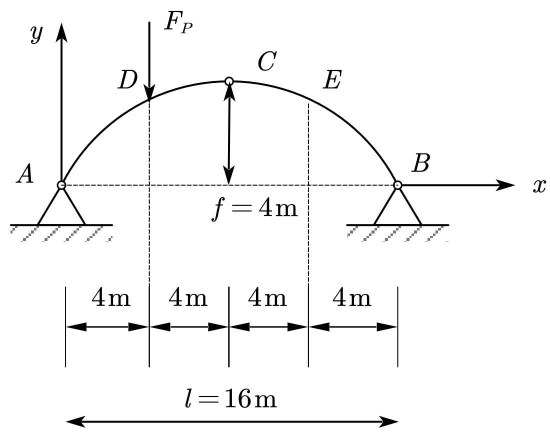
University of Maryland Fall 2018

ENCE353: Introduction to Structural Analysis

Homework #3 Solution

Problem 1: For the three-pin arc structure shown below, the profile is given by $y=\frac{4f}{l^2}x(l-x)$, where f = 4 m and l = 16 m.



(a) Calculate the reactions at A and B;

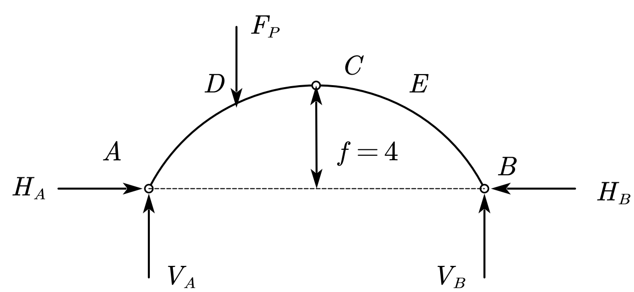
(b) Calculate the internal forces at point E (i.e. axial force F_N, shear force F_Q and bending moment M);

(c) Draw the moment diagram.

(d) if f = 8 m, redo the calculation (a), (b), and (c).

Solution:

(a) Draw the free body diagram of the arc structure:



$$\sum F_x=0,\ H_A=H_B$$

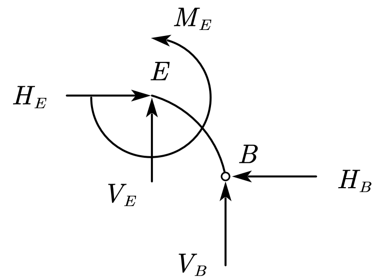
$$\sum F_y=0,\ V_A+V_B=F_p$$

$$\sum M_A=0,\ F_P\cdot 4-V_B\cdot 16=0\Rightarrow V_B=\frac{F_P}{4}(\uparrow)\ \Rightarrow V_A=\frac{3F_P}{4}(\uparrow)$$

Slice the arc structure at point C, and write the moment equilibrium about point C (left part) would get:

$$\sum M_C=0,\ -F_P\cdot 4+V_A\cdot 8-H_A\cdot 4=0\Rightarrow H_A=\frac{F_P}{2}(\rightarrow)\ \Rightarrow H_B=\frac{F_P}{2}(\leftarrow)$$

(b) Slice the arc structure at point E, and draw the free body diagram of the right part:

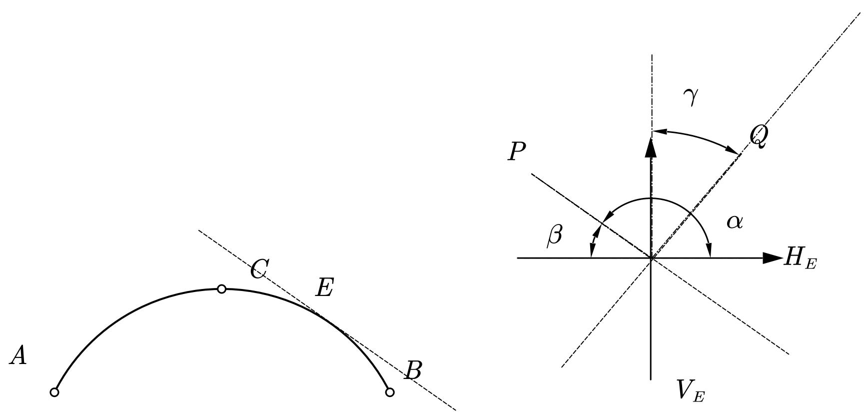


$$\sum F_x = 0,\ H_E = H_B \Rightarrow H_E = \frac{F_P}{2} (\rightarrow)$$

$$\sum F_y = 0,\ V_E + V_B = 0 \Rightarrow V_E = -\frac{F_P}{4} (\downarrow)$$

$$\sum M_E = 0,\ -M_E - V_B \cdot 4 + H_B \cdot y(12) = 0 \Rightarrow M_E = \frac{F_P}{2} (\curvearrowleft)$$

