Department of Civil and Environmental Engineering, Spring Semester, 2015

ENCE 353 Final Exam, Open Notes and Open Book

Name : ________________________

Exam Format and Grading. The exam will be 2 hrs plus five minutes to read the questions.

Partial credit will be given for partially correct answers, so please show all your working.

Question	Points	Score
1	20	
2	8	
3	12	
4	10	
Total	50	

Question 1: 20 points

COMPULSORY: Moment-Area, Virtual Work. Figure 1 is a front elevation view of a simple beam structure carrying two external loads P. The beam has section properties EI near the supports and 2EI in the center section.

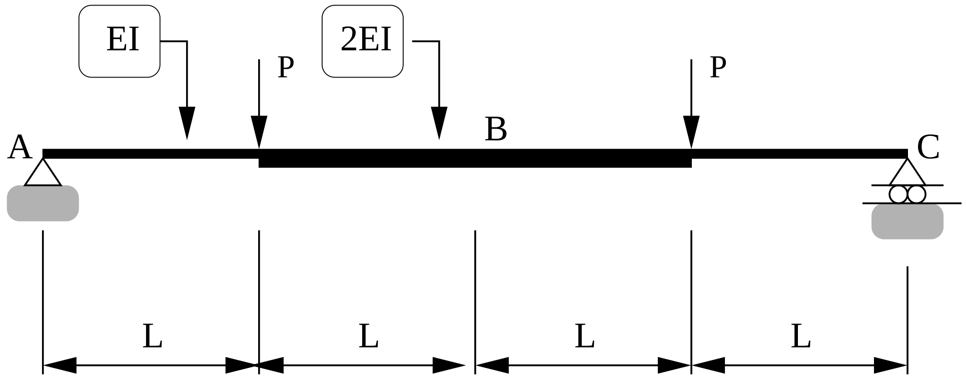

Figure 1: Simple beam structure (symmetric loads P).

[1a] (4 pts) Use the method of moment area to show that the end rotation at A (measured clockwise) is:

$$\theta_A = \frac{PL^2}{EI}. \tag{1}$$

[1b] (4 pts) Use the method of moment area to show that the vertical beam deflection at B is:

$$\triangle_B = \frac{13}{12}\frac{PL^3}{EI}. \tag{2}$$

[1c] (4 pts) A function is said to be even if it has the property f(x) = f(-x) (i.e., it is symmetric about the y axis). And a function is said to be odd if it has the property g(x) = - g(-x) (i.e., it is skew-symmetric about the y axis). One example of an even function is cos(x), and one example of an odd function is sin(x).

Using high-school-level calculus (or otherwise), show that:

$$\int_{-h}^{h} f(x)g(x)dx = 0. \tag{3}$$

Please show all of your working.

Figure 2 shows the same beam structure, but now the external loads are rearranged so that one load points down and one load points up.

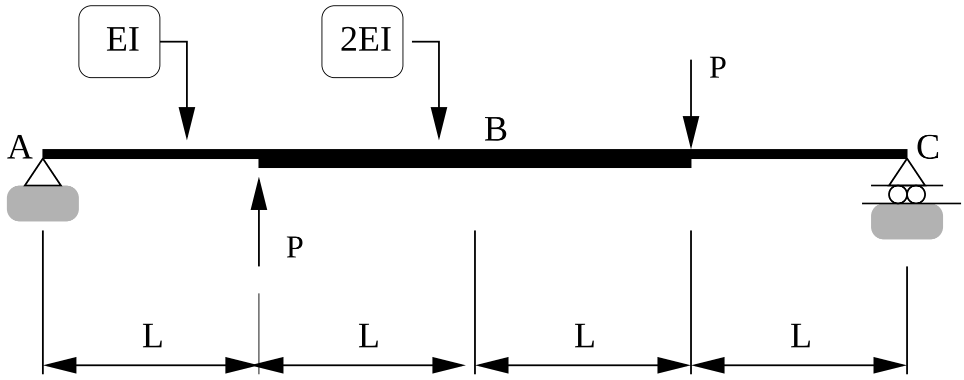

Figure 2: Simple beam structure (skew-symmetric loads P).

[1d] (4 pts) Use the method of virtual work and a coordinate system positioned at B to show that the vertical displacement of B is zero, i.e., $\triangle_B = 0$.

Now consider the problem.

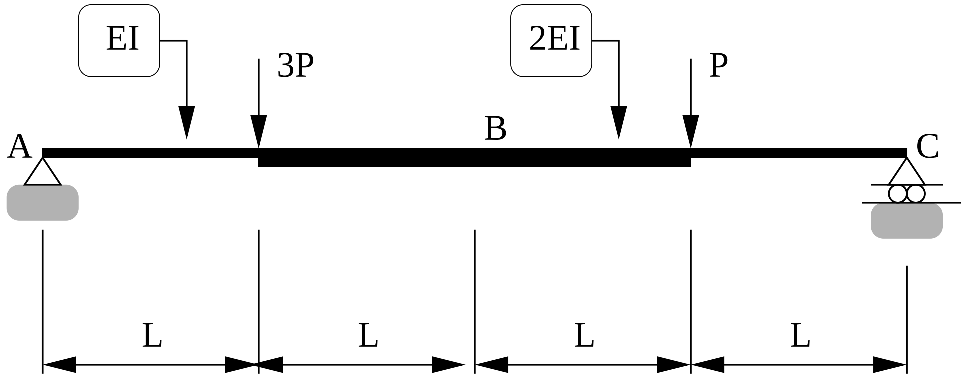

Figure 3: Simple beam structure (one external load 2P).

