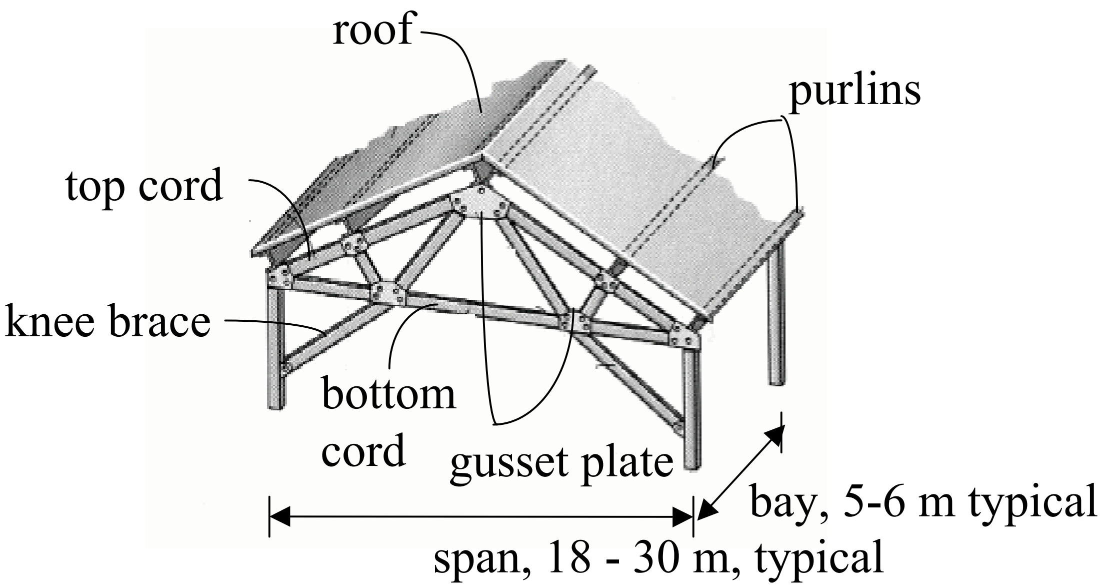
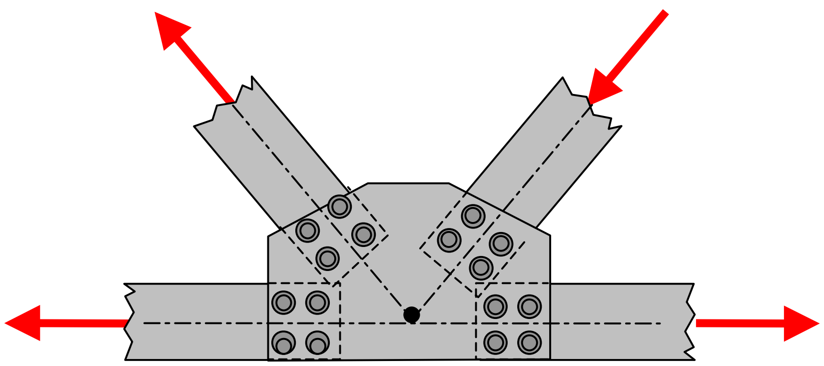
Analysis of Statically Determinate Trusses

- Common Types of Trusses
- Classification of Coplanar Trusses
- The Method of Joints
- Zero-Force Members
- The Method of Sections
- Compound Trusses
- Complex Trusses
- Space Trusses

