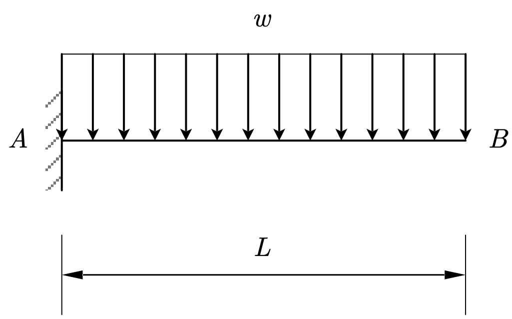
University of Maryland Fall 2017

ENCE353: Introduction to Structural Analysis

In-Class Problems #4 Solution

A cantilever beam is shown below, assuming the EI is constant along the beam:



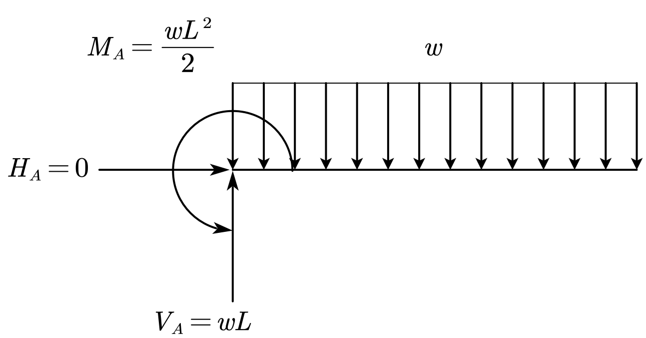
(1) Derive the deflection curve y(x) based on *elastic beam theory* (i.e. $d^2y/dx^2=M(x)/EI$);

(2) Based on the results of (1), calculate the deflection y(x=L) and the angle of tangent (or slope) at point B (θ_B);

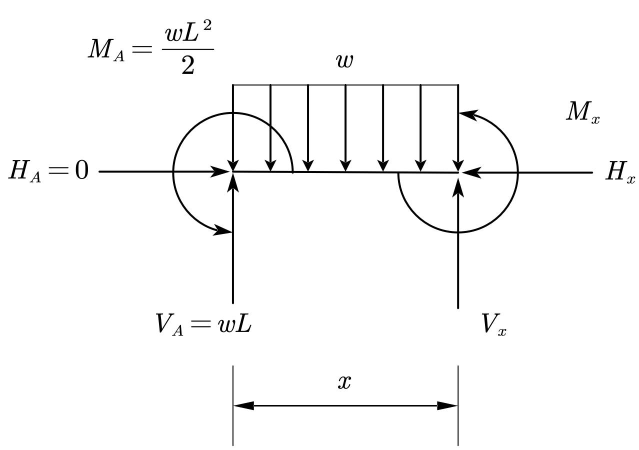
(3) Use *the method of moment-area* to verify your results from (2).

Solution:

(1) Draw the free body diagram of the beam and replace the boundary conditions with reactions:



Slice the beam in x=x, and draw the free body diagram, assuming x is 0 at point A and is L at point B:



Make moment equilibrium about pint x=x, assuming moment is positive in counter-clockwise:

$$M_A - V_A \cdot x + w \cdot x \cdot \frac{x}{2} + M(x) = 0$$

$$\Rightarrow M(x) = \frac{wL^2}{2} - wLx + \frac{wx^2}{2}$$

Recall elastic beam equation:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} = \frac{1}{EI} \cdot \left(\frac{wL^2}{2} - wLx + \frac{wx^2}{2}\right)$$

$$\frac{dy}{dx} = \frac{1}{EI} \cdot \left(\frac{wL^2}{2}x - wL\frac{x^2}{2} + \frac{wx^3}{6} + C_1\right)$$

$$y(x) = \frac{1}{EI} \cdot \left(\frac{wL^2x^2}{4} - wL\frac{x^3}{6} + \frac{wx^4}{24} + C_1x + C_2\right)$$

$$\text{Apply BC: } \frac{dy}{dx}|_{x=0} = 0,\ y(x=0) = 0$$

$$C_1 = C_2 = 0$$

$$\Rightarrow y(x) = \frac{1}{EI} \cdot \left(\frac{wL^2x^2}{4} - wL\frac{x^3}{6} + \frac{wx^4}{24}\right)$$

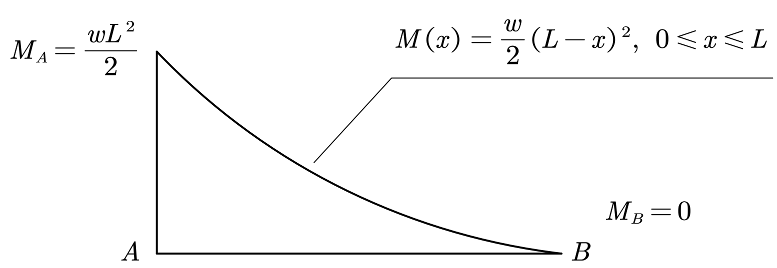
(2) Evaluate y(x) and dy/dx at x=L:

$$y(x=L) = \frac{1}{EI} \cdot \left(\frac{wL^2L^2}{4} - wL\frac{L^3}{6} + \frac{wL^4}{24}\right) = \frac{wL^4}{8EI}$$

$$\left.\frac{dy}{dx}\right|_{x=L} = \frac{1}{EI} \cdot \left(\frac{wL^2}{2}L - wL\frac{L^2}{2} + \frac{wL^3}{6}\right) = \frac{wL^3}{6EI}$$

$$\text{For small deflection, } \theta_B = \tan\theta_B = \frac{wL^3}{6EI}$$

(3) Draw the moment diagram:



$$\theta_B = \frac{1}{EI} \cdot \frac{1}{3} \cdot \frac{wL^2}{2} \cdot L = \frac{wL^3}{6EI}$$

$$\Delta_B = \frac{1}{EI} \cdot \frac{1}{3} \cdot \frac{wL^2}{2} \cdot L \cdot \frac{3L}{4} = \frac{wL^4}{8EI}$$