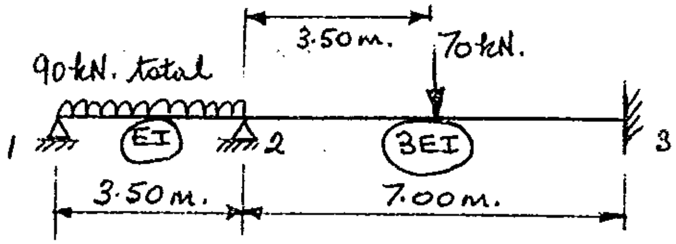
SLOPE DEFLECTION – Typical Examples.

Example. 1.

Determine the distribution of bending moment in the following two-span beam under the loading shown below.



F.E.Ms

$$M_{23}^F = -M_{32}^F = -\frac{70 \times 7.00}{8} = -61.25 \text{ kN.m.}$$

$$M_{12}^F = -M_{21}^F = -\frac{90 \times 3.50}{12} = -26.25 \text{ kN.m.}$$

Slope-deflection equations for the ends of each member :-

$$M_{12} = M_{12}^F + S_{12}\theta + c_{21} S_{21} \theta_2 + \text{No chord rotation}$$

ie $(S_{ij} + C_{ij} S_{ji})\phi = 0$ => $\phi = 0$

$$= -26.25 + \frac{2EI}{3.50}(2\theta_1 + \theta_2) \quad \text{..........(1)}$$

$$M_{21} = +26.25 + \frac{2EI}{3.50}(2\theta_2 + \theta_1) \quad \text{........(2)}$$

$$M_{23} = -61.25 + \frac{2 \times 3EI}{7.00}(2\theta_2 + \theta_3) \quad \text{.......(3)}$$

$$M_{32} = +61.25 + \frac{2 \times 3EI}{7.00}(2\theta_3 + \theta_2) \quad \text{........(4)}$$

We have 4 equations, and 7 unknowns. Derive the remaining equations from compatability and equilibrium at the joints.

Unknowns are, $\theta_1, \theta_2, \theta_3, M_{12}$, M_{21}, M_{23}, M_{32}

Compatability $\quad \theta_3 = 0 \quad$ ----. (5)

Equilibrium $\quad M_{12} = 0 \quad$ ------(6)

=> Use compatibility

$$M_{21} + M_{23} = 0. \text{---(7)}$$

Hence substitute equations (2) and (3) into (7) to get

(4)

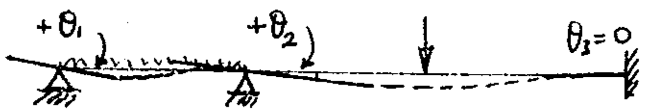
$$0.571\,EI\theta_1 + 2.857\,EI\theta_2 = 35.00 \quad \text{------(8)}$$

Equate equation (1) to zero, from the equilibrium equation (6) we ge

$$1.143\,EI\,\theta_1 + 0.571\,EI\,\theta_2 = 26.25 \quad \cdots (9)$$

Solving the simultaneous equations (8) and (9) gives

$$\theta_1 = +\frac{18.71}{EI} \quad ; \quad \theta_2 = +\frac{8.51}{EI}$$



Hence the joint (continuity) bending moments from equations (1) – (4) are

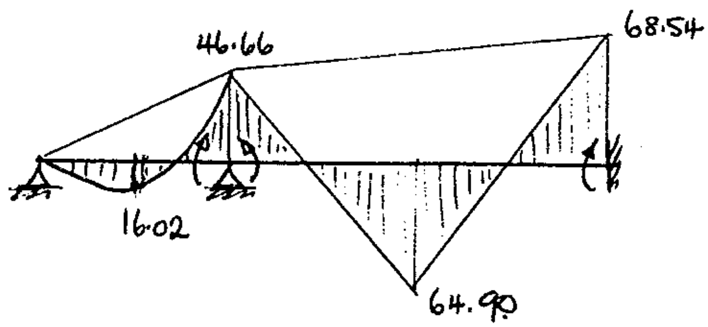
$$M_{12} = 0$$

$$M_{21} = +26.25 + \frac{2EI}{3.50}\left(2\times\frac{8.51}{EI} + \frac{18.71}{EI}\right) = +46.66\ \text{kN.m.}$$

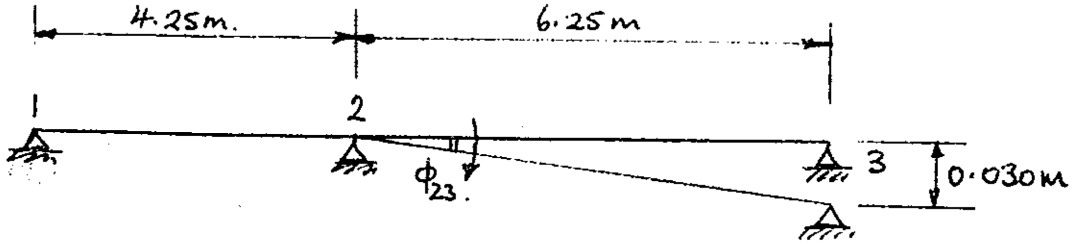
$$M_{23} = -61.25 + \frac{2\times 3EI}{7.00}\left(2\times\frac{8.51}{EI} + 0\right) = -46.66\ \text{kN.m.}$$

$$M_{32} = +61.25 + \frac{2\times 3EI}{7.00}\left(\frac{8.51}{EI}\right) = +68.54\ \text{kN.m.}$$

Bending moment diagram.



