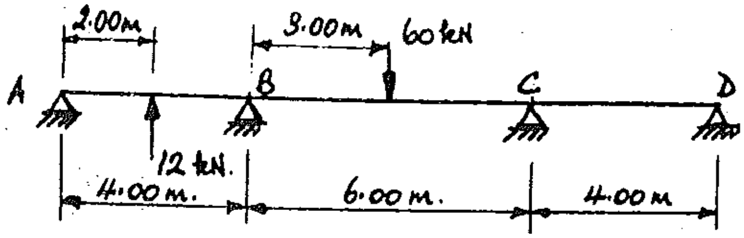
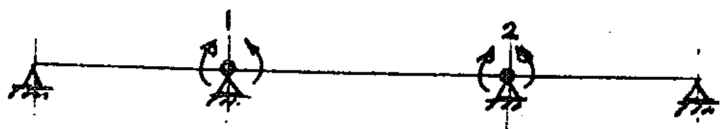
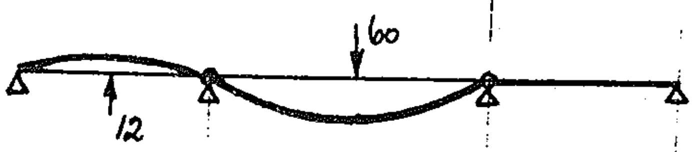
M Austin

Example of a continuous beam analysed by the flexibility coefficient method.



The section stiffness of spans AB and CD is EI, and BC is 2EI. Release the internal moments at supports B and C, and use point directions shown.



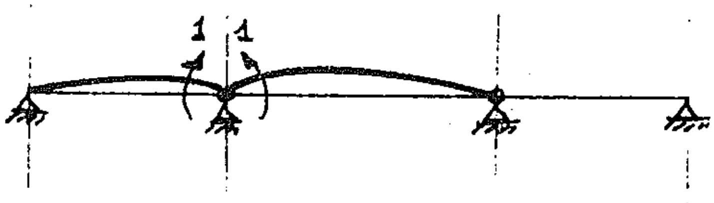


Rotations due to applied loads

Span AB, $\theta_B = \dfrac{12}{EI}$

Span BC, $\theta_B = -\dfrac{67.5}{EI}$; $\theta_C = -\dfrac{67.5}{EI}$

Total Rotations $\theta_B = -\dfrac{55.5}{EI}$; $\theta_C = -\dfrac{67.5}{EI}$



Rotations due to a unit moment at B

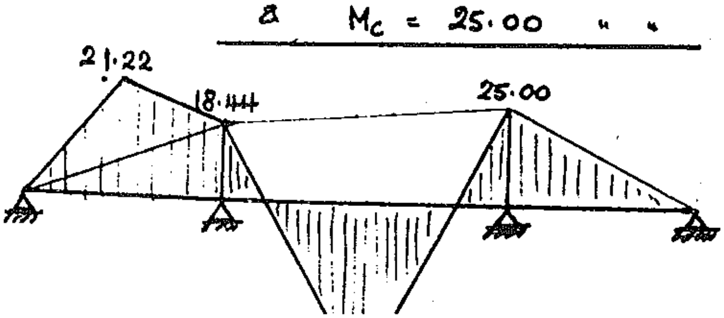
$f_{11} = \dfrac{2.33}{EI}$ $f_{21} = \dfrac{0.50}{EI}$.

Matrix relationship

$$-\begin{Bmatrix} -\dfrac{55.5}{EI} \\ -\dfrac{67.5}{EI} \end{Bmatrix} = \begin{bmatrix} \dfrac{2.33}{EI} & \dfrac{0.50}{EI} \\ \dfrac{0.50}{EI} & \dfrac{2.33}{EI} \end{bmatrix} \begin{Bmatrix} M_B \\ M_C \end{Bmatrix}$$

$\therefore$ $M_B = 18.44$ kN. m.

