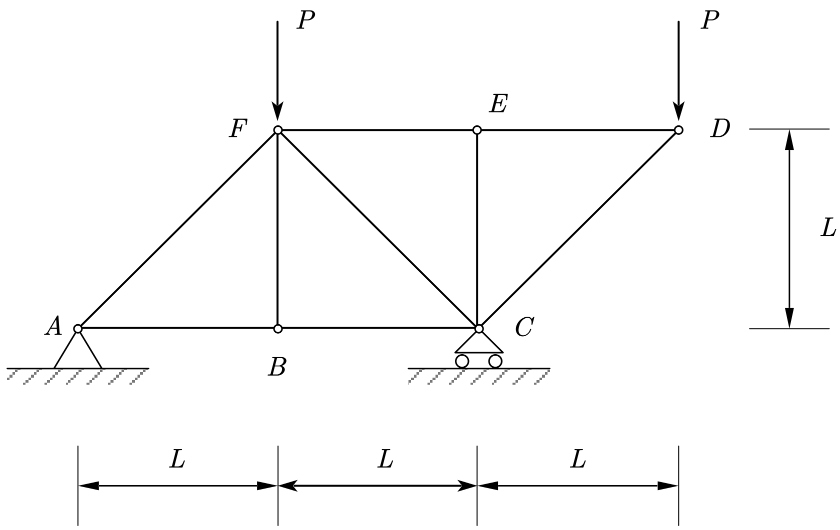
University of Maryland Fall 2018

ENCE353: Introduction to Structural Analysis

# Homework #5 Solution

**Problem 1:** Considering the truss structure shown below:




A vertical load of P kN is applied at nodes D and F. AE is constant for all truss members.

(1) Solve for the reactions at A and C.

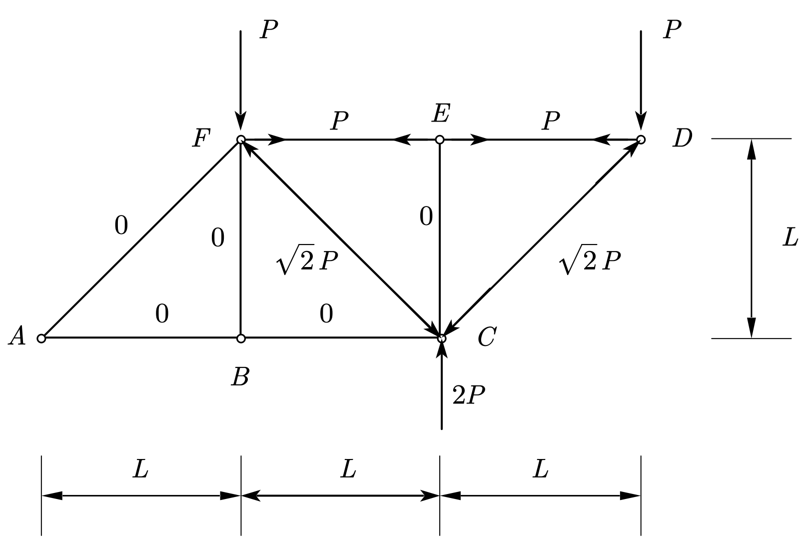
$$\sum F_x = 0,\ H_A = 0$$

$$\sum M = 0\ \ (@A),\ V_C \cdot 2L = P \cdot L + P \cdot 3L \Rightarrow V_C = 2P$$

