Department of Civil and Environmental Engineering, Fall Semester, 2015

ENCE 353 Final Exam, Open Notes and Open Book

Name : ________________________

Exam Format and Grading. The exam will be 2 hrs plus five minutes to read the questions.

Answer question 1. Then answer **three of the four** remaining questions. Questions 2, 3 and 4 are worth a maximum of 10 points each. Question 5 is a **super question** that is worth a maximum of 14 points out of 10. There is no penalty for not answering question 5. But if you choose to take it on, you could score as many as 44 points out of 40 on the exam, and have them all count toward your final grade.

Only the first four questions that you answer will be graded, so please cross out the question you do not want graded in the table below.

Partial credit will be given for partially correct answers, so please show all your working.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	14	
Total	40	

Question 1: 10 points

COMPULSORY: Moment-Area and Deflections. Consider the cantilevered beam structure shown in Figure 1.

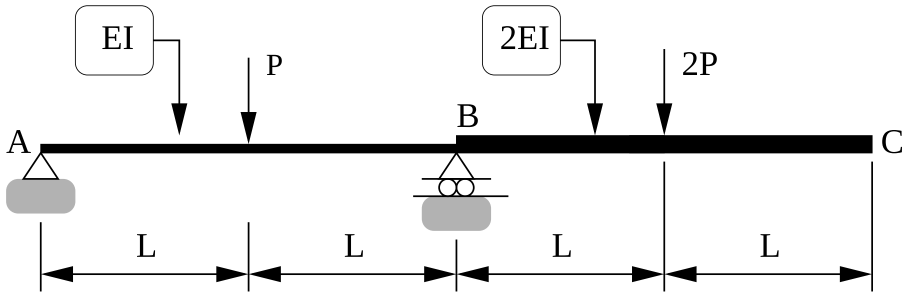

Figure 1: Front elevation view of a cantilevered beam structure.

Notice that segments A-B and B-C have cross-sectional properties EI and 2EI, respectively.

[1a] (2 pts) Compute and draw the M(x)/EI diagram for the complete beam A-B-C.

[1b] (2 pts) Draw and label a diagram of the deflected shape. Clearly indicate on your diagram regions of the beam having zero curvature.

[1c] (2 pts) Draw and label a diagram showing how the rotation at A is related to the beam deflections at points B and C.

[1d] (4 pts) Use the method of moment-area to compute the vertical deflection of the beam at point C.

Question 2: 10 points

OPTIONAL: Member forces in a Propped Cantilever. Figure 2 is an elevation view of a propped cantilever structure that carries external point loads P at points B and C. The cantilever support is fully fixed at point A.

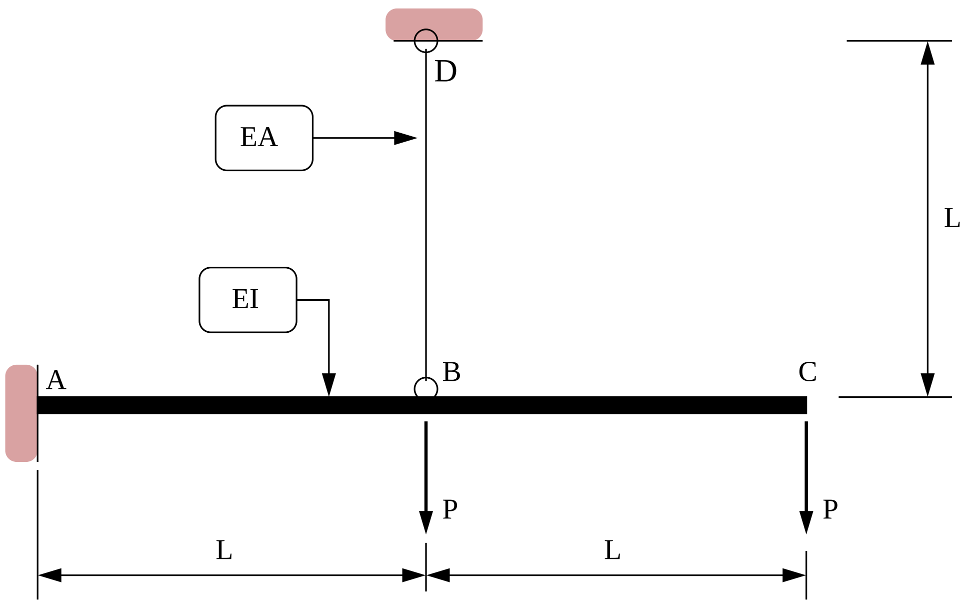

Figure 2: Elevation view of a propped cantilevel beam.