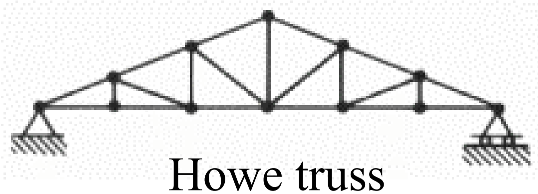
Common Types of Trusses

- **Roof Trusses**

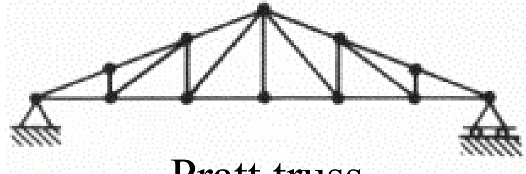




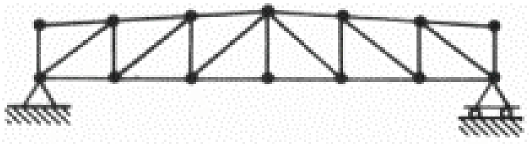
gusset plate



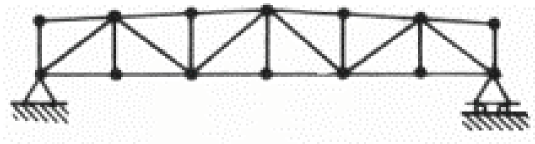
Howe truss
18 - 30 m



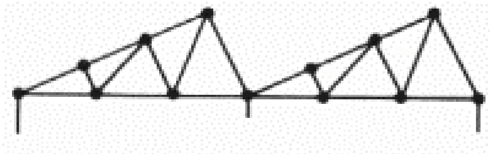
Pratt truss
18 - 30 m



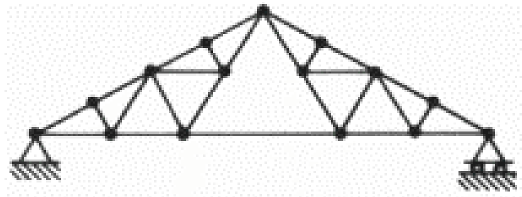
Howe truss
flat roof



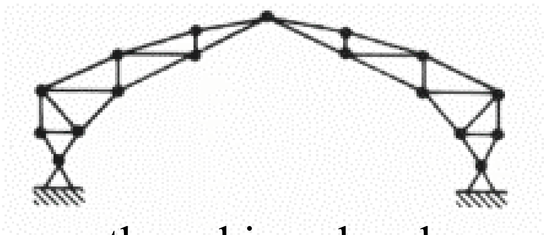
Warren truss
flat roof



saw-tooth truss
skylight

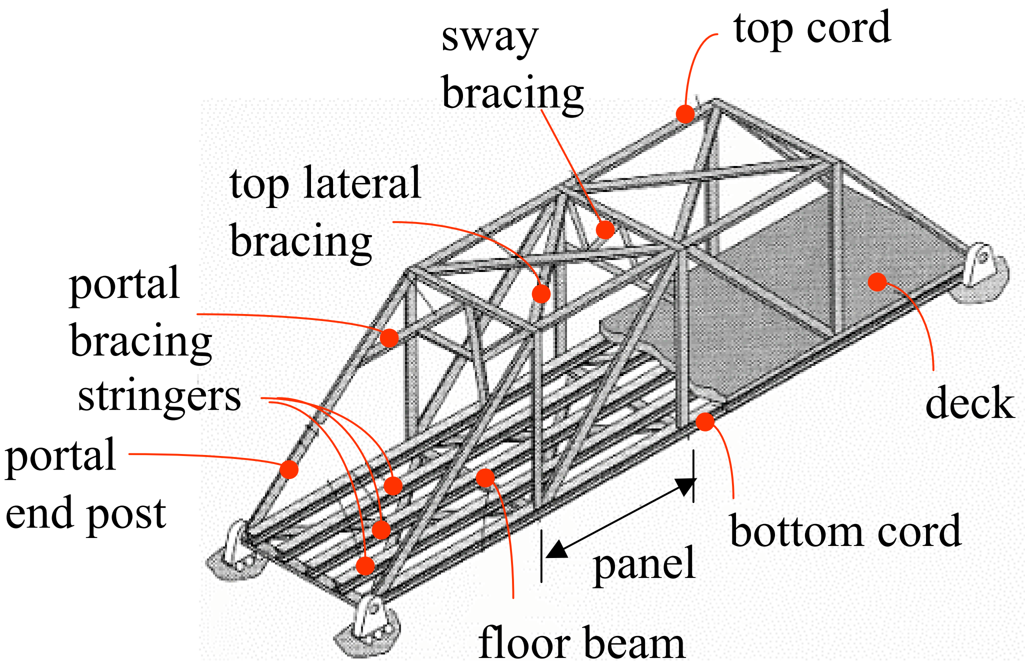


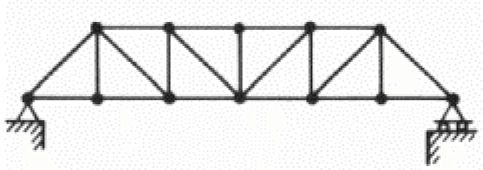
Fink truss
> 30 m



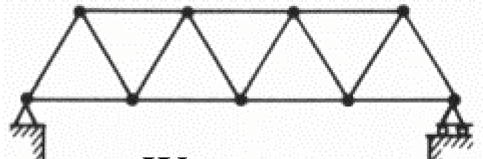
three-hinged arch
hangar, gymnasium

• **Bridge Trusses**

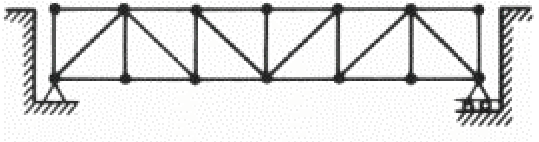




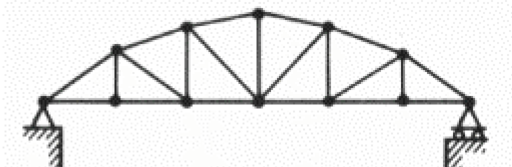
trough Pratt truss



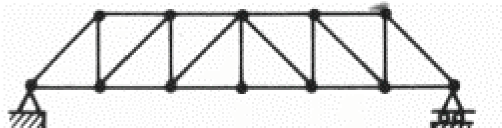
Warren truss



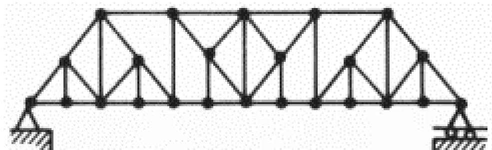
deck Pratt truss



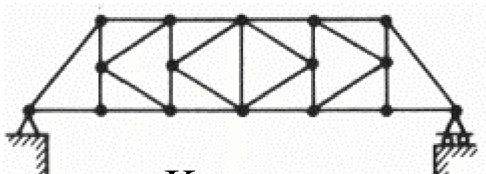
parker truss
(pratt truss with curved chord)