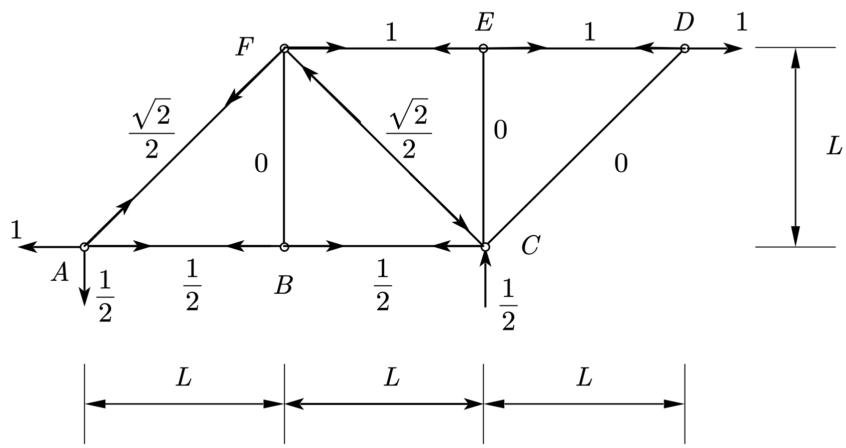
$$\sum F_x = 0,\ V_A + V_C = 2P \Rightarrow V_A = 0$$

(2) Use the method of virtual forces to show that the horizontal displacement at node D

(i) Draw the axial load diagram under real load:



(ii) Draw the axial load diagram under horizontal virtual load at D:




Use principle of virtual work to calculate the horizontal displacement at D:

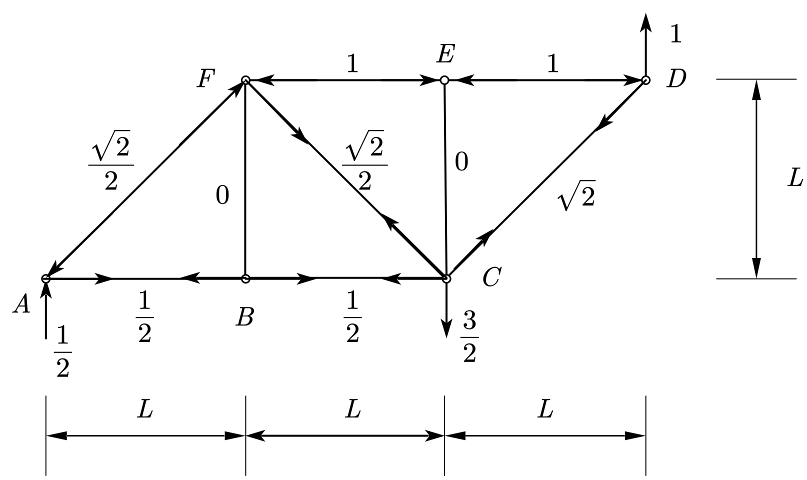
$$\Delta_D = \sum F_P \overline{F} L \frac{1}{EA}$$

$$= \frac{1}{EA} \cdot \left( P \cdot 1 \cdot L + P \cdot 1 \cdot L + \sqrt{2}\,P \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{2}\,L \right) = \frac{\left(2+\sqrt{2}\right)PL}{EA} \, (\rightarrow)$$

(3) Use the method of virtual forces to show that the vertical displacement at node D

Solution:

(iii) Draw the axial load diagram under vertical virtual load at D:

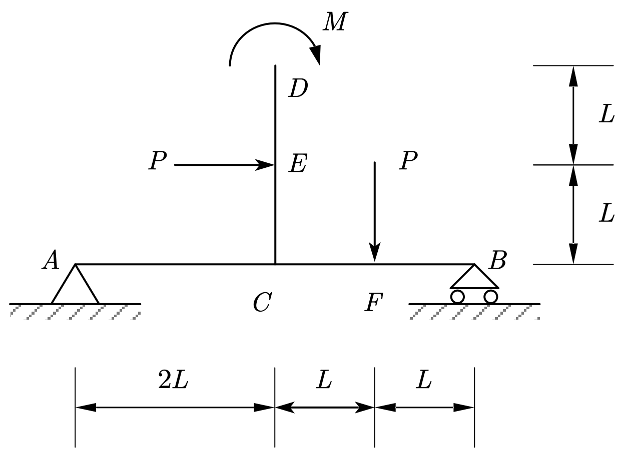



Use principle of virtual work to calculate the vertical displacement at D:

$$\Delta_D = \sum F_P \overline{F} L \frac{1}{EA}$$

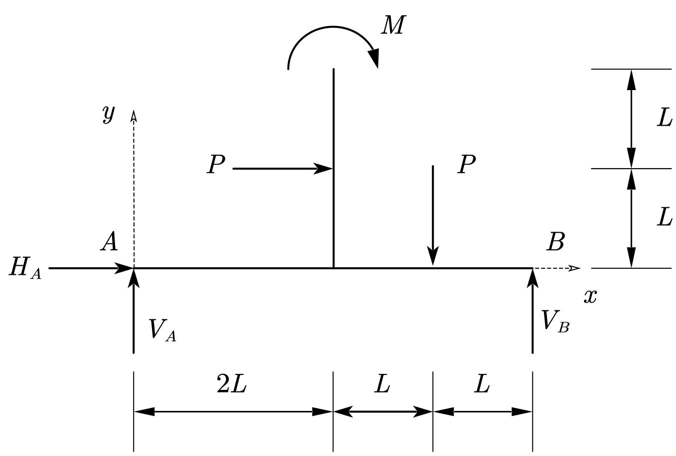
$$= \frac{1}{EA} \cdot \left( -P \cdot 1 \cdot L - P \cdot 1 \cdot L - \sqrt{2}\,P \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{2}\,L - \sqrt{2}\,P \cdot \sqrt{2} \cdot \sqrt{2}\,L \right) = - \frac{\left(2+3\sqrt{2}\right)PL}{EA} \, (\downarrow)$$

**Problem 2**: Consider the T-shaped structure shown below:




(1) Use static analysis method to calculate the vertical reactions at support A and B.

(i) Draw free body diagram, assuming the origin of the coordinate system is A:



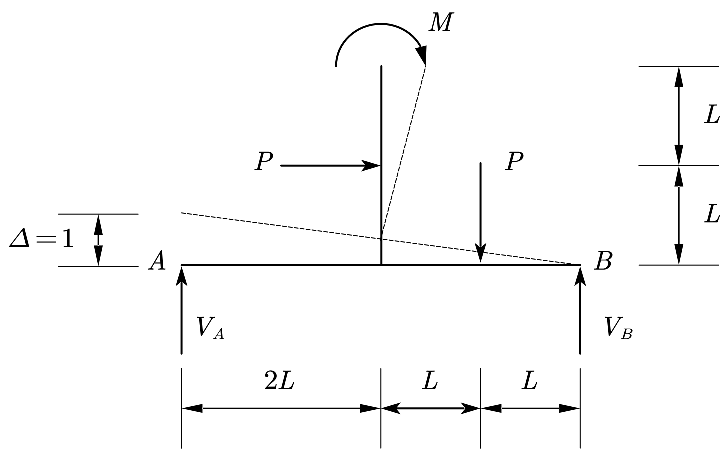

(ii) Equation of equilibrium:

$$\begin{cases} \sum F_x = 0, \ H_A + P = 0 \\ \sum F_y = 0, \ V_A + V_B - P = 0 \\ \sum M_A = 0, \ PL + P \cdot 3L + M - V_B \cdot 4L = 0 \end{cases}$$

$$\Rightarrow \begin{cases} H_A = -P \\ V_B = \dfrac{(4PL + M)}{4L} = P + \dfrac{M}{4L} \\ V_A = -\dfrac{M}{4L} \end{cases}$$

(2) Use the method of virtual displacements to calculate the vertical reactions at support A and B.