The structural system has constant section properties EI along the beam, and is supported by a truss element having section properties EA.

[2a] (1 pt) Compute the degree of indeterminacy for the propped cantilever beam.

[2b] (5 pt) Using the method of moment-area, or otherwise, show that the axial force T in the truss element is related to the externally applied loads P by the equation:

$$\frac{7}{6}\frac{PL^3}{EI} = \frac{TL^3}{3EI} + \frac{TL}{EA}. \tag{1}$$

Question 2b continued ...

[2c] (4 pt) Explain how the value of bending moment at the cantilever support (i.e., at point A) will change as a function of the problem parameters (i.e., P, T, E, I and A).

Question 3: 10 points

OPTIONAL: Flexibility Matrices. Consider the truss structure shown in Figure 3.

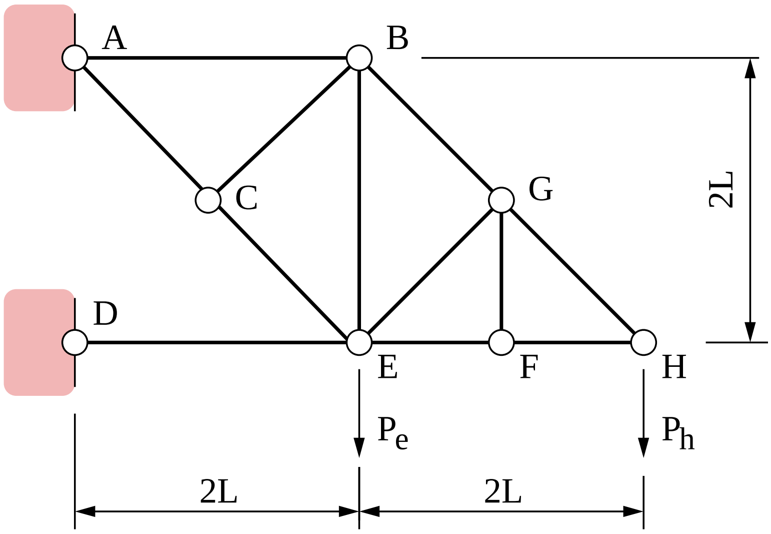

Figure 3: Elevation view of a pin-jointed truss.

The horizontal and vertical degrees of freedom are fully-fixed at supports A and D. The truss carries vertical loads P_e and P_h at nodes E and H, respectively. All frame members have cross section properties AE.

[3a] (2 pts) Use the method of joints to identify all of the zero-force members. Label these members on Figure 3.

[3b] (8 pts) Use the principle of virtual forces to compute the two-by-two flexibility matrix connecting the vertical displacements at points E and H to applied loads P_e and P_h, i.e., as a function of P_e, P_h, L and AE.

$$\begin{bmatrix} \triangle_e \\ \triangle_h \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} P_e \\ P_h \end{bmatrix}. \tag{2}$$

Question 3 continued ...

Question 4: 10 points

OPTIONAL: Support Reactions in a Two-Span Beam. Figure 4 is an elevation view of a two-span beam that carries a uniform load w (N/m) along its entire length.

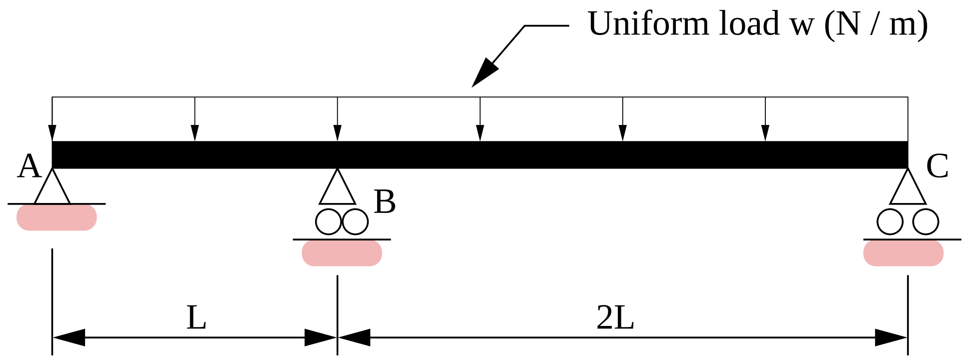

Figure 4: Elevation view of a two-span beam.