The axial force at point E is along with the tangent of the arc profile at point E and the shear force is perpendicular to the tangent of the arc profile. Thus, to calculate the axial and shear force, decomposition of H_E and V_E is needed:



$$\frac{dy}{dx}\Big|_{x=12} = \frac{4f}{l^2}(l - 2x)\Big|_{x=12} = -\frac{1}{2}$$

$$\tan\alpha = -\frac{1}{2} \Rightarrow \alpha = 153.43(\text{deg}) \Rightarrow \beta = 26.57(\text{deg}) \Rightarrow \gamma = \beta = 26.57(\text{deg})$$

$$\sin\beta = 0.447, \cos\beta = 0.894$$

$$P = V_E \sin\beta + H\cos\beta = \frac{F_P}{4} \cdot 0.447 + \frac{F_P}{2} \cdot 0.894 = 0.559F_P(\text{compression})$$

$$Q = -V_E\cos\beta + H\sin\beta = -\frac{F_P}{4} \cdot 0.894 + \frac{F_P}{2} \cdot 0.447 = 0$$

(c) Assume the moment at point x=x is positive in clockwise:

For section 0<=x<=4, write the moment equilibrium about point at x=x:

$$\sum M_{x=x}=0,\ V_A\cdot x-H_A\cdot y(x)+M(x)=0$$

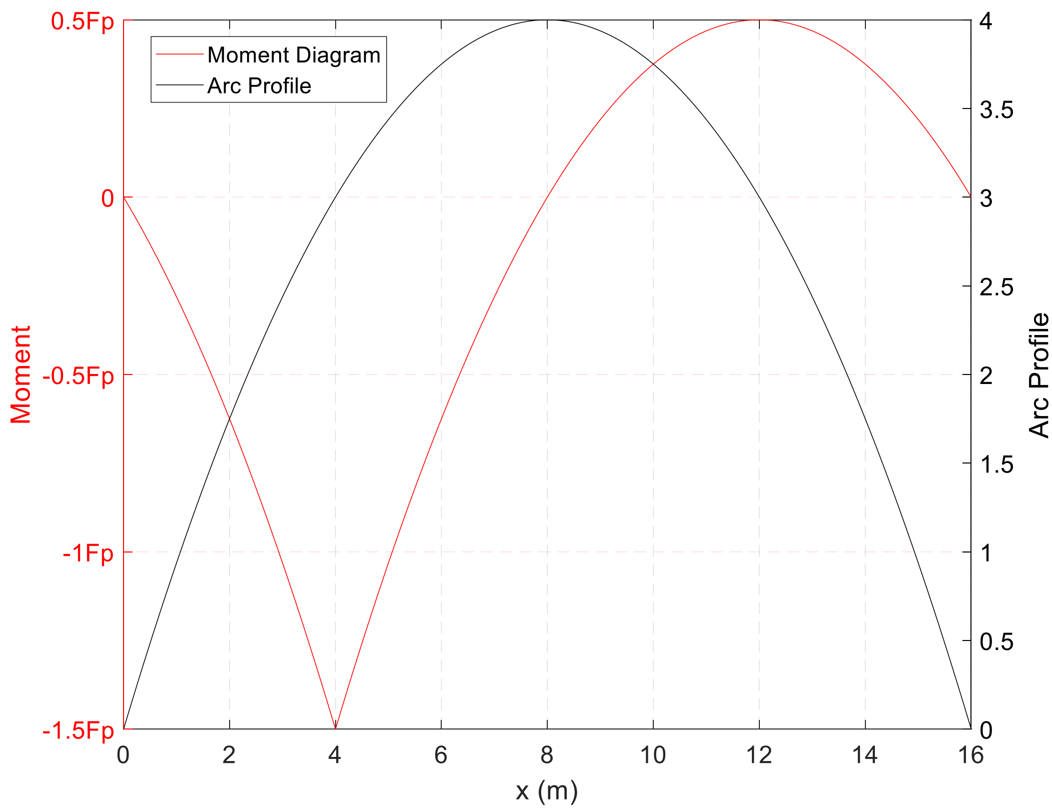
$$\Rightarrow M(x)=-\frac{3F_P}{4}x+\frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right)=-\frac{F_P}{32}x^2-\frac{F_P}{4}x$$

