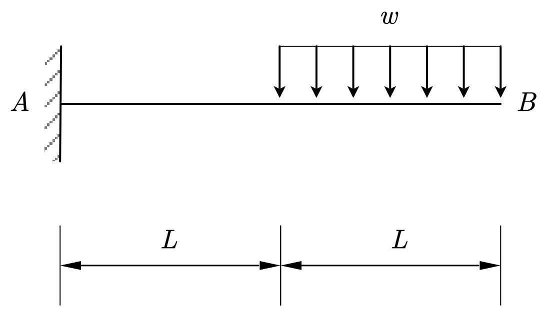
University of Maryland Fall 2017

ENCE353: Introduction to Structural Analysis

Homework #4 Solutions

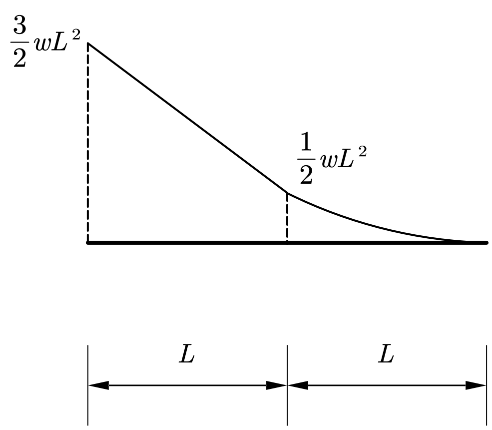
Problem 1: For a cantilever beam shown below:



Use the *method of moment-area* to calculate the vertical displacement at point B, assuming the EI is constant along the beam.

Solution:

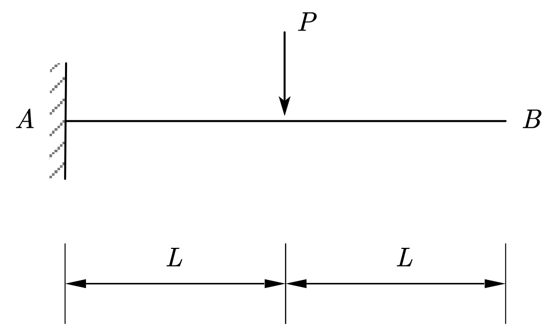
(1) Draw the moment diagram:



(2) Calculate the deflection at point B:

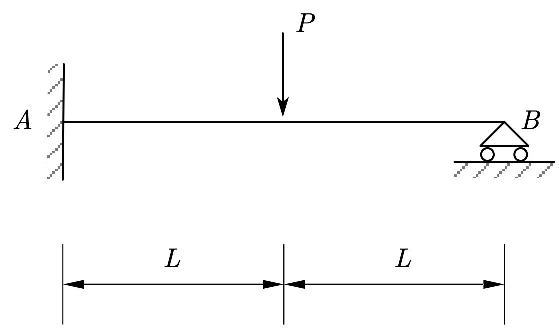
$$\Delta_B = \frac{\frac{1}{2}\cdot L\cdot wL^2}{EI}\cdot\left(\frac{2}{3}L+L\right)+\frac{L\cdot\frac{1}{2}wL^2}{EI}\cdot\left(\frac{1}{2}L+L\right)+\frac{\frac{1}{3}\cdot\frac{1}{2}wL^2\cdot L}{EI}\cdot\frac{3}{4}L=\frac{41}{24}\frac{wL^4}{EI}$$

Problem 2 For a cantilever beam shown below:



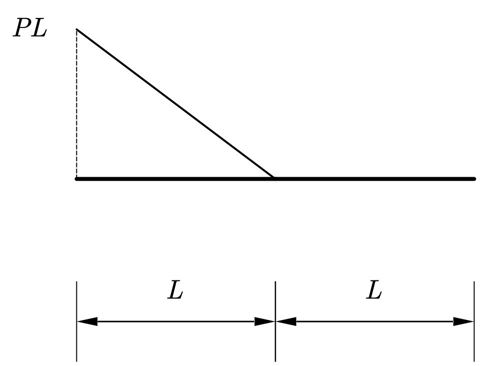
(1) Use the *method of moment-area* to calculate the vertical displacement at point B, assuming the EI is constant along the beam.

(2) Based on the results of (1), calculate the reaction force at point B for the following propped-cantilever beam, assuming the EI is constant along the beam:



Solution:

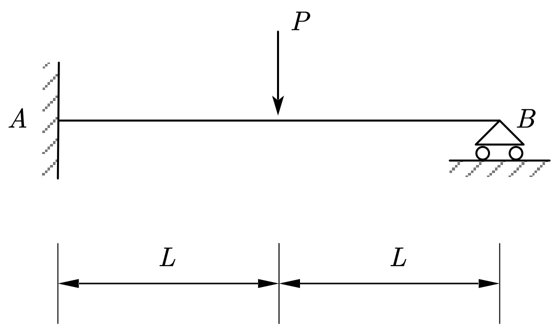
(1) Draw the moment diagram:



