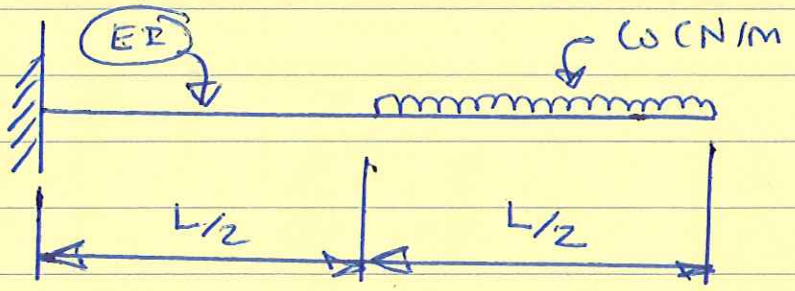
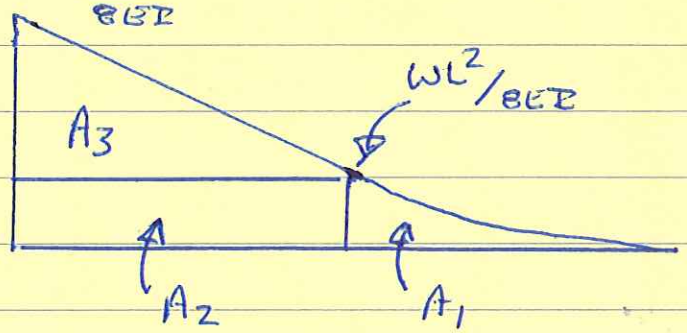
ENCE 353 : Homework 4 Soln's

Q1.



M/EI diagram.



$$A_1 = \frac{1}{3}\frac{L}{2}\cdot\frac{wL^2}{8EI} = \frac{wL^3}{48EI}$$

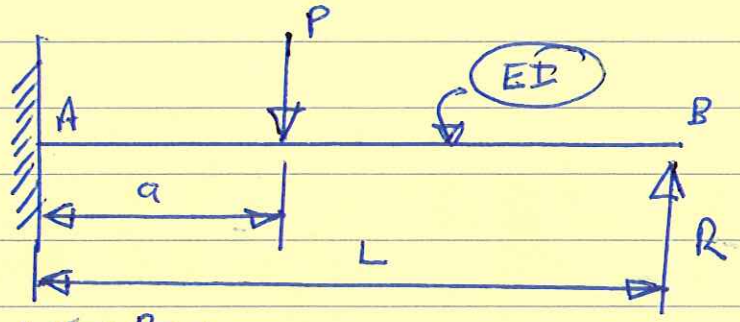
$$A_2 = \frac{wL^3}{16EI}$$

$$A_3 = \frac{wL^3}{16EI}$$

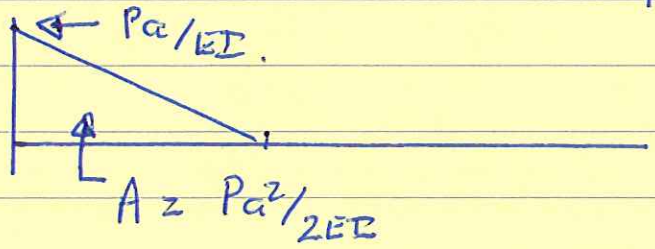
$$\Delta_B = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3$$

$$= \frac{WL^3}{EI}\left[\frac{1}{48}\cdot\frac{3L}{8} + \frac{1}{16}\cdot\frac{3L}{4} + \frac{1}{16}\cdot\frac{5L}{6}\right] = \frac{41wL^4}{384EI}.$$

Q2.