For section 4<=x<=16,

$$\sum M_{x=x}=0,\ V_A\cdot x-H_A\cdot y(x)-F_P\cdot(x-4)+M(x)=0$$

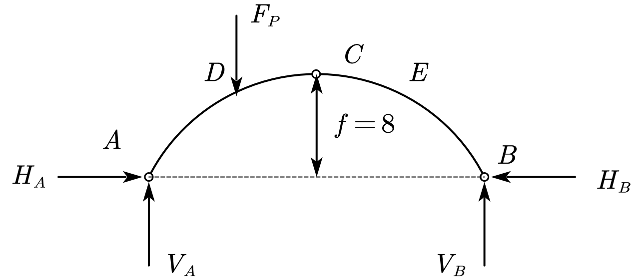
$$\Rightarrow M(x)=-\frac{3F_P}{4}x+\frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right)=-\frac{F_P}{32}x^2+\frac{3F_P}{4}x-4F_P$$

Draw the moment diagram (Assume the value of F_p >=0):



(d-a)

Draw the free body diagram of the arc structure:



$$\sum F_x = 0,\ H_A = H_B$$

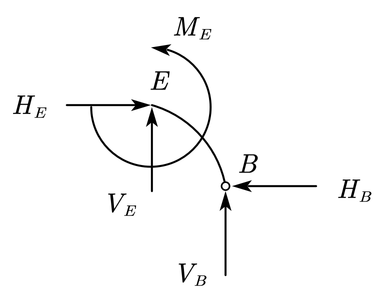
$$\sum F_y = 0,\ V_A + V_B = F_p$$

$$\sum M_A = 0,\ F_P \cdot 4 - V_B \cdot 16 = 0 \Rightarrow V_B = \frac{F_P}{4}(\uparrow) \Rightarrow V_A = \frac{3F_P}{4}(\uparrow)$$

Slice the arc structure at point C, and write the moment equilibrium about point C (left part) would get:

$$\sum M_C = 0,\ -F_P \cdot 4 + V_A \cdot 8 - H_A \cdot 8 = 0 \Rightarrow H_A = \frac{F_P}{4}(\rightarrow) \Rightarrow H_B = \frac{F_P}{4}(\leftarrow)$$

(d-b) Slice the arc structure at point E, and draw the free body diagram of the right part:

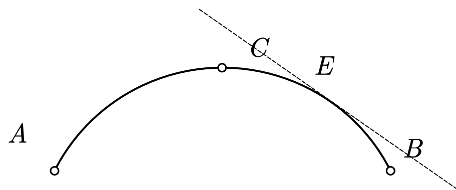


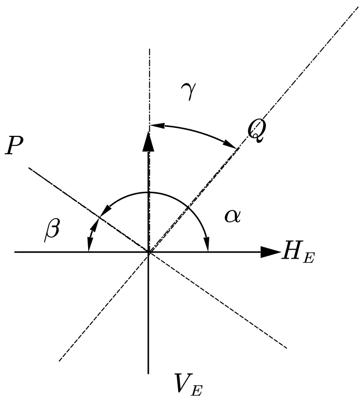
$$\sum F_x = 0,\ H_E = H_B \Rightarrow H_E = \frac{F_P}{4}(\rightarrow)$$

$$\sum F_y = 0,\ V_E + V_B = 0 \Rightarrow V_E = -\frac{F_P}{4}(\downarrow)$$

$$\sum M_E = 0,\ -M_E - V_B \cdot 4 + H_B \cdot y(12) = 0 \Rightarrow M_E = \frac{F_P}{2}(\curvearrowleft)$$

