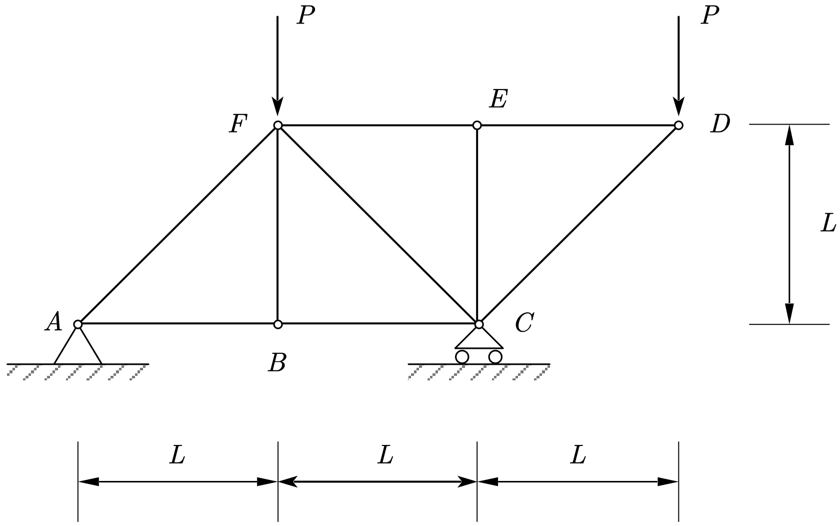
University of Maryland Fall 2017

ENCE353: Introduction to Structural Analysis

Homework #5 Solution

Problem 1: Considering the truss structure shown below:

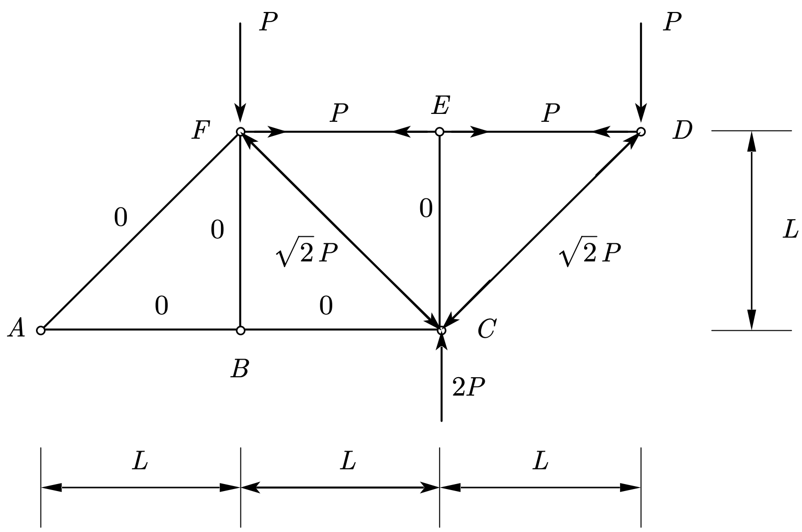


A vertical load of P kN is applied at nodes D and F. AE is constant for all truss members.

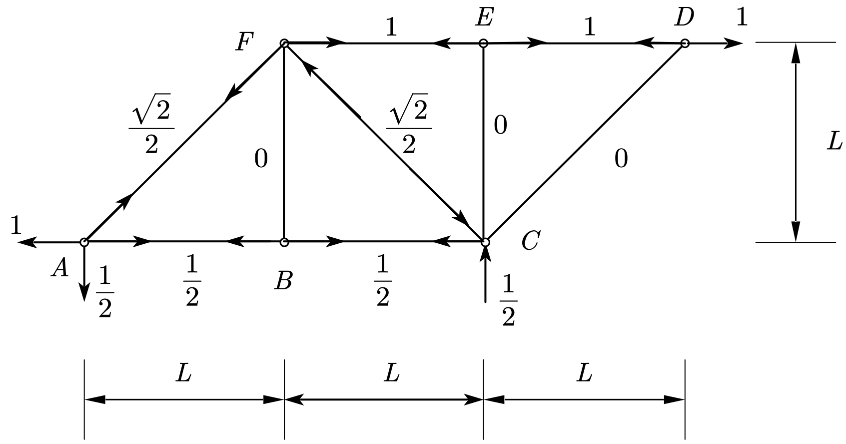
(1) Use the method of virtual forces to show that the horizontal displacement at node D

