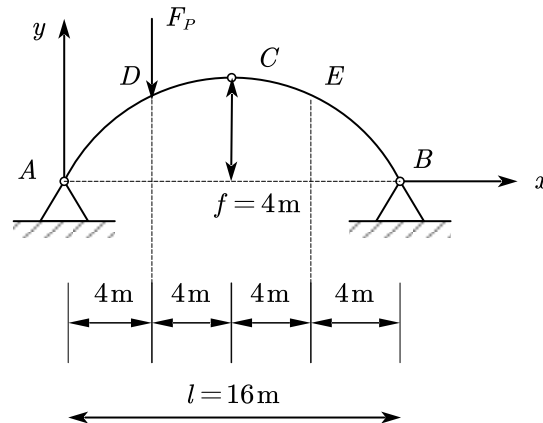


### Homework #3 Solution

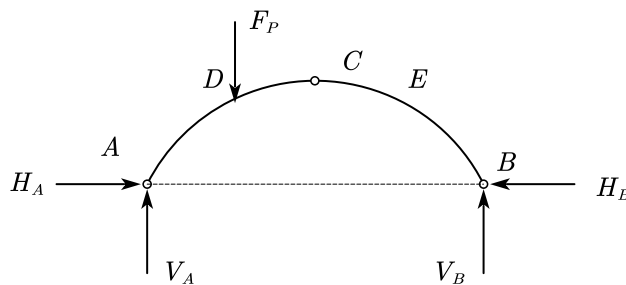
**Problem 1:** For the three-pin arc structure shown below, the profile is given by  $y = \frac{4f}{l^2} x(l - x)$ , where  $f = 4$  m and  $l = 16$  m.



- Calculate the reactions at A and B;
- Calculate the internal forces at point E (i.e. axial force  $F_N$ , shear force  $F_Q$  and bending moment  $M$ );
- Draw the moment diagram.

Solution:

- Draw the free body diagram of the arc structure:



$$\sum F_x = 0, H_A = H_B$$

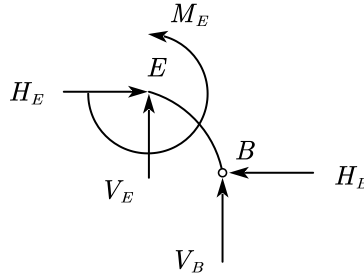
$$\sum F_y = 0, V_A + V_B = F_P$$

$$\sum M_A = 0, F_P \cdot 4 - V_B \cdot 16 = 0 \Rightarrow V_B = \frac{F_P}{4} (\uparrow) \Rightarrow V_A = \frac{3F_P}{4} (\uparrow)$$

Slice the arc structure at point C, and write the moment equilibrium about point C (left part) would get:

$$\sum M_C = 0, -F_P \cdot 4 + V_A \cdot 8 - H_A \cdot 4 = 0 \Rightarrow H_A = \frac{F_P}{2} (\rightarrow) \Rightarrow H_B = \frac{F_P}{2} (\leftarrow)$$

(b) Slice the arc structure at point E, and draw the free body diagram of the right part:

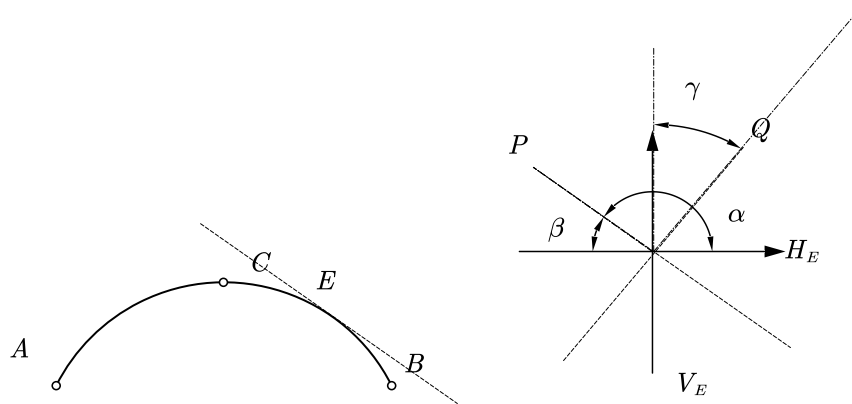


$$\sum F_x = 0, H_E = H_B \Rightarrow H_E = \frac{F_P}{2} (\rightarrow)$$

$$\sum F_y = 0, V_E + V_B = 0 \Rightarrow V_E = -\frac{F_P}{4} (\downarrow)$$

$$\sum M_E = 0, -M_E - V_B \cdot 4 + H_B \cdot y(12) = 0 \Rightarrow M_E = \frac{F_P}{2} (\curvearrowright)$$