Determine the distribution of bending moments induced in the two-span beam when joint 3 settles by 30 mm. EI is constant along the full length of the beam.



The chord rotation ϕ_{23} is clockwise and positive, so the term $\left(\frac{3u}{L}\right)$ is negative. The F.E.M. are all zero - no applied loads.

Slope-deflection equations.

$$M_{12} = \frac{2EI}{4.25}\left(2\theta_1 + \theta_2\right) \quad \text{------------(1)}$$

$$M_{21} = \frac{2EI}{4.25}\left(2\theta_2 + \theta_1\right) \quad \text{------------(2)}$$

$$M_{23} = \frac{2EI}{6.25}\left(2\theta_2 + \theta_3 - \frac{3\times 0.030}{6.25}\right) \quad \text{------------(3)}$$

$$M_{32} = \frac{2EI}{6.25}\left(2\theta_3 + \theta_2 - \frac{3\times 0.030}{6.25}\right) \quad \text{------------(4)}$$

7 unknowns - 4 equations. Three further equations come from joint equilibrium.

$$M_{12} = 0 \quad \text{-------(5)}$$

$$M_{21} + M_{23} = 0 \quad \text{------(6)}$$

$$M_{32} = 0. \quad \text{------------(7)}$$

Solve :- From equations (1) and (5) $\theta_1 = -\theta_2/2$ -------(8)

" " (4) " (7) $\theta_3 = 0.0072 - \theta_2/2$. ---(9)

Substituting equations (2) and (3) into (6) gives

$$\frac{1}{4.25}\left(2\theta_2 + \theta_1\right) + \frac{1}{6.25}\left(2\theta_2 + \theta_3 - 0.0144\right) = 0 \quad \text{----(10)}$$

Using the relationships derived in (8) and (9) we now get

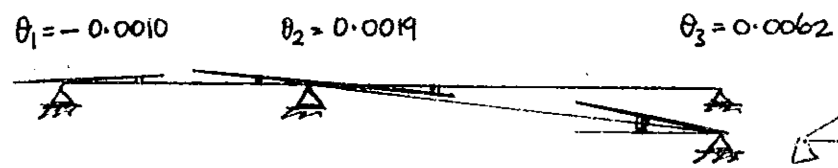
It follows that $\theta_1 = -0.00097$ radians.

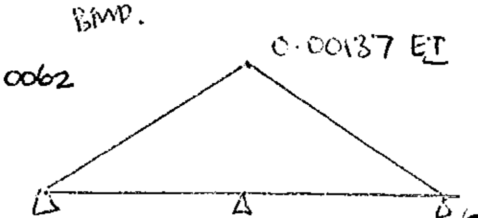
and $\theta_3 = +0.0062$ radians.

Backsubstitute these values into the Slope-Deflection equation.

$$M_{21} = \frac{2EI}{4.25}\left(2\times 0.00194 - 0.0010\right) = 0.00137\,EI$$

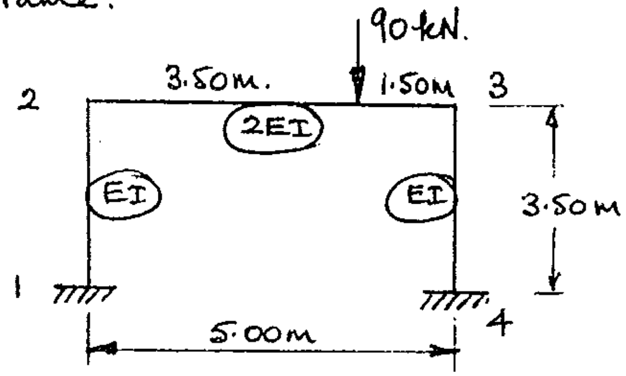
$$M_{23} = \frac{2EI}{6.25}\left(2\times 0.00194 + 0.0062 - 0.0144\right) = -0.00137\,EI.$$

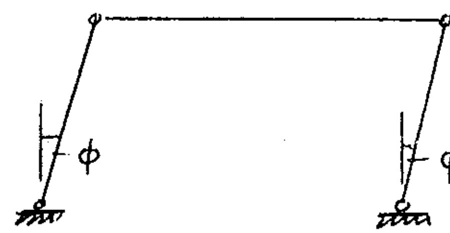




Example. 3.