(i) Assign virtual displacement at A:

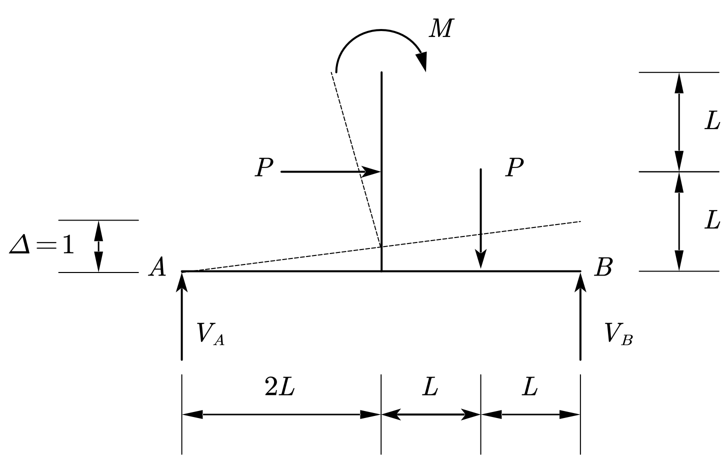



(ii) Use principle of virtual work to calculate the vertical reaction at A:

$$V_A \cdot 1 + P \cdot 1 \cdot \frac{L}{4L} - P \cdot 1 \cdot \frac{L}{4L} + M \cdot \frac{1}{4L} = 0$$

$$\Rightarrow V_A = -\frac{M}{4L}$$

(iii) Assign virtual displacement at B:



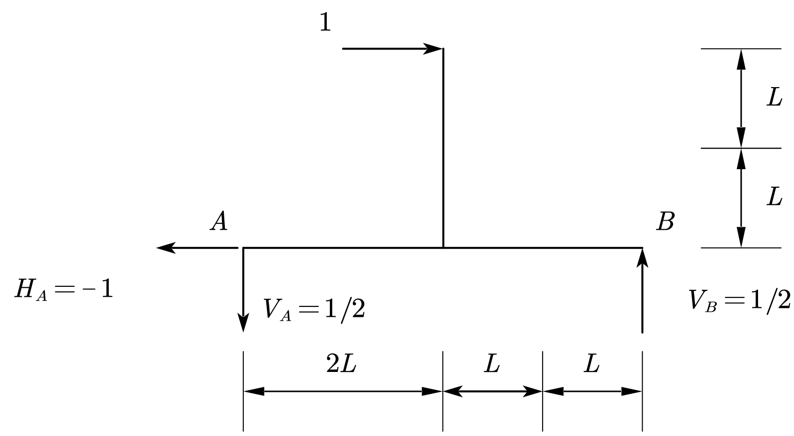

(iv) Use principle of virtual work to calculate the vertical reaction at B:

$$V_B \cdot 1 - P \cdot 1 \cdot \frac{L}{4L} - P \cdot 1 \cdot \frac{3L}{4L} - M \cdot \frac{1}{4L} = 0$$

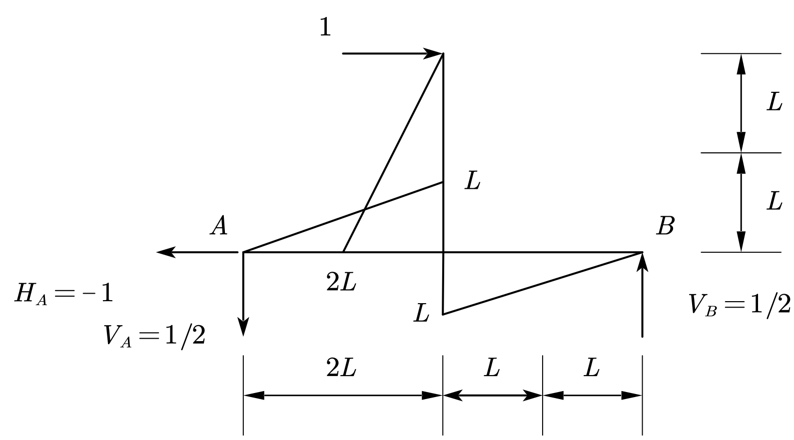
$$\Rightarrow V_B = P + \frac{M}{4L}$$

(3) Use the method of virtual force to calculate the horizontal displacement at D.