& $M_C = 25.00$ " "



Example (1). Temperature problem (suitable for lack-of-fit, shrinkage, creep a

A portal frame shown in Figure 1 is constructed from concrete ($E = 26000$ MPa) and has a constant cross section ($I = 700 \times 10^{-6}\ m^4$). Obtain the distribution of bending moments introduced when the temperature of the left-hand column rises $10°C$, the beam $20°C$ and the right-hand column $0°C$. The coefficient of thermal expansion of concrete α is $10^{-5}/°C$.

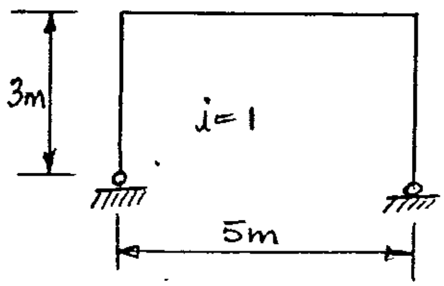
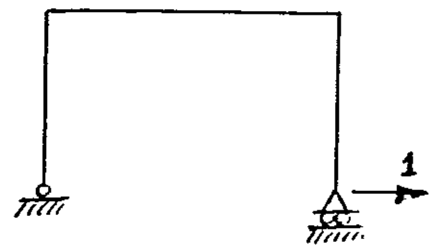
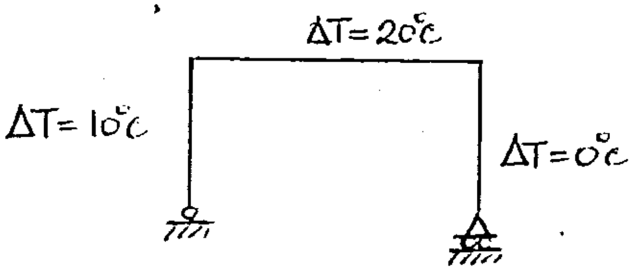


Figure 1



Point direction for horizontal redundant reaction release.

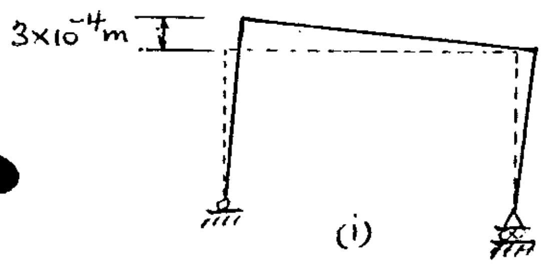


Expansion of

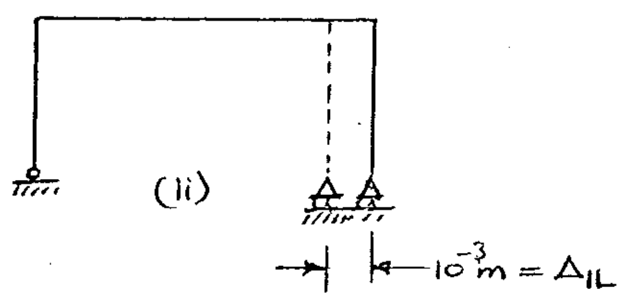
(i) left-hand column $= 10 \times 10^{-5} \times 3 = 3 \times 10^{-4}$ m

(ii) beam $= 20 \times 10^{-5} \times 5 = 10^{-3}$ m

(iii) right-hand column $= 0$

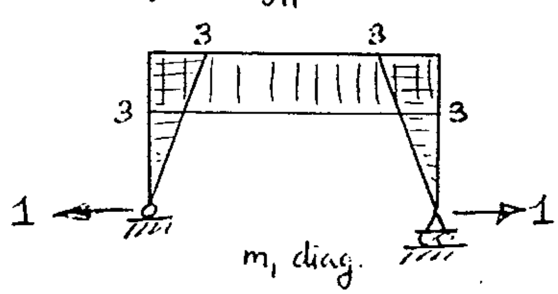


(i)