Howe truss



baltimore truss



K truss

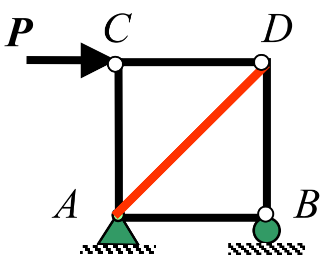
Assumptions for Design

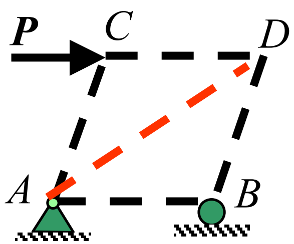
1. All members are connected at both ends by smooth frictionless pins.
2. All loads are applied at joints (member weight is negligible).

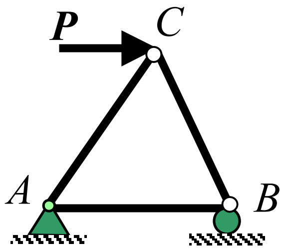
Notes: Centroids of all joint members coincide at the joint.
All members are straight.
All load conditions satisfy Hooke's law.

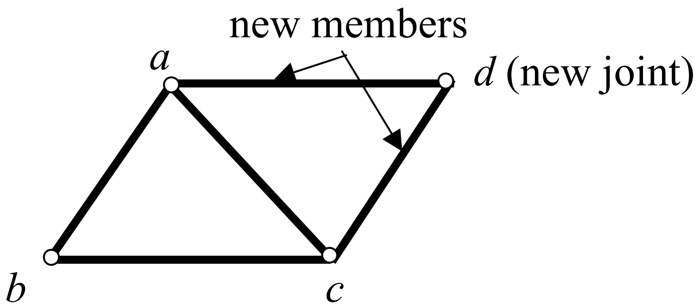
Classification of Coplanar Trusses

• **Simple Trusses**

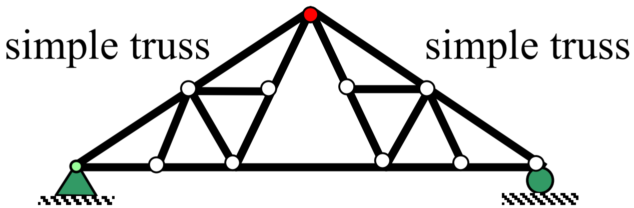




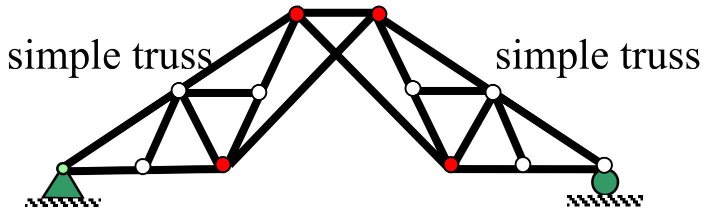




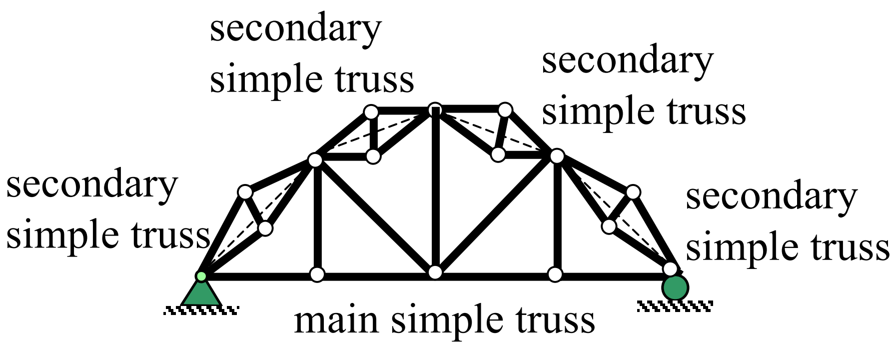
• **Compound Trusses**



Type 1

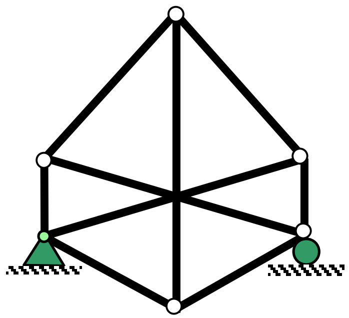


Type 2



Type 3

• **Complex Trusses**



• **Determinacy**

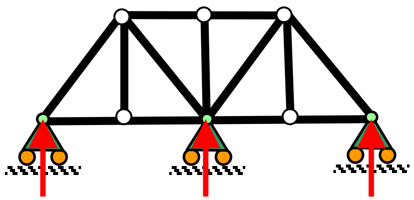
$b + r = 2j$	statically determinate
$b + r > 2j$	statically indeterminate

In particular, the degree of indeterminacy is specified by the difference in the numbers $(b + r) - 2j$.