Determine the distribution of bending moments in the following frame.





Sway Mechanism.

F.E.M.

$$M^F_{23} = -28.35 \text{ kN.m.}$$

$$M^F_{32} = +66.15 \text{ kN.m.}$$

Slope-deflection equations.

$$M_{12} = \frac{2EI}{3.50}\left(2\theta_1 + \theta_2 - 3\phi\right) \quad \text{------ (1)}$$

$$M_{21} = \frac{2EI}{3.50}\left(2\theta_2 + \theta_1 - 3\phi\right) \quad \text{------ (2)}$$

$M_{12}^F = M_{21}^F = 0$

$$M_{23} = \frac{2\times 2EI}{5.00}\left(2\theta_2 + \theta_3\right) - 28.35 \quad \text{---- (3)}$$

$$M_{32} = +66.15 + \frac{2\times 2EI}{5.00}\left(2\theta_3 + \theta_2\right) \quad \text{---- (4)}$$

$$M_{34} = \frac{2EI}{3.50}\left(2\theta_3 + \theta_4 - 3\phi\right) \quad \text{--------- (5)}$$

$$M_{43} = \frac{2EI}{3.50}\left(2\theta_4 + \theta_3 - 3\phi\right) \quad \text{--------- (6)}$$

11 unknowns — 6 equations.

Comparability $\quad \theta_1 = 0 \quad$ ---------- (7)

$\theta_4 = 0 \quad$ ------ (8)

Equilibrium $\quad M_{21} + M_{23} = 0 \quad$ ------ (9)

$M_{32} + M_{34} = 0. \quad$ ------ (10)

One more equation is required namely the "Moment-Sway" equation of equilibrium.

$$M_{12} + M_{21} + M_{34} + M_{43} = 0 \quad \text{------ (11).}$$

Solve :-

(a) Substitute equations (2) and (3) into equation (9), equating $\theta_1 = 0$ (7)

$$2.743\,\theta_2 + 0.8\,EI\,\theta_3 - 1.714\,\phi = 28.35/EI.$$

(b) Substitute equations (4) and (5) into equation (10), equating $\theta_4 = 0$ (8)

$$0.8\,\theta_2 + 2.743\,\theta_3 - 1.714\,\phi = -66.15/EI.$$

(c) Substitute equations (1), (2), (5) and (6) into equation (11)

$$\theta_2 + \theta_3 - 4\phi = 0.$$

In matrix form

$$\begin{bmatrix} +2.743 & +0.80 & -1.714 \\ +0.8 & +2.743 & -1.714 \\ +1 & +1 & -4 \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_3 \\ \phi \end{Bmatrix} = \begin{Bmatrix} 28.35/EI \\ -66.15/EI \\ 0 \end{Bmatrix}$$

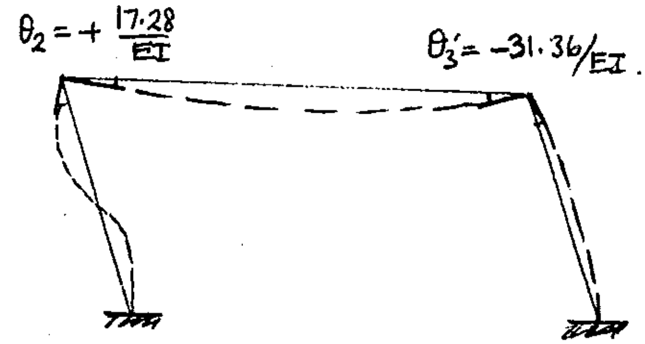
Solve to get $\quad \theta_2 = \dfrac{17.28}{EI}$

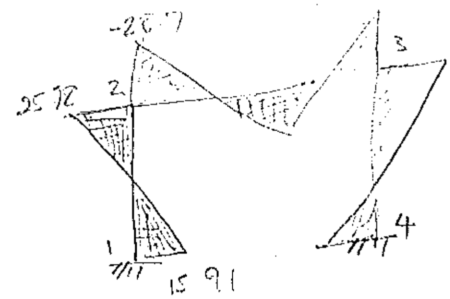
$$\theta_3 = -\frac{31.36}{EI}$$

8

+ 3.52

Note that ϕ is negative so the frame sways in a direction opposite to the sway mechanism selected on page 4.





Backsubstitute to get the joint moments.

$$M_{12} = \frac{2EI}{3.5}\left(\frac{17.28}{EI} + 3\times\frac{3.52}{EI}\right) = +15.91 \text{ kN.m.}$$

$M_{21} = +25.78$ kN.m.

$M_{23} = -25.78$ kN.m.

$M_{32} = +29.81$ kN.m.

$M_{34} = -29.80$ kN.m.

$M_{43} = -11.89$ kN.m.