The axial force at point E is along with the tangent of the arc profile at point E and the shear force is perpendicular to the tangent of the arc profile. Thus, to calculate the axial and shear force, decomposition of H_E and V_E is needed:





$$\frac{dy}{dx}\Big|_{x=12} = \frac{4f}{l^2}(l-2x)\Big|_{x=12} = -1$$

$$\tan\alpha = -1 \Rightarrow \alpha = 135\,(\text{deg}) \Rightarrow \beta = 45\,(\text{deg}) \Rightarrow \gamma = \beta = 45\,(\text{deg})$$

$$\sin\beta = 0.707\,, \cos\beta = 0.707$$

$$P = V_E \sin\beta + H\cos\beta = \frac{F_P}{4}\cdot 0.707 + \frac{F_P}{4}\cdot 0.707 = 0.353F_P\,(\text{compression})$$

$$Q = -V_E\cos\beta + H\sin\beta = -\frac{F_P}{4}\cdot 0.707 + \frac{F_P}{4}\cdot 0.707 = 0$$

(d-c) Assume the moment at point x=x is positive in clockwise:

For section 0<=x<=4, write the moment equilibrium about point at x=x:

$$\sum M_{x=x} = 0,\ V_A\cdot x - H_A\cdot y(x) + M(x) = 0$$

$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right) = -\frac{F_P}{32}x^2 - \frac{F_P}{4}x$$

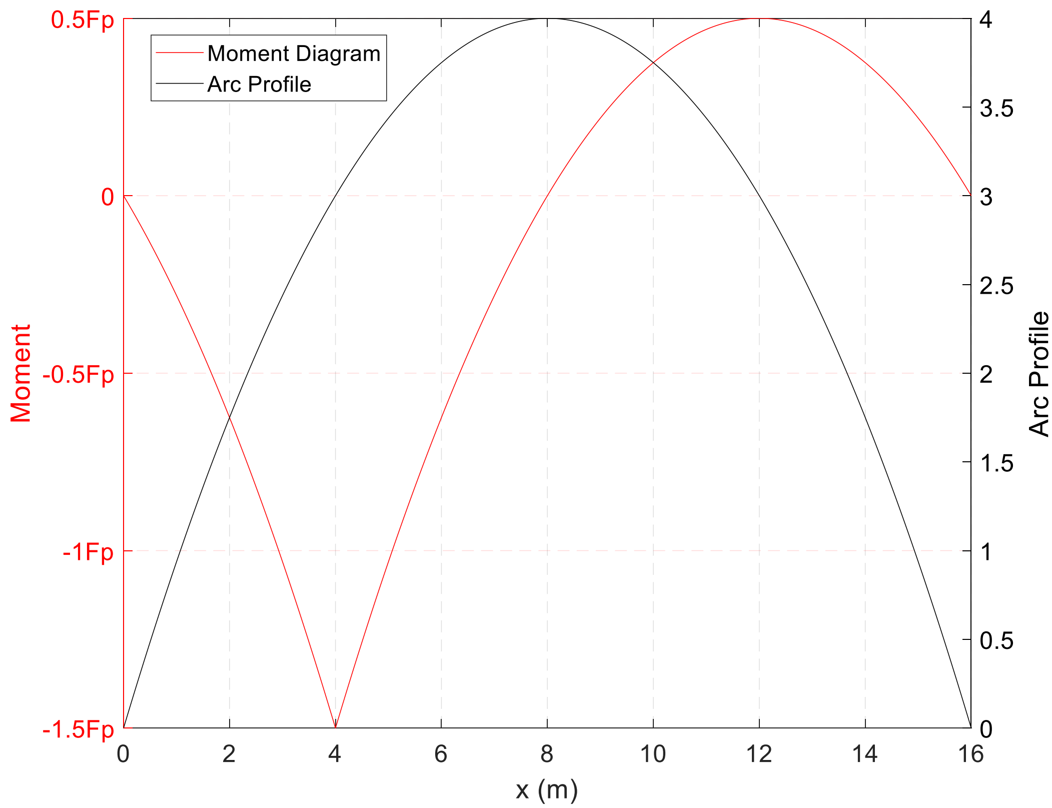
For section 4<=x<=16,

$$\sum M_{x=x} = 0,\ V_A\cdot x - H_A\cdot y(x) - F_P\cdot(x-4) + M(x) = 0$$

$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2}\left(\frac{4f}{l^2}x(l-x)\right) - F_P\cdot(x-4)$$