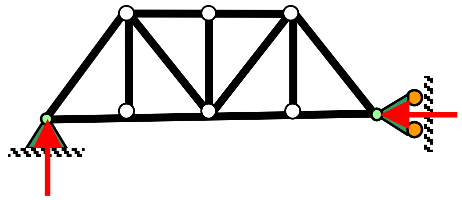
- **Stability**

$b + r < 2j$	unstable
$b + r \geqslant 2j$	unstable if truss support reactions are concurrent or parallel or if some of the components of the truss form a collapsible mechanism

External Unstable

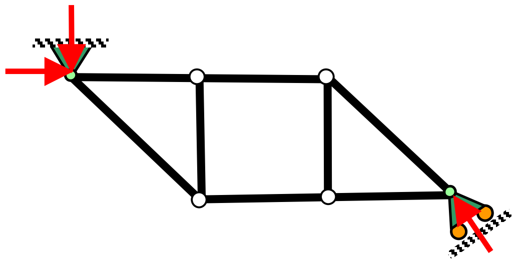


Unstable-parallel reactions

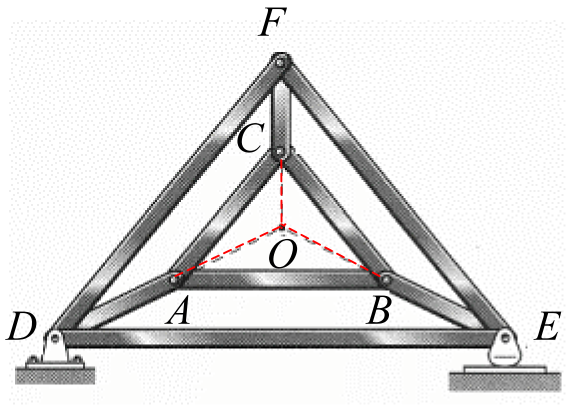


Unstable-concurrent reactions

Internal Unstable



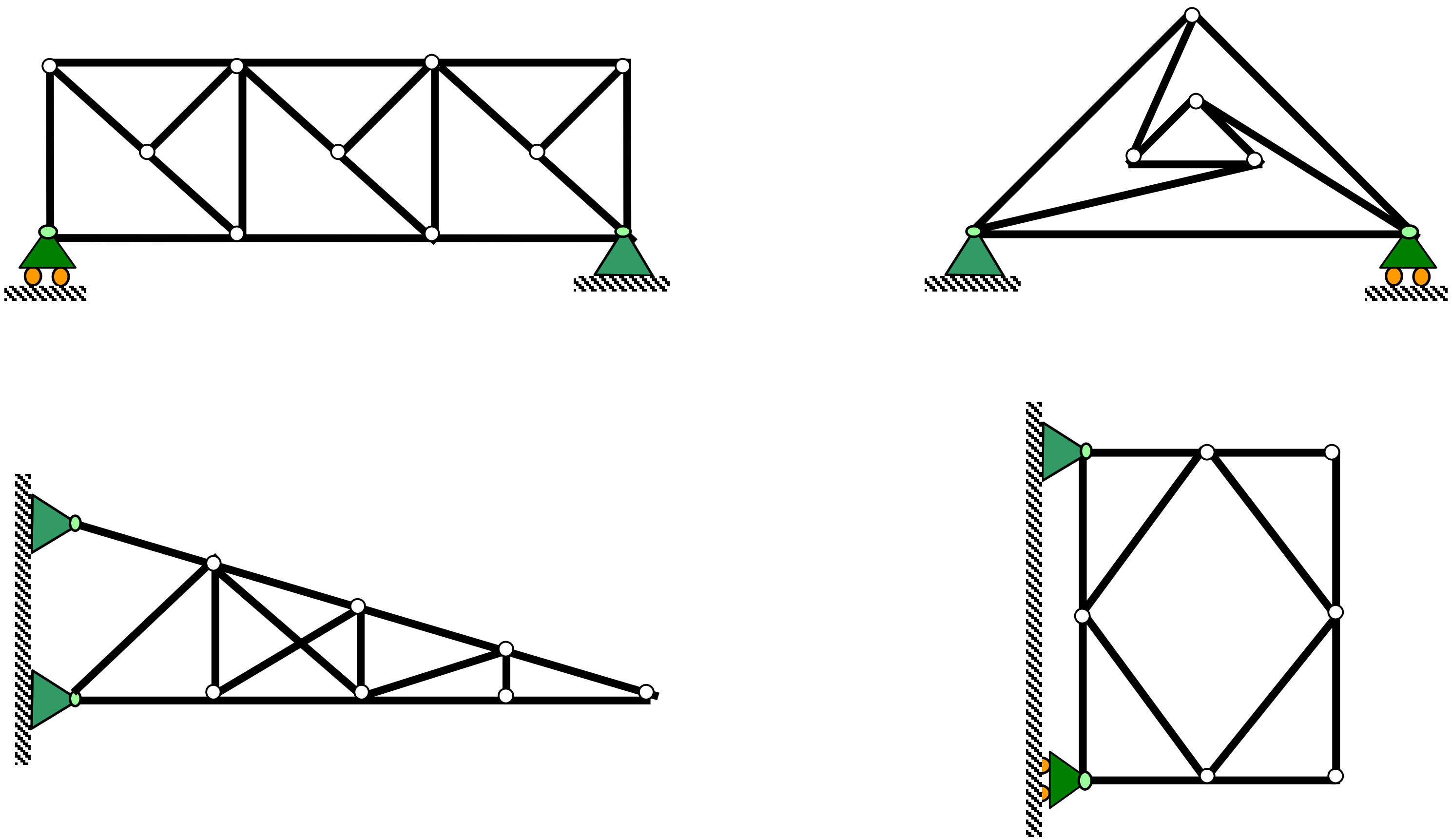
$$8 + 3 = 11 < 2(6)$$



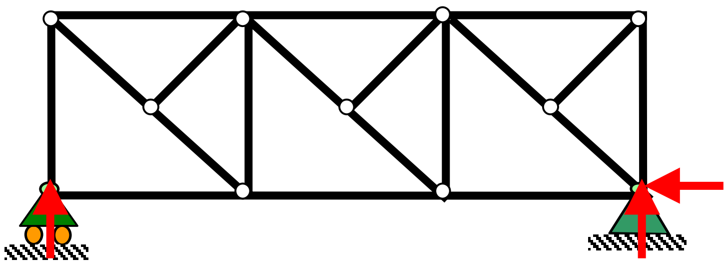
AD, *BE*, and *CF* are concurrent at point *O*

Example 3-1

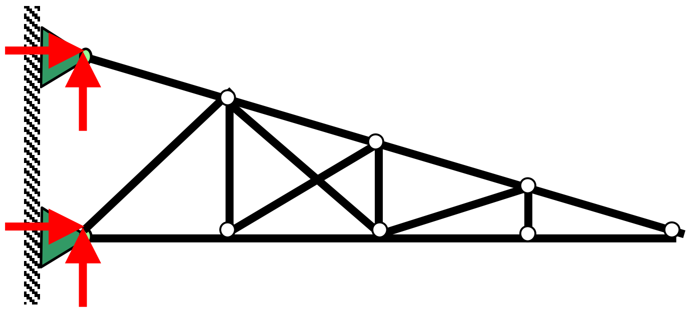