(ii)

Now obtain f_{11}



m_1 diag.

$$f_{11} = \int_0^L \frac{m_1^2}{EI} ds$$

$$= \frac{2}{EI}\int_0^3 x^2\,dx + \frac{1}{EI}\int_0^5 3^2\,dx.$$

$$= \frac{2}{EI}\left[\frac{x^3}{3}\right]_0^3 + \frac{9}{EI}\left[x\right]_0^5$$

$$= \frac{63}{EI}.$$

$f_{11} P_1 = -\Delta_{1L} \quad \therefore \quad P_1 = -0.0000158\,EI = -288$ N

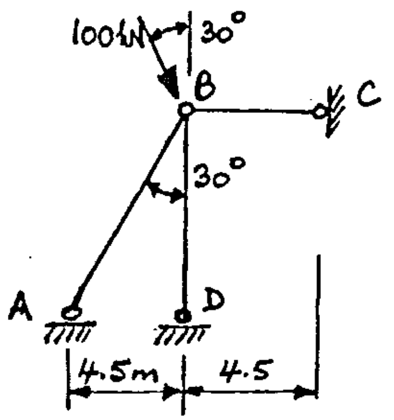
863 ... 863 N.m.

1) Member shortening.

$E = 20\ \text{kN/mm}^2$

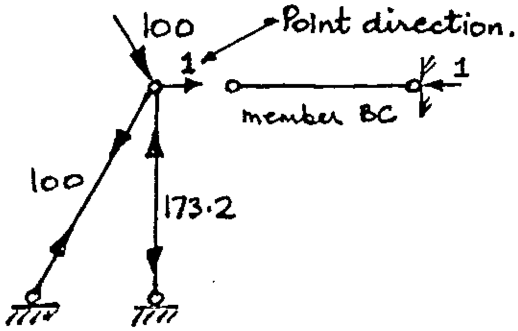
$A_{AB} = A_{BD} = 1000\ \text{mm}^2$

$A_{BC} = 1500\ \text{mm}^2$



$i = 6+3-(2\times4) = 1$

∴ Release one member force – eg. $\overline{BC}$. Apply 100kN load and determine the deflection in point direction 1 of the primary structure