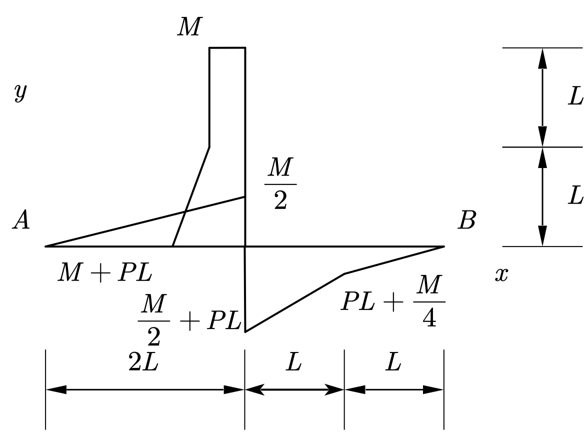
(i) Assign virtual force at D:




(ii) Draw the virtual moment diagram:




(iii) Draw the real moment diagram:



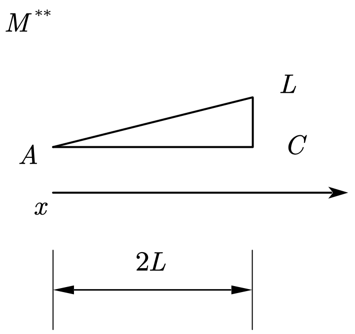

(iv) apply virtual force method:

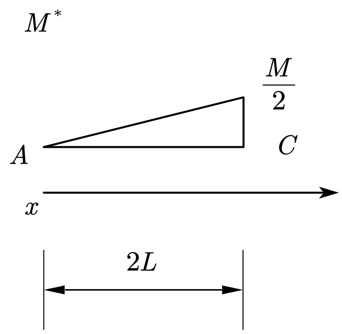
$$\Delta_D = \int M^* \frac{M^{**}}{EI}$$

$$= \int_{AB} M^* \frac{M^{**}}{EI} + \int_{CD} M^* \frac{M^{**}}{EI}$$

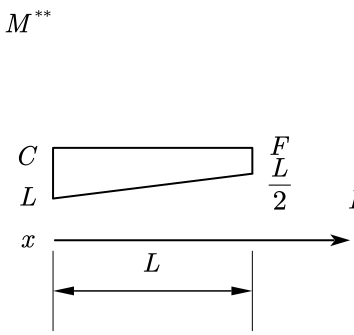
$$= \left(\int_{AC} M^* \frac{M^{**}}{EI} + \int_{CF} M^* \frac{M^{**}}{EI} + \int_{FB} M^* \frac{M^{**}}{EI}\right) + \left(\int_{CE} M^* \frac{M^{**}}{EI} + \int_{ED} M^* \frac{M^{**}}{EI}\right)$$

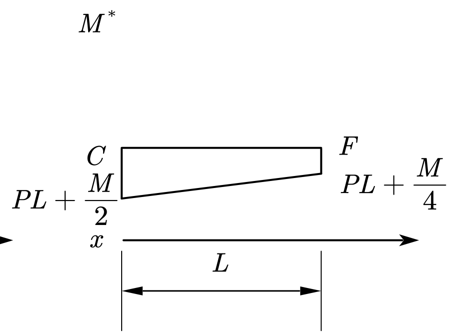
Solve for the integral one-by-one:



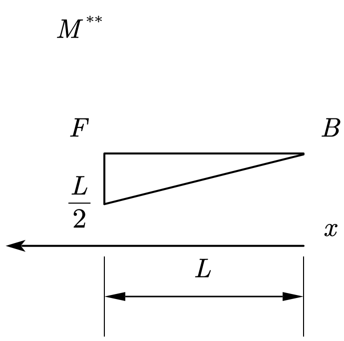



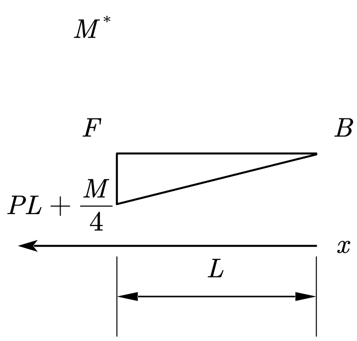

$$\int_{AC} M^* \frac{M^{**}}{EI} = \int_0^{2L} \frac{\frac{M}{2}}{2L} x \cdot \frac{\frac{x}{2}}{EI} dx = \frac{ML^2}{3EI}$$