The axial force at point E is along with the tangent of the arc profile at point E and the shear force is perpendicular to the tangent of the arc profile. Thus, to calculate the axial and shear force, decomposition of  $H_E$  and  $V_E$  is needed:



$$\frac{dy}{dx} \Big|_{x=12} = \frac{4f}{l^2} (l - 2x) \Big|_{x=12} = -\frac{1}{2}$$

$$\tan \alpha = -\frac{1}{2} \Rightarrow \alpha = 153.43(\text{deg}) \Rightarrow \beta = 26.57(\text{deg}) \Rightarrow \gamma = \beta = 26.57(\text{deg})$$

$$\sin \beta = 0.447, \cos \beta = 0.894$$

$$P = V_E \sin \beta + H \cos \beta = \frac{F_P}{4} \cdot 0.447 + \frac{F_P}{2} \cdot 0.894 = 0.559F_P (\text{compression})$$

$$Q = -V_E \cos \beta + H \sin \beta = -\frac{F_P}{4} \cdot 0.894 + \frac{F_P}{2} \cdot 0.447 = 0$$

(c) Assume the moment at point  $x=x$  is positive in clockwise:

For section  $0 \leq x \leq 4$ , write the moment equilibrium about point at  $x=x$ :

$$\sum M_{x=x} = 0, V_A \cdot x - H_A \cdot y(x) + M(x) = 0$$

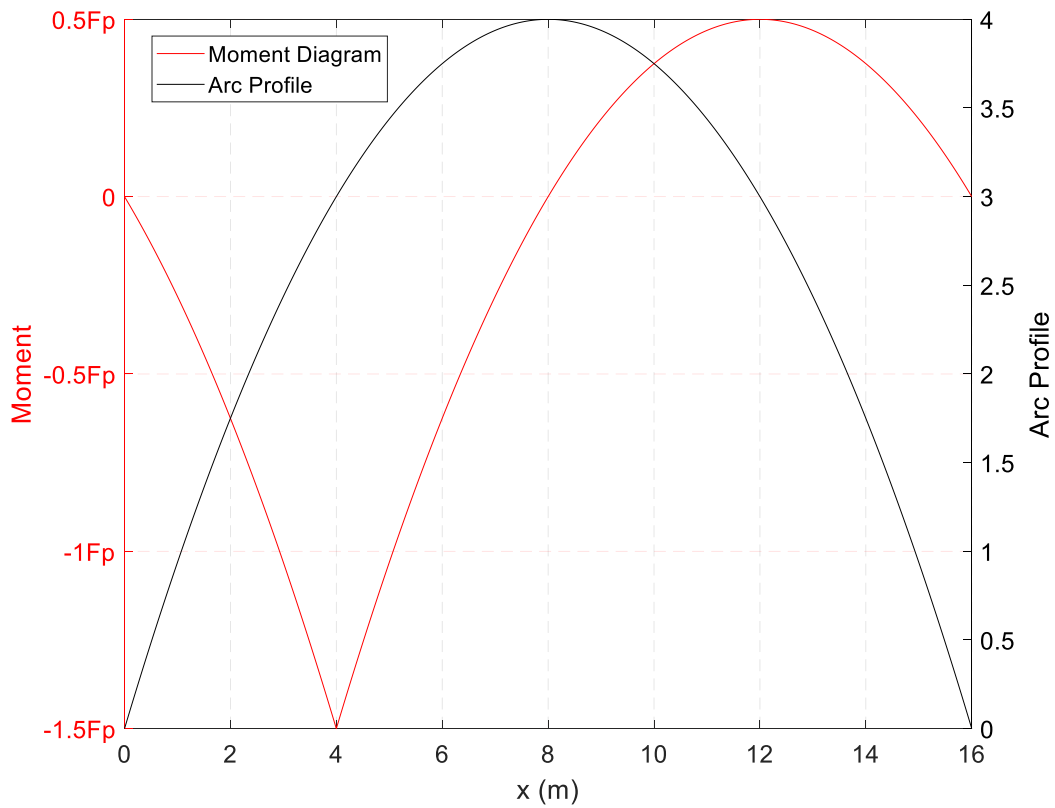
$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2} \left( \frac{4f}{l^2} x(l-x) \right) = -\frac{F_P}{32}x^2 - \frac{F_P}{4}x$$