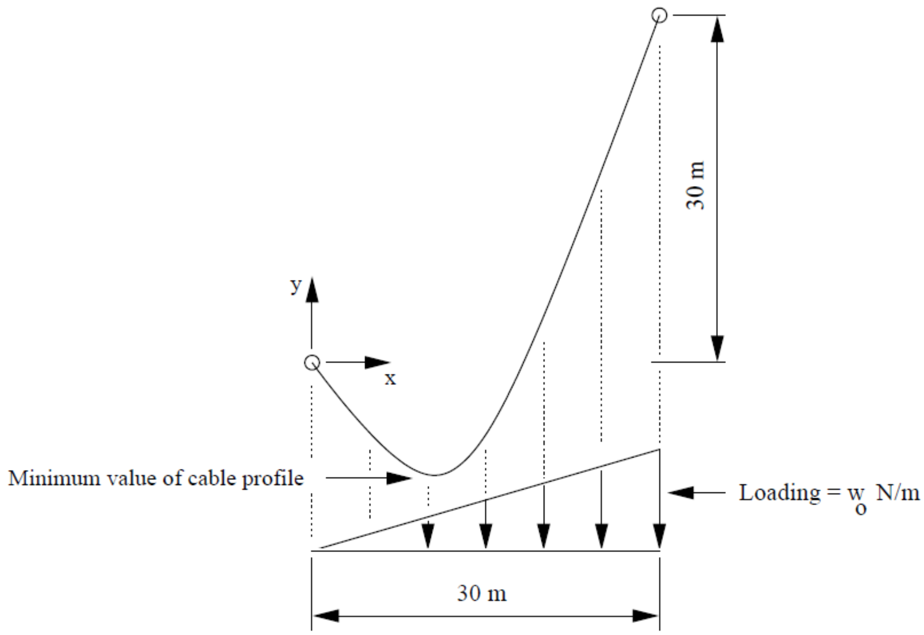
$$= -\frac{F_P}{32}x^2 + \frac{3F_P}{4}x - 4F_P$$

Draw the moment diagram (Assume the value of F_p >=0):



Moment Diagram
Arc Profile
0.5Fp
0
-0.5Fp
-1Fp
-1.5Fp
Moment
4
3.5
3
2.5
2
1.5
1
0.5
0
Arc Profile
0
2
4
6
8
10
12
14
16
x (m)

Problem 2 The cable structure shown below carries a triangular load that is zero at the left-hand support and increases to wo N/m at the right-hand support.



(a) Prove that the cable profile is govern by the equation:

$$y(x) = \frac{w_0 x^3}{180H} + \left(1 - \frac{5w_0}{H}\right)x$$

Solution:

$$\frac{d^2y}{dx^2} = \frac{w(x)}{H} = \frac{w_0 x}{30H}$$

$$\Rightarrow \frac{dy}{dx} = \frac{w_0 x^2}{60H} + A$$

$$\Rightarrow y(x) = \frac{w_0 x^3}{180H} + Ax + B$$

Apply $y(0) = 0,\ y(30) = 30$:

$$\begin{cases} B = 0 \\ A = 1 - \dfrac{5w_0}{H} \end{cases}$$

$$\Rightarrow y(x) = \frac{w_0 x^3}{180H} + \left(1 - \frac{5w_0}{H}\right)x$$

(b) Assume, the minimum value of the cable profile occurs at x=10 m, calculate the reactions at both ends.

Solution:
Note: Point A is the left end, and point B is the right end.

$$\frac{dy}{dx} = \frac{w_0 x^2}{60H} + 1 - \frac{5w_0}{H}$$

$$\left.\frac{dy}{dx}\right|_{x=10} = 0 \Rightarrow \frac{100w_0}{60H} + 1 - \frac{5w_0}{H} = 0 \Rightarrow H = \frac{10}{3} w_0$$

$$\sum M@A = 0, V_B \cdot 30 - H \cdot 30 - \frac{1}{2} \cdot 30 \cdot w_0 \cdot \frac{2}{3} \cdot 30 = 0 \Rightarrow V_B = \frac{40}{3} w_0$$

$$\sum F_y = 0,\ V_A = 15w_0 - V_B = \frac{5}{3} w_0$$