[1e] (4 pts) Use your answers from parts [1b] and [1d] to write down an expression for the vertical deflection at B due to the loading pattern shown in Figure 3. `Note:` You should find this is a one line answer.

Question 2: 8 points

Consider the two-span beam structure shown in Figure 1.

Figure 4: Front elevation view of a cantilevered beam structure.

[2a] (4 pts) Use the Muller-Breslau Principle to compute the influence line diagram for the vertical reaction **at A**.

[2b] (4 pts) Now suppose that span B-C carries a uniform load of w_o/L N/m. Using your influence line diagram from Part [2a], compute the vertical reaction **at A**.

Question 3: 12 points

Consider the truss structure shown in Figure 5.

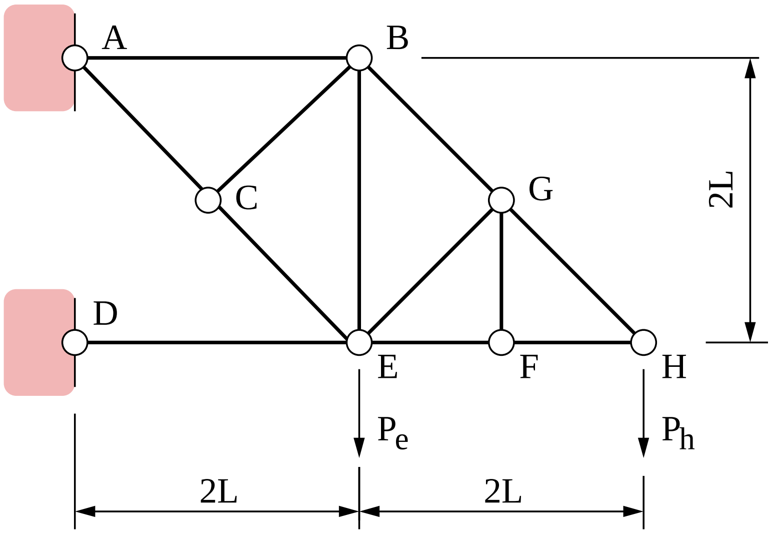

Figure 5: Elevation view of a pin-jointed truss.

The horizontal and vertical degrees of freedom are fully-fixed at supports A and D. The truss carries vertical loads P_e and P_h at nodes E and H, respectively. All frame members have cross section properties AE.

[3a] (2 pts) Use the method of joints to identify all of the zero-force members. Label these members on Figure 5.

[3b] (3 pts) Use the principle of virtual forces to compute the vertical deflection at node E due to load P_e alone (i.e., $P_h = 0$).

[3c] (3 pts) Use the principle of virtual forces to compute the vertical deflection at node H due to load P_h alone (i.e., $P_e = 0$).

[3d] (4 pts) Use the principle of virtual forces to compute the two-by-two flexibility matrix connecting the vertical displacements at points E and H to applied loads P_e and P_h, i.e., as a function of P_e, P_h, L and AE.

$$\begin{bmatrix} \triangle_e \\ \triangle_h \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} P_e \\ P_h \end{bmatrix}. \tag{4}$$

Question 3 continued ...

Question 4: 10 points

Figure 6 is an elevation view of a propped cantilever structure that carries an external point load P at its tip.

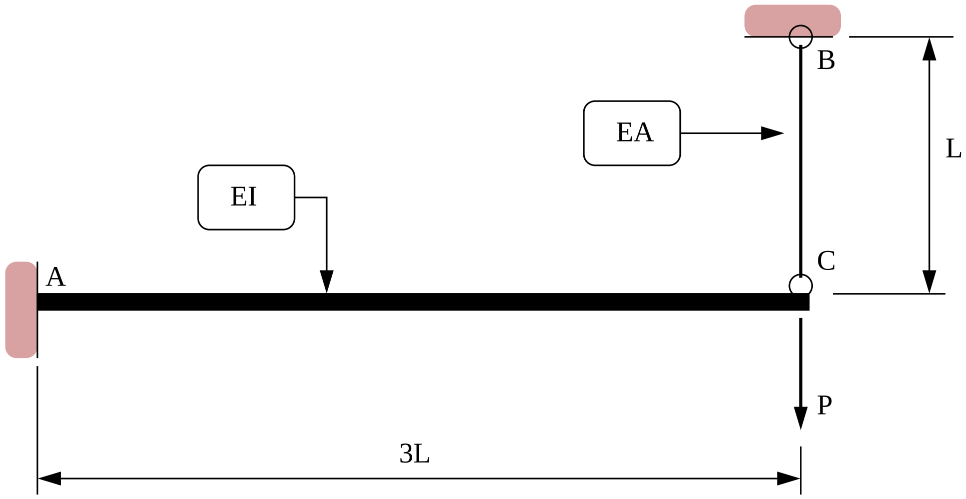

Figure 6: Elevation view of a propped cantilevel beam.

The structural system has constant section properties EI along the beam, and is supported by a truss element having section properties EA.

[4a] (1 pt) Compute the degree of indeterminacy for the propped cantilever beam.

[4b] (6 pt) Show that the axial force in the truss element, P_o, is given by:

$$P_o = P\left(\frac{9AL^2}{I + 9AL^2}\right). \qquad (5)$$

Question 4b continued ...

[4c] (3 pt) Explain how the value of bending moment at the cantilever support (i.e., at point A) will change as: (1) the truss element cross section area $A \rightarrow 0$, and (2) the truss element cross section area $A \rightarrow \infty$.