Classify each of the trusses in the figure below as stable, unstable, statically determinate, or statically indeterminate. The trusses are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the trusses.



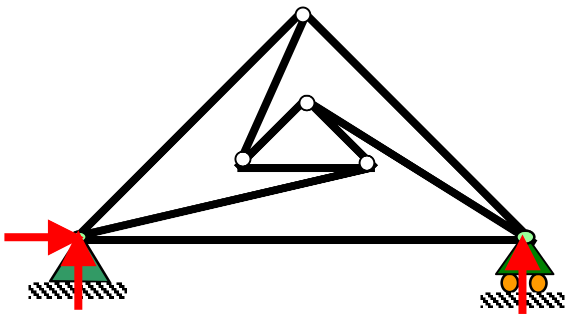
SOLUTION



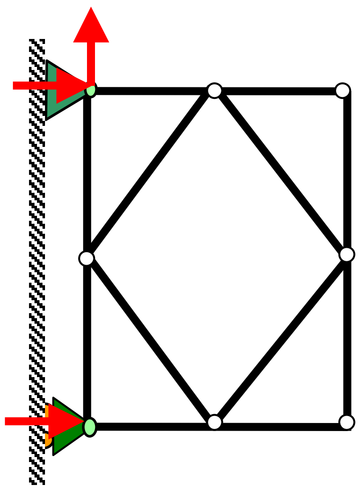
Externally stable, since the reactions are not concurrent or parallel. Since $b = 19$, $r = 3$, $j = 11$, then $b + r = 2j$ or $22 = 22$. Therefore, the truss is *statically determinate*. By inspection the truss is *internally stable.*



Externally stable. Since $b = 15$, $r = 4$, $j = 9$, then $b + r > 2j$ or $19 > 18$. The truss is *statically indeterminate* to the first degree. By inspection the truss is *internally stable.*