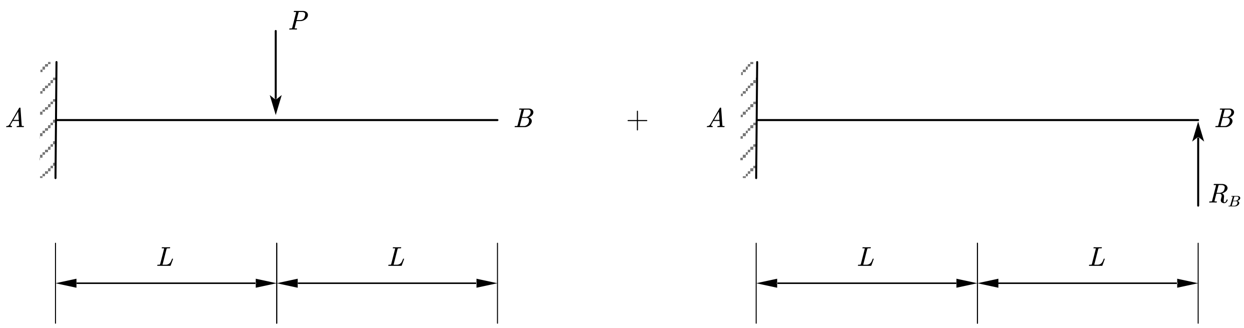
Calculate the deflection at point B:

$$\Delta_B = \frac{\frac{1}{2} \cdot L \cdot PL}{EI} \cdot \left(\frac{2}{3}L + L\right) = \frac{5}{6}\frac{PL^3}{EI} (\downarrow)$$

(2) The propped-cantilever beam can be decomposed by the following two simple beams:

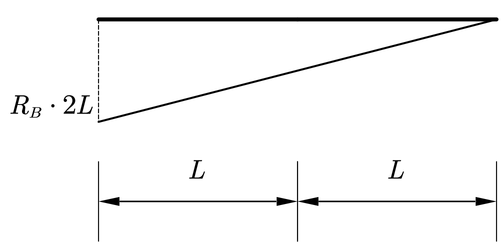


$$\parallel$$



For the first beam, we already have the solutions in (1). Now, we are dealing with the second one:

Draw the moment diagram with unknown reaction force R_B:



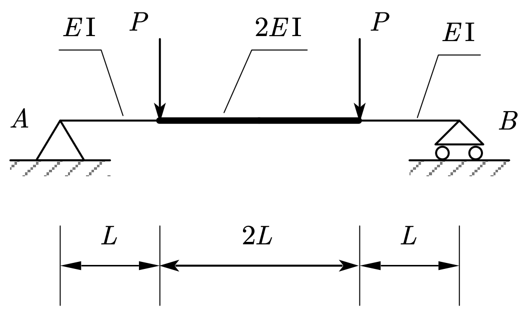
Calculate the deflection at B:

$$\Delta_B = \frac{\frac{1}{2} \cdot 2L \cdot R_B \cdot 2L}{EI} \cdot \left(\frac{2}{3} \cdot 2L\right) = \frac{8}{3}\frac{R_B L^3}{EI}\,(\uparrow)$$

Because the real structure will have zero deflection at point B, thus:

$$\Delta_B = \frac{5}{6}\frac{PL^3}{EI} - \frac{8}{3}\frac{R_B L^3}{EI} = 0 \Rightarrow R_B = \frac{5}{16}P\,(\uparrow)$$

Problem 3 For a simple supported beam shown below:

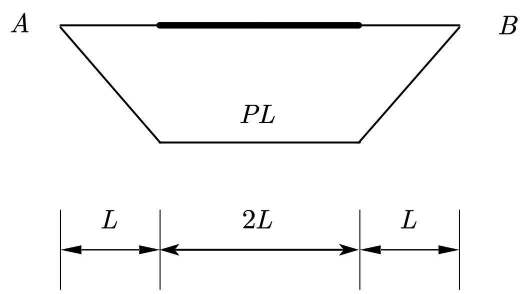


(1) Use the *method of moment-area* to calculate the rotation at point A.

(2) Use the *method of moment-area* to calculate the vertical deflection at mid-span.

Solution:

(1) Draw the moment diagram:

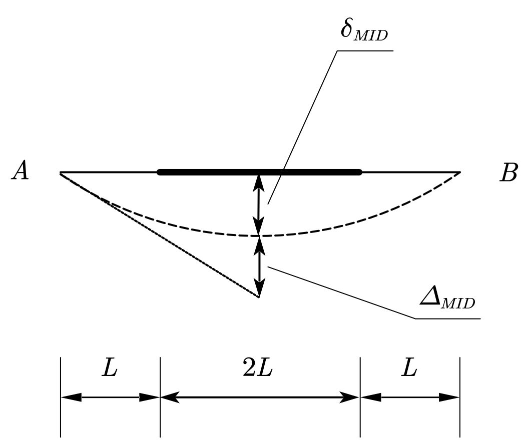


Calculate the rotation at A (note: the rotation at mid span will be 0 due to symmetry):

$$\theta_{MID} - \theta_A = \frac{\frac{1}{2} \cdot L \cdot PL}{EI} + \frac{L \cdot PL}{2EI} = \frac{PL^2}{EI}$$

$$\Rightarrow \theta_A = -\frac{PL^2}{EI}$$

(2) Calculate the deflection at mid span:



$$\Delta_{MID} = \frac{\frac{1}{2} \cdot L \cdot PL}{EI} \cdot \left(\frac{1}{3}L + L\right) + \frac{L \cdot PL}{2EI} \cdot \frac{1}{2}L = \frac{11}{12}\frac{PL^3}{EI}$$

According to geometry:

$$\theta_A = \frac{\Delta_{MID} + \delta_{MID}}{2L}$$

$$\Rightarrow \delta_{MID} = \theta_A \cdot 2L - \Delta_{MID} = \frac{PL^2}{EI} \cdot 2L - \frac{11}{12}\frac{PL^3}{EI} = \frac{13}{12}\frac{PL^3}{EI}$$