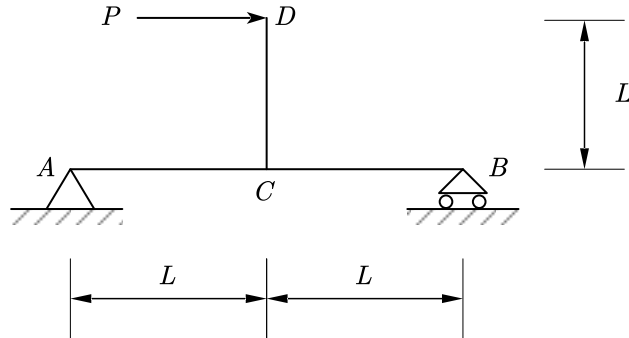## In-Class Problems #5 Solution
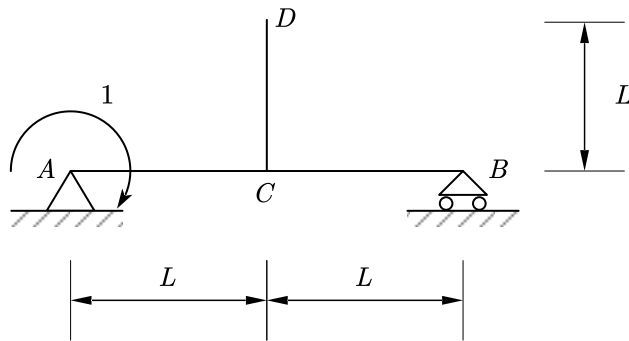
Consider a T shape structure shown below:



Assuming EI is constant for the structure and EA is sufficient large.
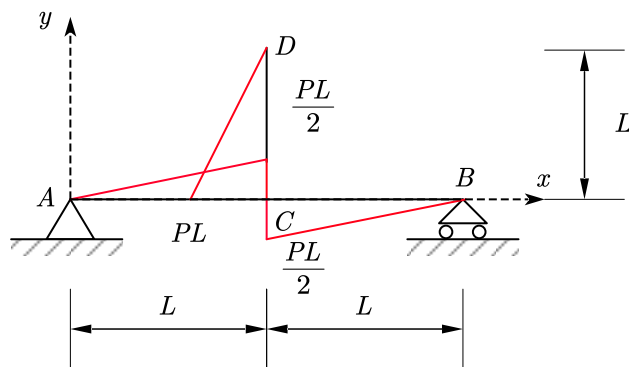
(1) Use principle of virtual work to calculate the rotation at A;
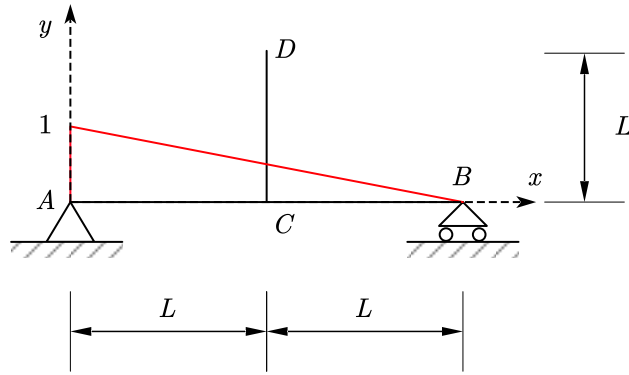
Solution:

Apply a virtual unit moment at A and draw the moment diagram for real load and virtual load respectively:



Bending Moment Diagram (real load):



Bending Moment Diagram (virtual load):

Assume the origin of the coordinate system is at A:
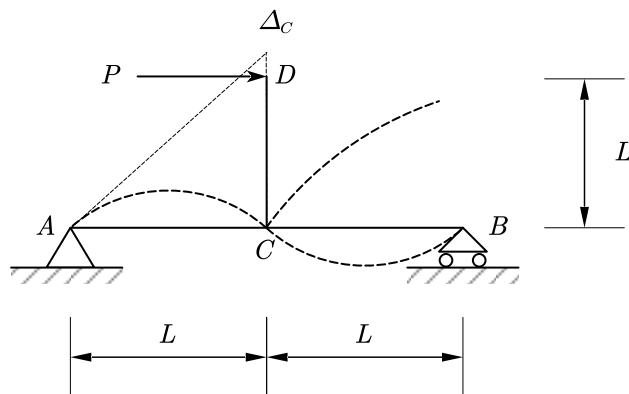
$$m(x) = \frac{-1}{2L}x + 1, \ \ 0 \leqslant x \leqslant 2L$$

$$M(x) = \begin{cases} \dfrac{P}{2}x, \ \ 0 \leqslant x \leqslant L \\[2mm] \dfrac{-P}{2}x + PL, \ \ L \leqslant x \leqslant 2L \end{cases}$$

$$\theta_A = \int_0^{2L} \frac{m(x)M(x)}{EI}dx$$

$$= \frac{1}{EI}\left(\int_0^L \left(\frac{-1}{2L}x + 1\right) \cdot \left(\frac{P}{2}x\right)dx - \int_L^{2L} \left(\frac{-1}{2L}x + 1\right) \cdot \left(\frac{-P}{2}x + PL\right)dx\right)$$

$$= \frac{1}{EI}\left(\frac{PL^2}{6} - \frac{PL^2}{12}\right) = \frac{PL^2}{12EI}$$

(2) Use method of moment area to calculate the rotation at A.

Solution:



$$\Delta_C = \frac{\dfrac{1}{2} \cdot \dfrac{PL}{2} \cdot L \cdot \dfrac{L}{3}}{EI} = \frac{PL^3}{12EI}$$

$$\theta_A = \frac{\Delta_C}{L} = \frac{PL^2}{12EI}$$