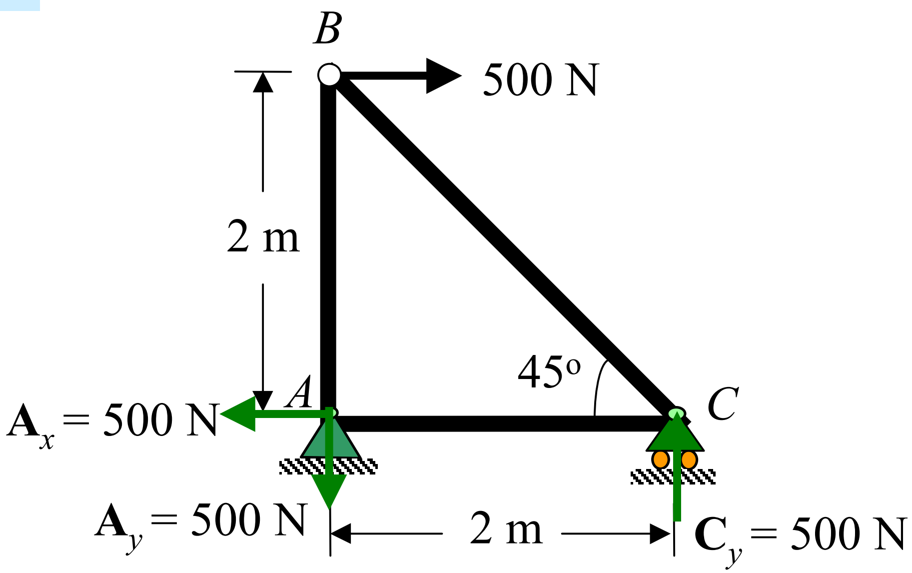


Externally stable. Since $b = 9$, $r = 3$, $j = 6$, then $b + r = 2j$ or $12 = 12$. The truss is *statically determinate*. By inspection the truss is *internally stable.*

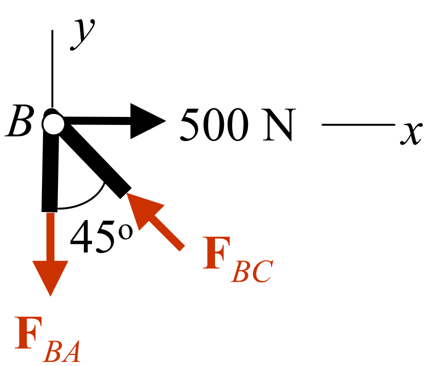


Externally stable. Since $b = 12$, $r = 3$, $j = 8$, then $b + r < 2j$ or $15 < 16$. The truss is *internally unstable.*

