Department of Civil and Environmental Engineering, Fall Semester, 2017

ENCE 353 Midterm 2, Open Notes and Open Book

Name : ________________________

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please show all of your working.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 1: 15 points

Deriving Formulae for the Deflection of a Cantilever Beam. Consider the cantilever beam structure shown in Figure 2.

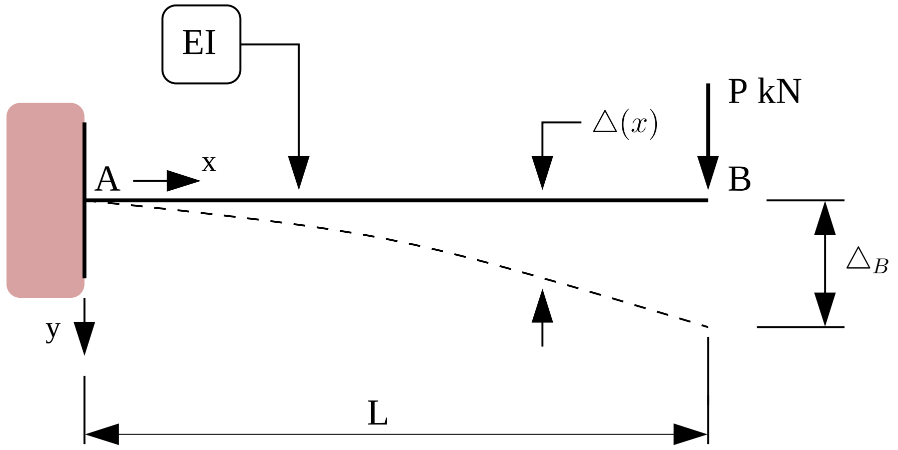

Figure 1: Front elevation view of a cantilever.

In this problem setup: (1) the beam is fully fixed at point A and the flexural stiffness EI is constant along the beam, (2) there is a vertical load P kN acting downwards at point B, and (1) the coordinate system is positioned at point A.

[1a] (6 pts) Use the method of **moment area** to show that:

$$\triangle(x) = \left(\frac{P}{6EI}\right) x^2 \left(3L - x\right). \tag{1}$$

Notice that when x = L, equation 1 gives the formula we have used in class.

Question 1a continued ....

[1b] (6 pts) Starting from the differential equation,

$$\frac{d^2y}{dx^2} = \left[\frac{M(x)}{EI}\right], \tag{2}$$

and appropriate boundary conditions, show that:

$$\triangle(x) = \left(\frac{P}{6EI}\right) x^2 \left(3L - x\right). \tag{3}$$

[1c] (3 pts) Derive a formula for the slope of the beam as a function of x. This is a one line solution. Use your formula to verify that the beam rotation at B is:

$$\theta_B = \left(\frac{PL^2}{2EI} \right). \tag{4}$$

Question 2: 15 points

Moment-Area and Deflections. Consider the cantilevered beam structure shown in Figure 2.

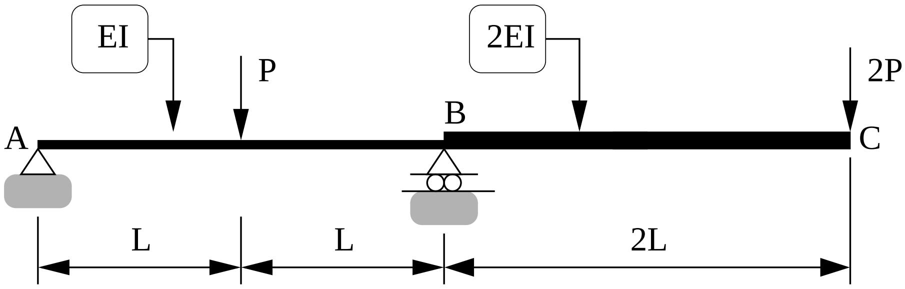

Figure 2: Front elevation view of a cantilevered beam structure.

Notice that segments A-B and B-C have cross-sectional properties EI and 2EI, respectively.

[2a] (3 pts) Compute and draw the M(x)/EI diagram for the complete beam A-B-C.

[2b] (4 pts) Draw and label a diagram of the deflected shape. Clearly indicate on your diagram regions (or points) of the beam having zero curvature.

[2c] (4 pts) Draw and label a diagram showing how the rotation at A is related to the beam deflections at points B and C.

[2d] (4 pts) Use the method of moment-area to compute the vertical deflection of the beam at point C.

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch. A vertical load P is applied at node D.

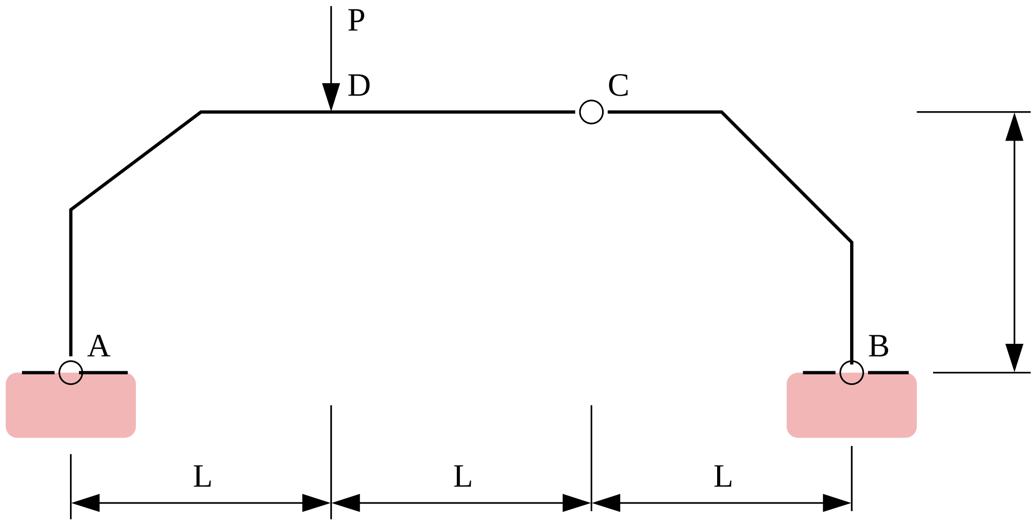

Figure 3: Front elevation view of a simple three-pinned arch.

[3a] (5 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of L and P.

[3b] (3 pts) Compute the magnitude and orientation of the **<u>total reaction force vector</u>** at support B. Show that it passes through the hinge at C. You can annotate Figure 3 if you think it will help to explain your solution.

[3c] (2 pts) Suppose that your calculations indicated that the "total reaction force at support B" **<u>did not</u>** pass through the hinge at C. What would that mean?