Solution:

(i) Draw the axial load diagram under real load:



(ii) Draw the axial load diagram under horizontal virtual load at D:



Use principle of virtual work to calculate the horizontal displacement at D:

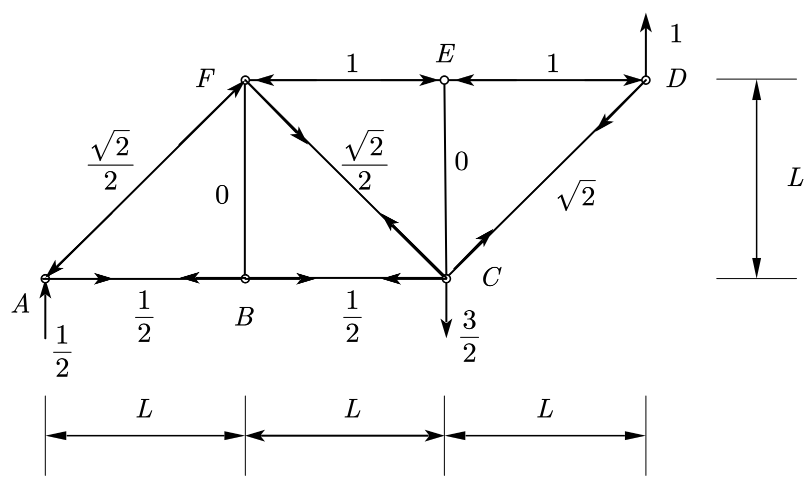
$$\Delta_D = \sum F_P \overline{F} L \frac{1}{EA}$$

$$= \frac{1}{EA} \cdot \left(P \cdot 1 \cdot L + P \cdot 1 \cdot L + \sqrt{2} P \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{2} L \right) = \frac{(2+\sqrt{2})PL}{EA} (\rightarrow)$$

(2) Use the method of virtual forces to show that the vertical displacement at node D

Solution:

(iii) Draw the axial load diagram under vertical virtual load at D:

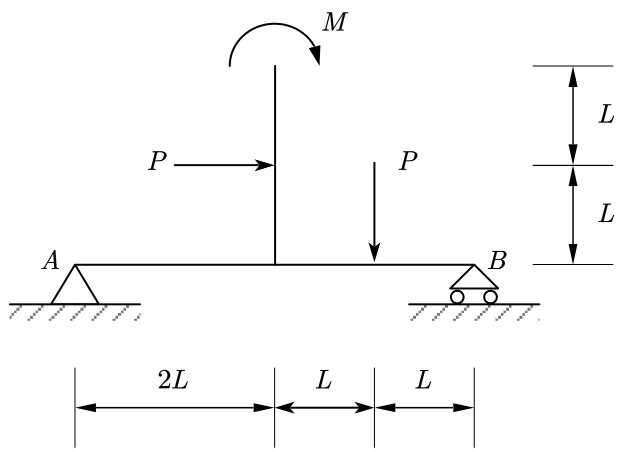


Use principle of virtual work to calculate the vertical displacement at D:

$$\Delta_D = \sum F_P \overline{F} L \frac{1}{EA}$$

$$= \frac{1}{EA} \cdot \left(-P \cdot 1 \cdot L - P \cdot 1 \cdot L - \sqrt{2} P \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{2} L - \sqrt{2} P \cdot \sqrt{2} \cdot \sqrt{2} L \right) = - \frac{(2+3\sqrt{2})PL}{EA} (\downarrow)$$

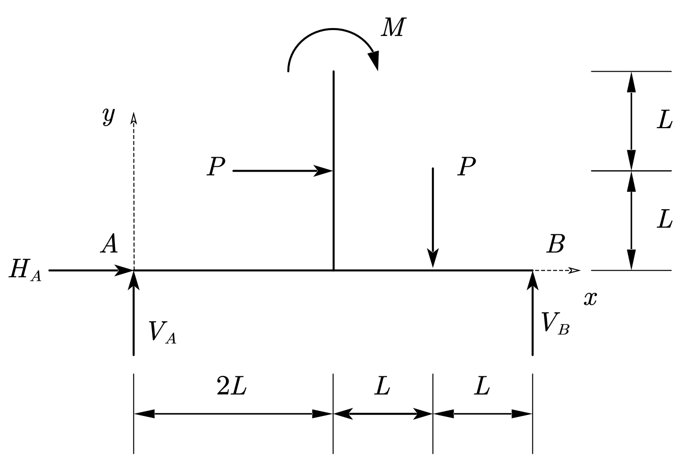
Problem 2: Consider the T-shaped structure shown below:



(1) Use static analysis method to calculate the vertical reactions at support A and B.