The Method of Joints



Joint B



$$\xrightarrow{+} \Sigma F_x = 0:$$

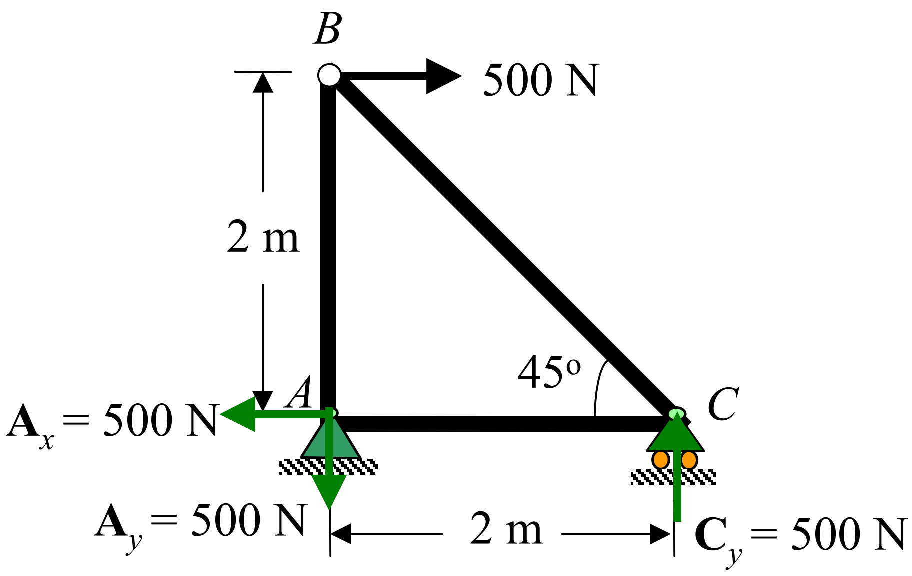
$$500 - F_{BC}\sin45^\circ = 0$$

$$F_{BC} = 707 \text{ N (C)}$$

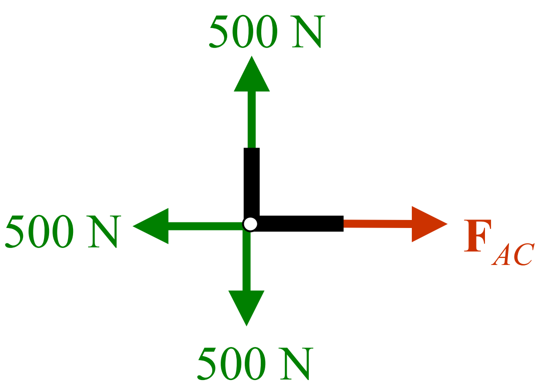
$$+\uparrow \Sigma F_y = 0:$$

$$-F_{BA} + F_{BC}\cos45^\circ = 0$$

$$F_{BA} = 500 \text{ N (T)}$$



Joint A

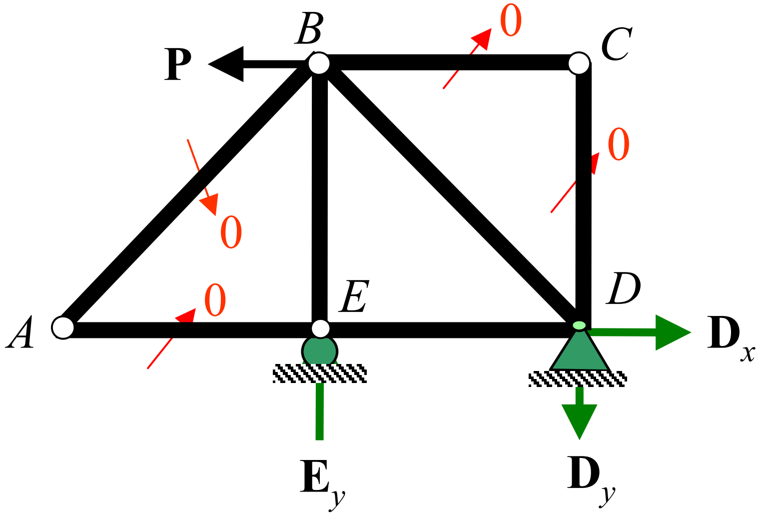


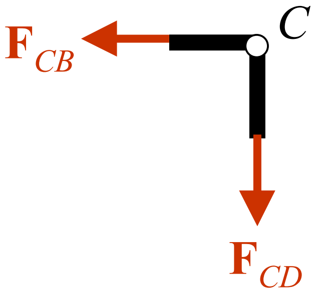
$\xrightarrow{+} \Sigma F_x = 0:$

$500 - F_{AC} = 0$

$F_{AC} = 500 \text{ N (T)}$

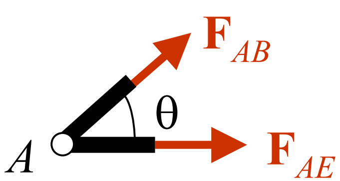
Zero-Force Members





$\xrightarrow{+} \Sigma F_x = 0$: $F_{CB} = 0$

$+\uparrow \Sigma F_y = 0$: $F_{CD} = 0$

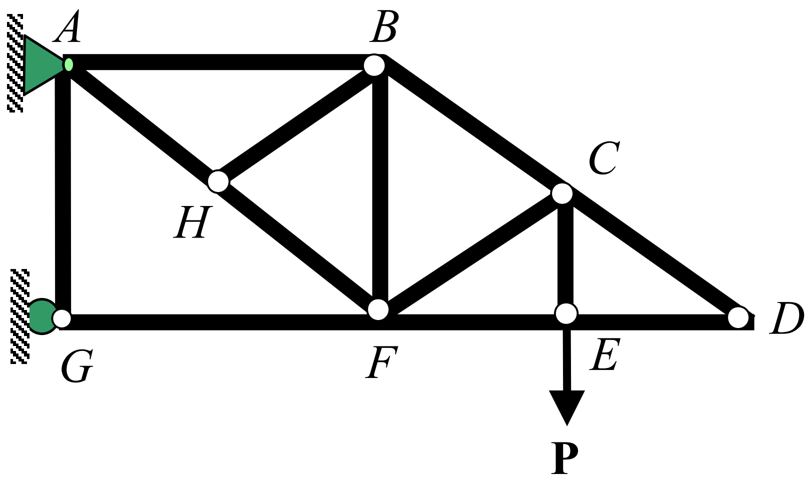


$+\uparrow \Sigma F_y = 0$: $F_{AB}\sin\theta = 0$, $F_{AB} = 0$