M/EI



(2)

Apply Moment-area to get Δ_B

$$\Delta_B = A\left[L - a + \frac{2}{3}a\right]$$

$$= \frac{Pa^2}{2EI}\left[L - a/3\right]$$

The vertical displacement of the cantilever tip due to a reaction force R

$$\Delta_R = \frac{-RL^3}{3EI}$$

Compatibility of displacements

$$\Delta_B + \Delta_R = 0 \Rightarrow R = \frac{3}{2}P\left[\frac{a}{L}\right]^2\left[1 - \frac{1}{3}\left(\frac{a}{L}\right)\right].$$

Moment at cantilever support

$$M(a) = Pa - RL = Pa - \frac{3}{2}PL\left[\frac{a}{L}\right]^2\left[1 - \frac{1}{3}\left(\frac{a}{L}\right)\right]$$

Notice that $M(0) = M(L) = 0$, Max moment occurs when

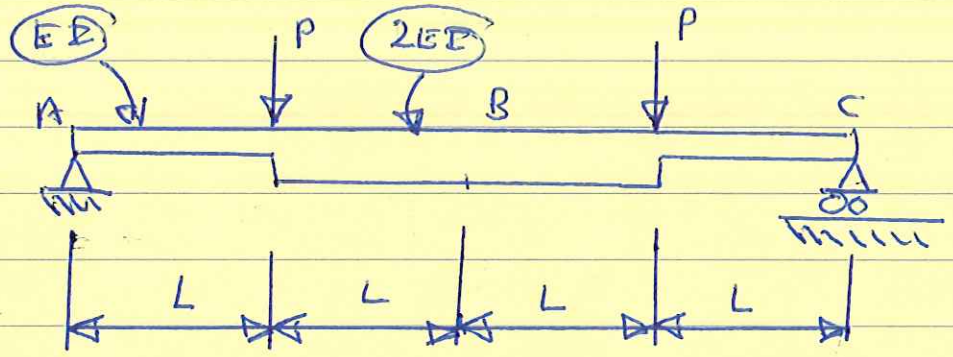
$$dM/da = 0 \Rightarrow \frac{dM}{da} = P\left[1 - \frac{3}{2}\left(2\left(\frac{a}{L}\right) - \left(\frac{a}{L}\right)^2\right]\right) = 0$$

if $P \neq 0 \Rightarrow 2(a/L) - (a/L)^2 = 2/3$

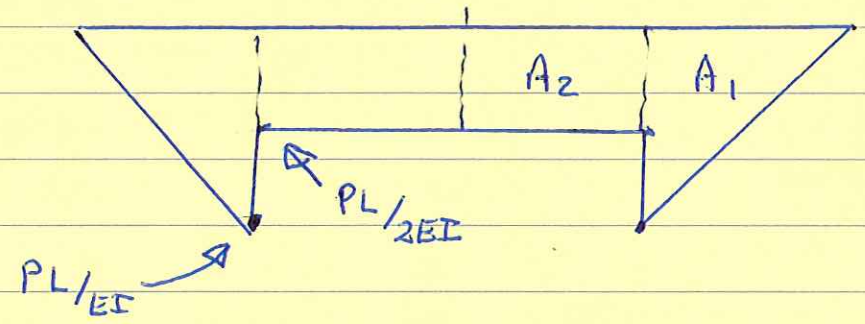
$$\Rightarrow \boxed{a = \left(1 - 1/\sqrt{3}\right)L}$$

(3)

Q3



M/EI diagram



$A_1 = \frac{1}{2}\frac{PL^2}{EI} = A_2$

θ_A = Area of M/EI diagram between A & B

$$= \left(\tfrac{1}{2} PL^2 + \frac{PL^2}{2}\right) \tfrac{1}{EI}$$

$$= \frac{PL^2}{EI}.$$

A_B = first moment of M/EI diagram between A & B, evaluated about A.

$$= A_1 \cdot \left(\tfrac{2}{3}L\right) + A_2 \left(L + \tfrac{L}{2}\right)$$

$$= \left(\tfrac{1}{3} + \tfrac{3}{4}\right)\frac{PL^3}{EI} = \frac{13}{12}\frac{PL^3}{EI}$$