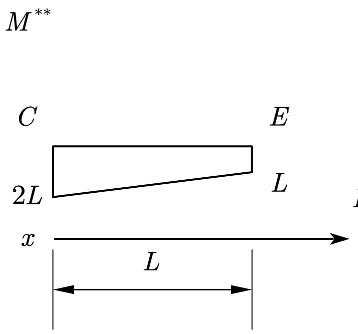



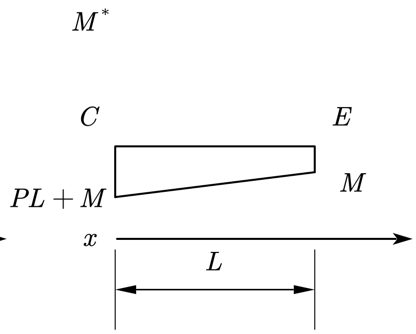

$$\int_{CF} M^* \frac{M^{**}}{EI} = \int_0^{L} \left(PL + \frac{M}{2} - \frac{M}{4L} x\right) \cdot \frac{L - \frac{x}{2}}{EI} dx = \frac{3PL^3}{4EI} + \frac{7ML^2}{24EI}$$



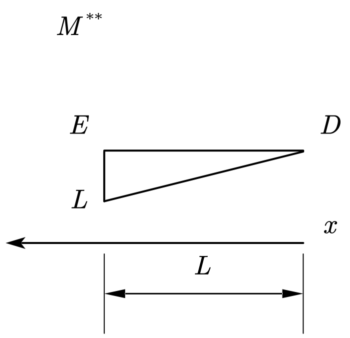



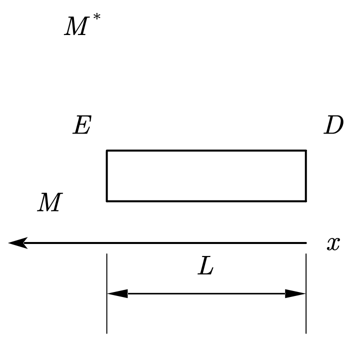

$$\int_{FB} M^* \frac{M^{**}}{EI} = \int_0^{L} \left(P + \frac{M}{4L}\right) x \cdot \frac{\frac{x}{2}}{EI} dx = \frac{PL^3}{6EI} + \frac{ML^2}{24EI}$$




$$\int_{CE} M^* \frac{M^{**}}{EI} = \int_0^L (PL + M - Px) \cdot \frac{(2L - x)}{EI} dx = \frac{5PL^3}{6EI} + \frac{3ML^2}{2EI}$$




$$\int_{ED} M^* \frac{M^{**}}{EI} = \int_0^L M \cdot \frac{x}{EI} dx = \frac{ML^2}{2EI}$$

As a result:

$$\Delta_D = \int M^* \frac{M^{**}}{EI}$$

$$= \int_{AB} M^* \frac{M^{**}}{EI} + \int_{CD} M^* \frac{M^{**}}{EI}$$

$$= \left(\int_{AC} M^* \frac{M^{**}}{EI} + \int_{CF} M^* \frac{M^{**}}{EI} + \int_{FB} M^* \frac{M^{**}}{EI}\right) + \left(\int_{CE} M^* \frac{M^{**}}{EI} + \int_{ED} M^* \frac{M^{**}}{EI}\right)$$

$$= \left(\frac{ML^2}{3EI} + \frac{3PL^3}{4EI} + \frac{7ML^2}{24EI} + \frac{PL^3}{6EI} + \frac{ML^2}{24EI}\right) + \left(\frac{5PL^3}{6EI} + \frac{3ML^2}{2EI} + \frac{ML^2}{2EI}\right)$$

$$= \frac{7PL^3}{4EI} + \frac{8ML^2}{3EI}$$