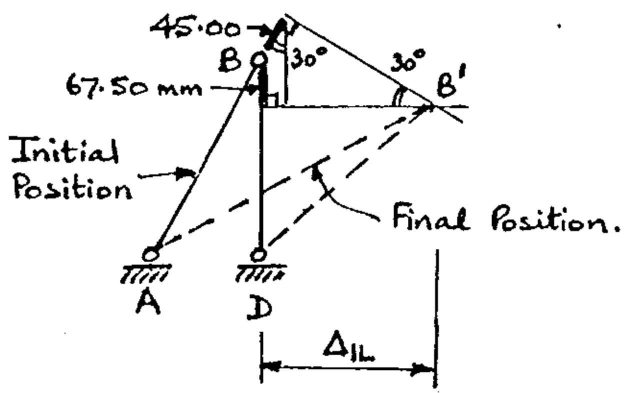
Shortening of BD

$$= \frac{173.2\times\sqrt{3}\times4500}{1000\times20} = 67.50\ \text{mm}$$

Lengthening of AB

$$= \frac{100\times2\times4500}{1000\times20} = 45.00\ \text{mm}$$

Displacement diagram :-



$$\Delta_{1L} = 45\sin30° + \left(\frac{67.50 + 45\cos30°}{\tan30°}\right)$$

$$\therefore \Delta_{1L} = 207.2\ \text{mm.}$$

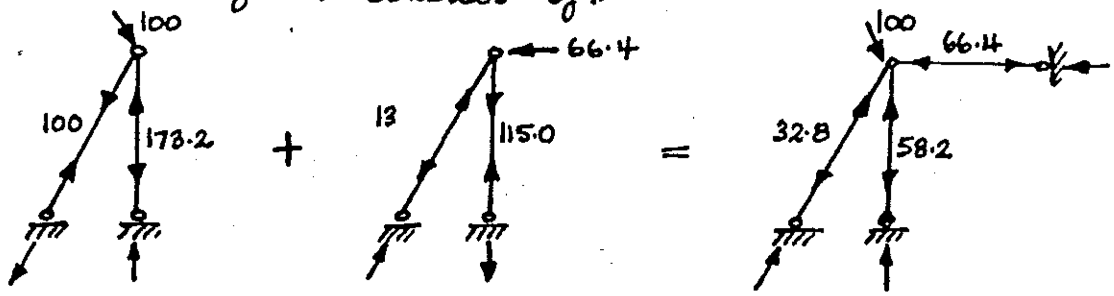
Shortening of the released member $= \dfrac{\overline{BC}\times4500}{1500\times20} = 0.15\,\overline{BC}$

The flexibility coefficient f_{11} has been determined in an earlier problem and is $13.2L/AE$ where L is the distance AD and AE are the member props. AB, BD $\therefore f_{11} = \dfrac{13.2\times4500}{1000\times20} = 2.97$

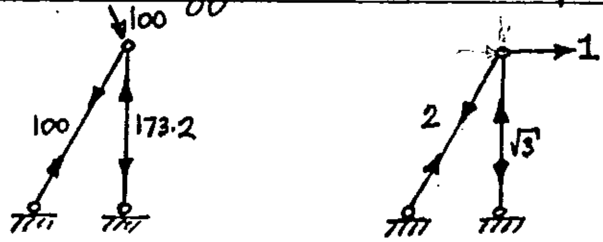
Eqn. compat.: $2.97\,\overline{BC} + 0.15\,\overline{BC} = -207.2$

$$\therefore \overline{BC} = -66.4\ \text{kN}$$

The final member forces consists of:-



<u>Using the Energy theorem "Principle of Virtual Forces",</u>

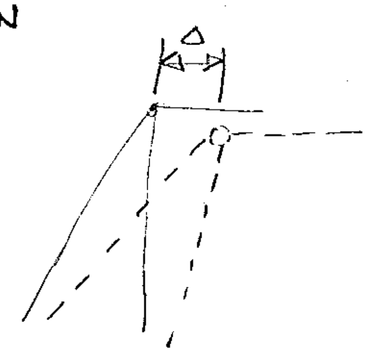


'F' forces   'f' forces.

Member	L/AE	F	f	$\frac{FfL}{AE}$	$\frac{f^2L}{AE}$	$f.\overline{BC}+F$
AB	0.4500	100	2	90	1.800	−32.6
BD	0.3897	−173.2	$-\sqrt{3}$	116.9	1.169	−58.4
BC	0.1500		1		0.150	−66.3
			Σ =	206.9	3.119	
				Δ	f_{11}	

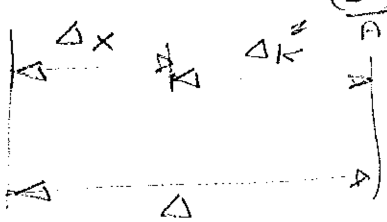
$\frac{L}{AE}$ Term

Redundant Force $\quad \overline{BC} = -\dfrac{206.9}{3.119} = -66.3$ kN



$$BC = \frac{-\Delta}{f_{11}}$$

$$\Leftrightarrow \quad f_{11}\,\underline{BC} + \Delta = 0$$



$$\Delta K = \frac{(BC)L}{AE}$$

$$\Delta K = \frac{(\overline{BC})L}{AE} =$$

$$\Delta_K = \frac{(\overline{BC})L}{AE} \qquad \Delta =$$

$$-\Delta = f_{11}P_1 = \Delta$$

$$\Delta X = \Delta - \Delta K$$
$$= f_{11}P_1 - BC\left(\frac{L}{AE}\right)$$