For section  $4 \leq x \leq 16$ ,

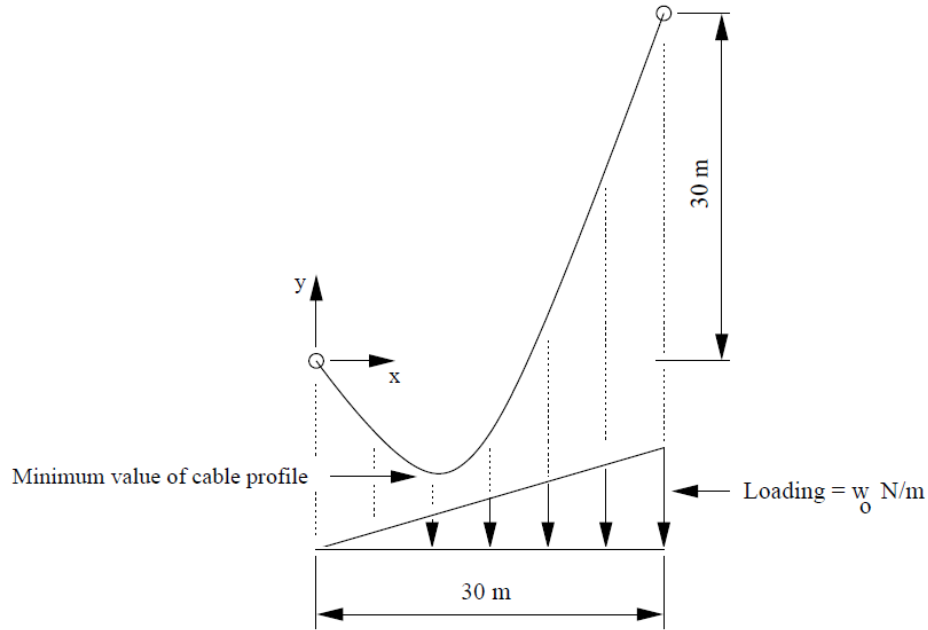
$$\sum M_{x=x} = 0, V_A \cdot x - H_A \cdot y(x) - F_P \cdot (x-4) + M(x) = 0$$

$$\Rightarrow M(x) = -\frac{3F_P}{4}x + \frac{F_P}{2} \left( \frac{4f}{l^2} x(l-x) \right) = -\frac{F_P}{32}x^2 + \frac{3F_P}{4}x - 4F_P$$

Draw the moment diagram (Assume the value of  $F_p \geq 0$ ):



**Problem 2** The cable structure shown below carries a triangular load that is zero at the left-hand support and increases to  $w_0$  N/m at the right-hand support.



(a) Prove that the cable profile is governed by the equation:

$$y(x) = \frac{w_0 x^3}{180H} + \left(1 - \frac{5w_0}{H}\right)x$$

Solution:

$$\frac{d^2 y}{dx^2} = \frac{w(x)}{H} = \frac{w_0 x}{30H}$$

$$\Rightarrow \frac{dy}{dx} = \frac{w_0 x^2}{60H} + A$$

$$\Rightarrow y(x) = \frac{w_0 x^3}{180H} + Ax + B$$

Apply  $y(0) = 0$ ,  $y(30) = 30$ :

$$\begin{cases} B = 0 \\ A = 1 - \frac{5w_0}{H} \end{cases}$$

$$\Rightarrow y(x) = \frac{w_0 x^3}{180H} + \left(1 - \frac{5w_0}{H}\right)x$$

(b) Assume, the minimum value of the cable profile occurs at  $x=10$  m, calculate the reactions at both ends.

Solution:

Note: Point A is the left end, and point B is the right end.

$$\frac{dy}{dx} = \frac{w_0 x^2}{60H} + 1 - \frac{5w_0}{H}$$

$$\left. \frac{dy}{dx} \right|_{x=10} = 0 \Rightarrow \frac{100w_0}{60H} + 1 - \frac{5w_0}{H} = 0 \Rightarrow H = \frac{10}{3}w_0$$

$$\sum M@A = 0, V_B \cdot 30 - H \cdot 30 - \frac{1}{2} \cdot 30 \cdot w_0 \cdot \frac{2}{3} \cdot 30 = 0 \Rightarrow V_B = \frac{40}{3}w_0$$

$$\sum F_y = 0, V_A = 15w_0 - V_B = \frac{5}{3}w_0$$