The beam has constant section properties EI along its length.

[4a] (10 pts) Compute the vertical support reactions at A, B and C, and the distribution of bending moments along the beam. When you are finished, check that the sum of the vertical reactions = $3wL$. Draw and label the bending moment diagram.

Question 4a continued ...

Question 5: 14 points

OPTIONAL: Compute Support Reactions in a Suspension Bridge. Figure 5 is a front elevation view of a suspension bridge that has three spans, two support towers, and anchor supports at the bridge endpoints.

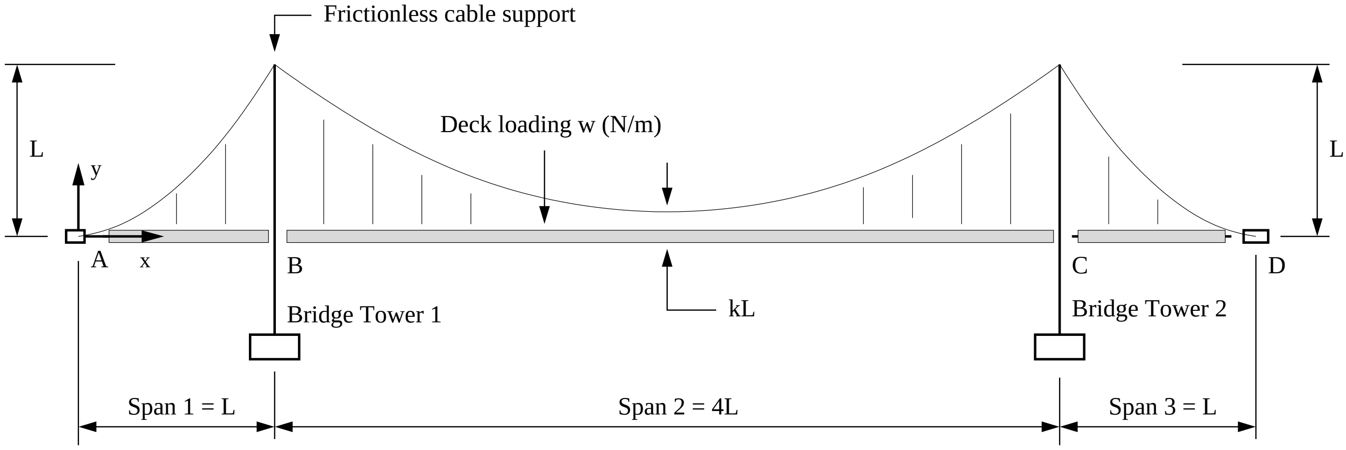

Figure 5: Elevation view of a three-span suspension bridge.

Spans 1, 2 and 3 have lengths L, $4L$ and L, respectively, and carry a uniform load w (N/m). The towers have height L above the bridge deck level. At the mid-point of Span 2, the lowest point of the cable profile is kL above the bridge deck.

The purpose of this question is to work step by step toward the computation of vertical support reactions at the cable anchor points (i.e., at points A and D), and at the towers (i.e., at points B and C). For the purposes of this analysis, assume that the bridge cable weight is bundled into the applied loads w, and can otherwise be ignored. Also, assume that the cable passes through the top of the towers on a frictionless support and, as a result, the horizontal component of cable force will be constant along the entire bridge.

[5a] (3 pts) By examining equilibrium of the cable profile in Span 2, show that the horizontal component of cable force is:

$$H = \left[\frac{2}{1-k}\right] wL. \tag{3}$$

[5b] (3 pts) Show that the equation of the cable profile in Span 1 is:

$$y(x) = \left[\frac{1-k}{4L}\right]x^2 + \left[\frac{3+k}{4}\right]x \tag{4}$$

[5c] (3 pts) Use the results from parts [5a] and [5b] to show that the vertical reaction at the anchor support (i.e., at point A) is:

$$V_A = \left[\frac{3+k}{2-2k}\right] wL \tag{5}$$

acting downwards.

[5d] (3 pts) Hence, show that the vertical support reaction at Tower 1 is:

$$V_B = \left[\frac{9 - 5k}{2 - 2k}\right] wL. \tag{6}$$

acting upwards.

[5e] (2 pts) Write down a simple formula for the **total reaction force** at the cable anchor point A. (I am looking for a one-line answer here).