Solution:

(i) Draw free body diagram, assuming the origin of the coordinate system is A:



(ii) Equation of equilibrium:

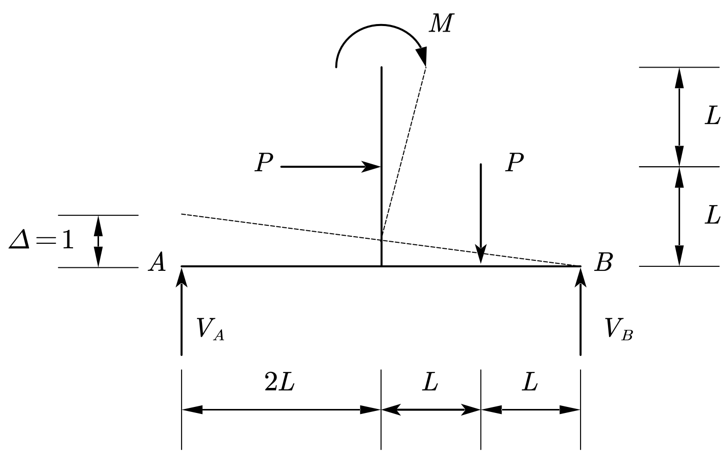
$$\begin{cases} \sum F_x = 0, \ H_A + P = 0 \\ \sum F_y = 0, \ V_A + V_B - P = 0 \\ \sum M_A = 0, \ PL + P \cdot 3L + M - V_B \cdot 4L = 0 \end{cases}$$

$$\Rightarrow \begin{cases} H_A = -P \\ V_B = \dfrac{(4PL + M)}{4L} = P + \dfrac{M}{4L} \\ V_A = -\dfrac{M}{4L} \end{cases}$$

(2) Use the method of virtual displacements to calculate the vertical reactions at support A and B.

Solution:

(i) Assign virtual displacement at A:

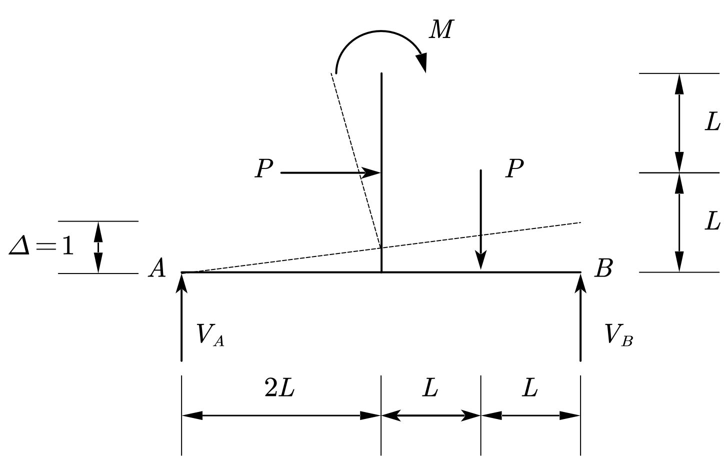


(ii) Use principle of virtual work to calculate the vertical reaction at A:

$$V_A \cdot 1 + P \cdot 1 \cdot \frac{L}{4L} - P \cdot 1 \cdot \frac{L}{4L} + M \cdot \frac{1}{4L} = 0$$

$$\Rightarrow V_A = - \frac{M}{4L}$$

(iii) Assign virtual displacement at B:



(iv) Use principle of virtual work to calculate the vertical reaction at B:

$$V_B \cdot 1 - P \cdot 1 \cdot \frac{L}{4L} - P \cdot 1 \cdot \frac{3L}{4L} - M \cdot \frac{1}{4L} = 0$$

$$\Rightarrow V_A = P + \frac{M}{4L}$$