University of Maryland Fall 2017

ENCE353: Introduction to Structural Analysis

In-Class Problems #3 Solution

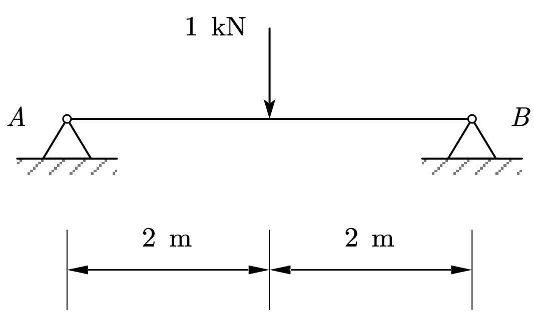
A cable structure is shown below; the original length of the cable is 4 m. Assume the cable will behave as linear elastic and the stiffness k = 1 kN/m (i.e. the axial force of the cable $F_N = k*D$, where D is the deformation of the cable). A unit point-load is applied on the cable as shown in the figure.

(1) What is the total length of the cable after the load is applied.

(2) Draw the cable profile after the load is applied.

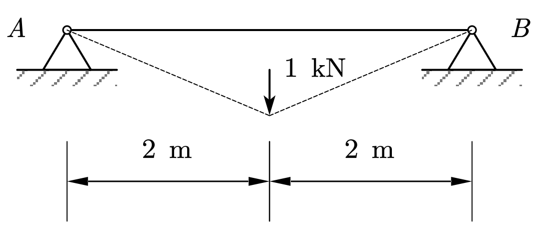
(3) Draw the moment diagram.

(Math tool: for equation $4x^4+16x^3-x^2-4x-4=0 \ (x\in R^+)$, the solution is x=0.7335.)

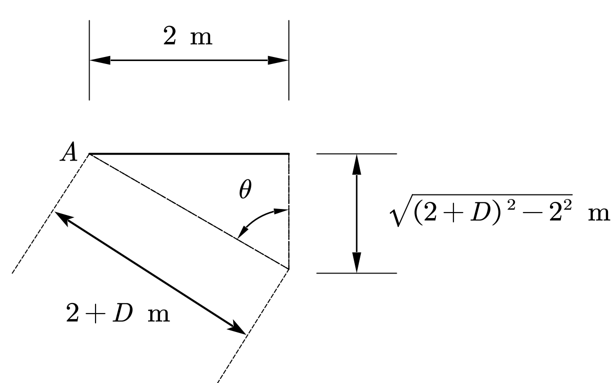


Solution:

As the cable can only carry tension force, the deformed shape of the structure will be achieved as the following figure:



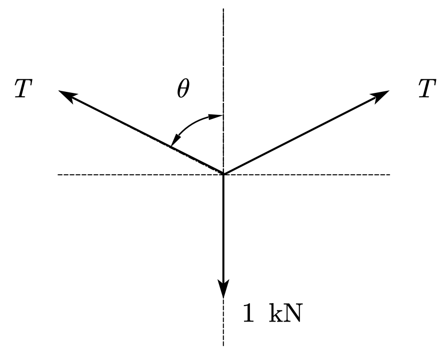
Assume the elongation of the cable is D, the compatibility structure would have:



So, there is a relationship:

$$\cos\theta = \frac{\sqrt{(2+D)^2 - 2^2}}{2+D}$$

The force equilibrium in vertical direction at midpoint of the cable (due to symmetry, the tension force of the cable are equal at left and right side of the midpoint):



$$T\cos\theta + T\cos\theta = 1, \text{ where } T = kD, \text{ and } k = 1 \text{ kN/m}$$

$$\Rightarrow T\cos\theta = \frac{1}{2}$$

$$\Rightarrow kD \cdot \frac{\sqrt{(2+D)^2 - 2^2}}{2+D} = \frac{1}{2}$$

$$\Rightarrow D \cdot \frac{\sqrt{4D + D^2}}{2+D} = \frac{1}{2}$$

$$\Rightarrow \sqrt{4D + D^2} = \frac{2+D}{2D}$$

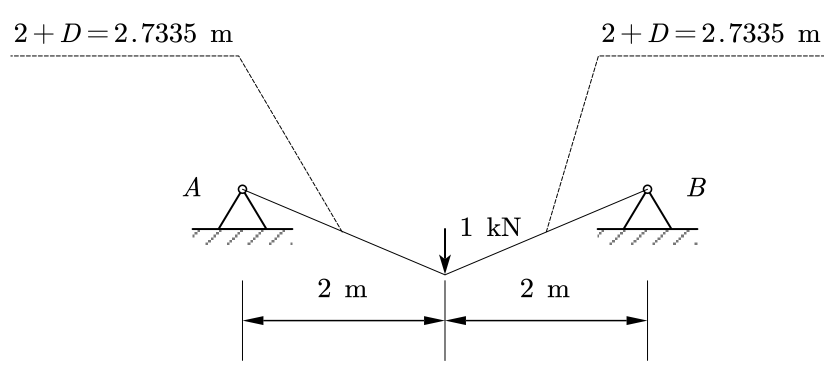
$$\Rightarrow 4D + D^2 = \frac{(2+D)^2}{4D^2}$$

$$\Rightarrow 4D^4 + 16D^3 - D^2 - 4D - 4 = 0$$

$$\Rightarrow D = 0.7335 \text{ m}$$

(1) So, the total elongation of the cable would be 2D=2*0.7335=1.467 m; thus, the total length of the cable after the load is applied would be 4+2D=5.467 m.

(2)



(3) There is no moment on the cable.