Department of Civil and Environmental Engineering,   Spring Semester, 2015

ENCE 353 Midterm 2, Open Notes and Open Book

Name : _____

Exam Format and Grading. Answer question 1. Then answer either question two or question three. Cross out the question you do not want graded in the table below. Partial credit will be given for partially correct answers, so please show all your working.

Question	Points	Score
1	25	
2	15	
3	15	
Total	40	

Question 1: 25 points

COMPULSORY: Moment-Area, Virtual Work. Figure 1 is a front elevation view of a simple beam structure carrying two external loads P. EI is constant along the beam.
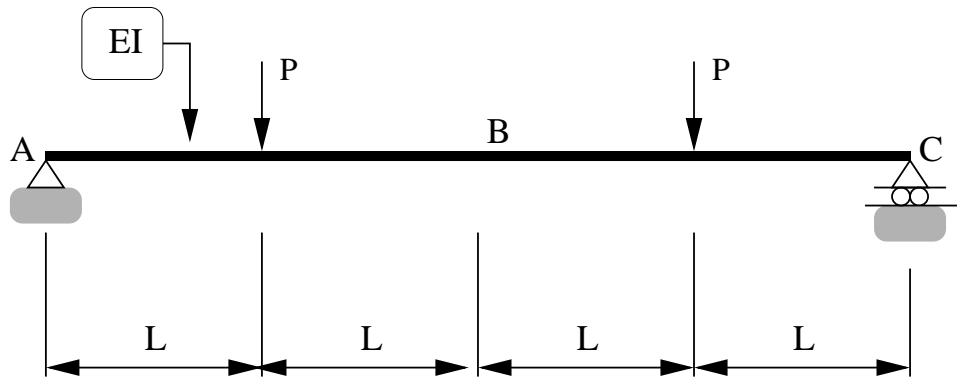

Figure 1: Simple beam structure (symmetric loads P).

[1a] (5 pts) Use the <u>method of moment area</u> to show that the end rotation at A is:

$$\theta_A = \frac{3}{2}\frac{PL^2}{EI}. \tag{1}$$

[1b] (5 pts) Use the <u>method of moment area</u> to show that the vertical beam deflection at B is:

$$\triangle_B = \frac{11}{6}\frac{PL^3}{EI}. \tag{2}$$

Figure 2 shows the same beam structure, but now the external loads are rearranged so that one load points down and one load points up.
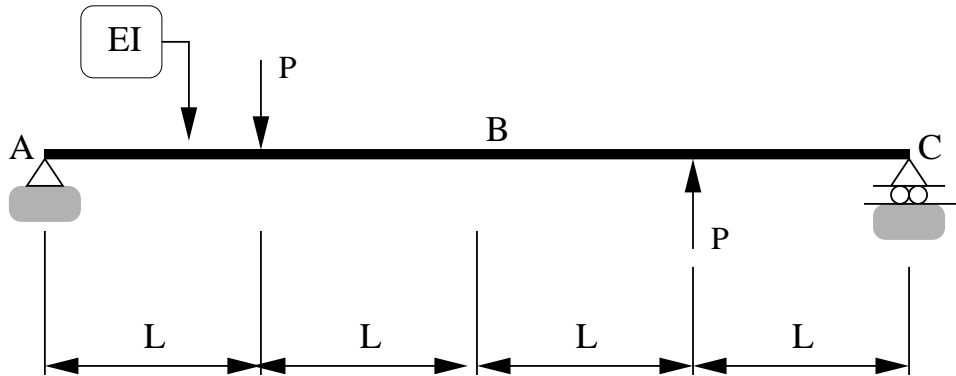

Figure 2: Simple beam structure (skew-symmetric loads P).

[1c] (5 pts) A function is said to be even if it has the property f(x) = f(-x) (i.e., it is symmetric about the y axis). And a function is said to be odd if it has the property g(x) = - g(-x) (i.e., it is skew-symmetric about the y axis). Using high school level calculus (or otherwise), show that:

$$\int_{-h}^{h} f(x)g(x)dx = 0. \tag{3}$$

[1d] (5 pts) Use the <u>method of virtual work</u> and a coordinate system positioned at B to show that the vertical displacement of B is zero, i.e., $\triangle_B = 0$.
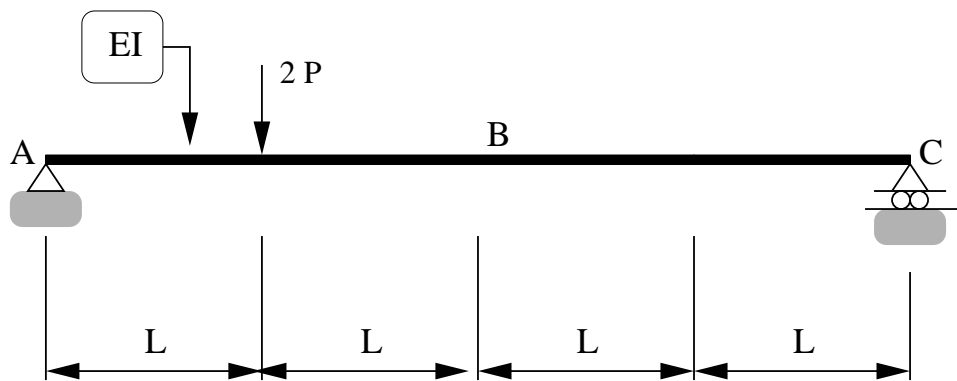
Now consider the problem.

Figure 3: Simple beam structure (one external load 2P).

[**1e**] (5 pts) Use your answers from parts [1b] and [1d] to write down an expression for the vertical deflection at B due to the loading pattern shown in Figure 3. `Note:` You should find this is a one line answer.

Question 2: 15 points

OPTIONAL: Virtual Work. Figure 4 is a front elevation view of a simple two-bar truss. A horizontal load P is applied at node B.
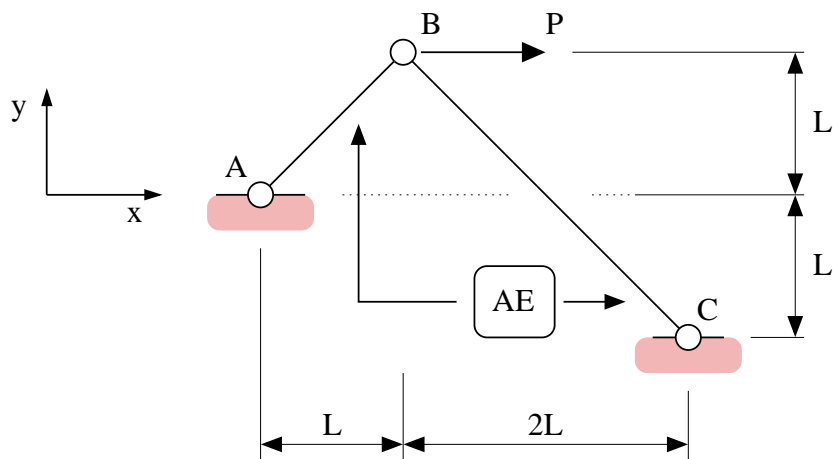

Figure 4: Front elevation view of a simple two-bar truss.

[2a] (5 pts) Use the <u>method of virtual work</u> to show that the horizontal displacement at node B is:

$$\triangle_x = \frac{3}{\sqrt{2}} \frac{PL}{AE}.$$ (4)

respectively.

[2b] (5 pts) Use the method of virtual work to show that the vertical displacement at node B is:

$$\triangle_y = \frac{-1}{\sqrt{2}} \frac{PL}{AE}. \tag{5}$$

[2c] (5 pts) Hence, show that:

$$\text{Total displacement at node B} = \sqrt{5} \frac{PL}{AE} \tag{6}$$

Question 3: 15 points

OPTIONAL: Virtual Displacements. Consider the S-shaped beam structure shown in Figure 5.
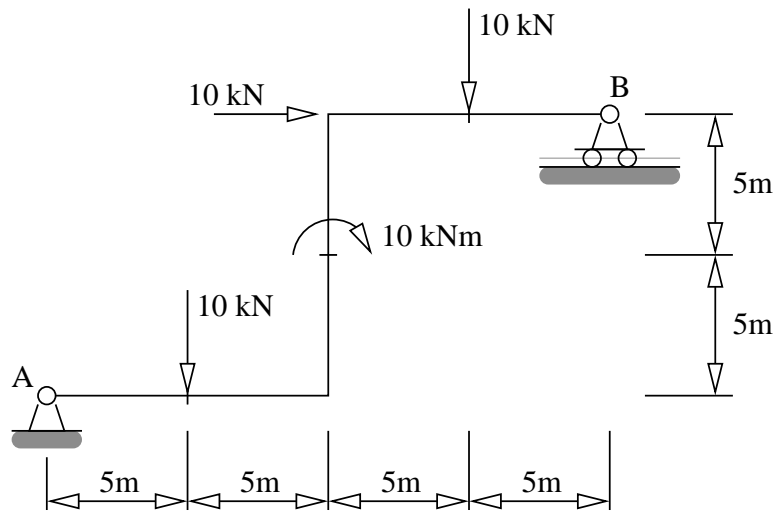

Figure 5: S-shaped beam structure.

[3a] (5 pts) Compute the vertical support reactions at A and B.

[3b] (10 pts) Use the <u>principle of virtual displacements</u> to compute the vertical reaction at B.