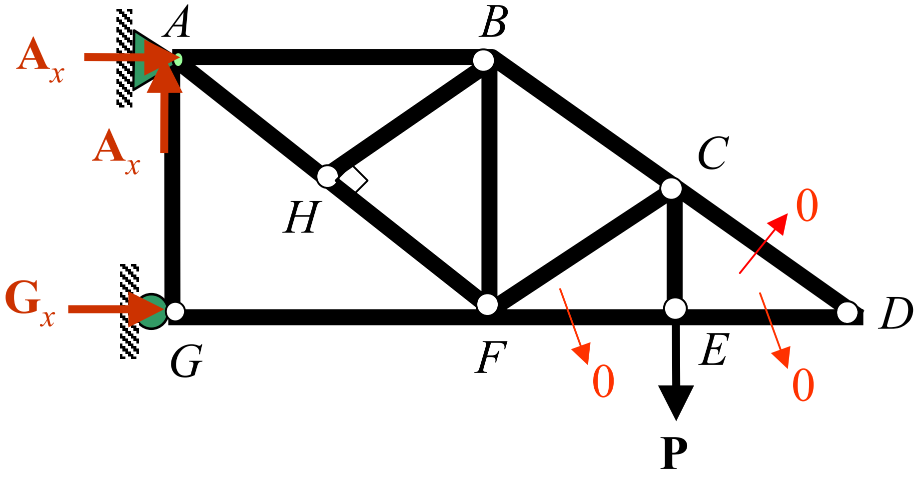
$\xrightarrow{+} \Sigma F_x = 0$: $F_{AE} + 0 = 0$, $F_{AE} = 0$

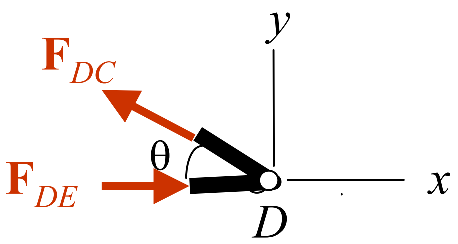
Example 3-4

Using the method of joints, indicate all the members of the truss shown in the figure below that have zero force.



SOLUTION

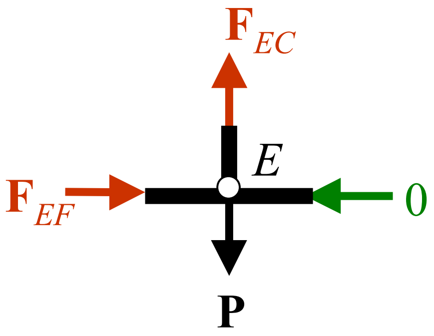




Joint D

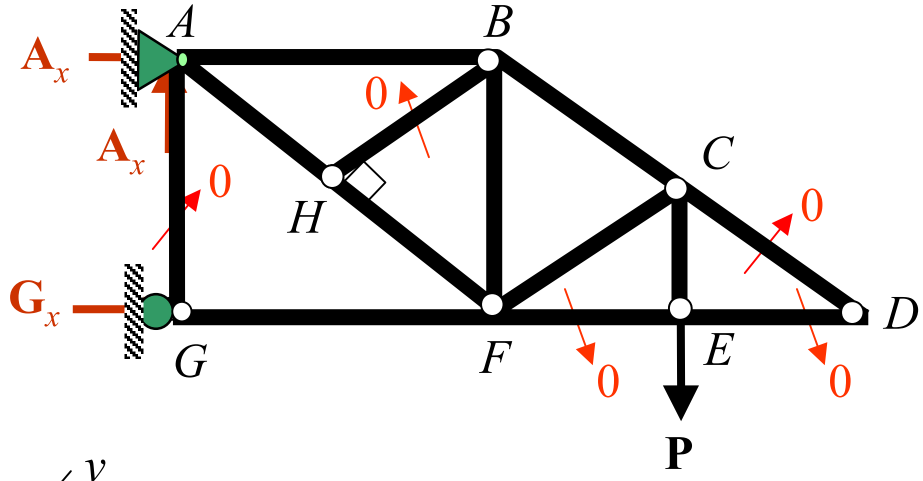
$$+\uparrow \Sigma F_y = 0: \quad F_{DC}\sin\theta = 0, \qquad F_{DC} = 0$$

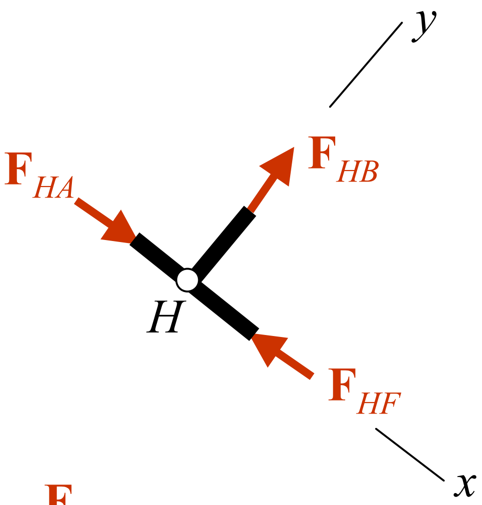
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{DE} + 0 = 0, \qquad F_{DE} = 0$$

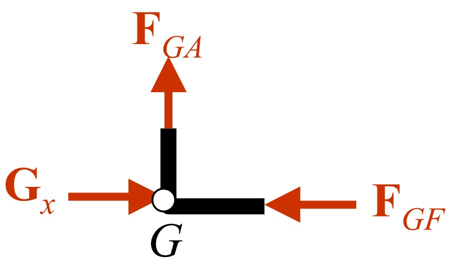


Joint E

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{EF} = 0$$







Joint H

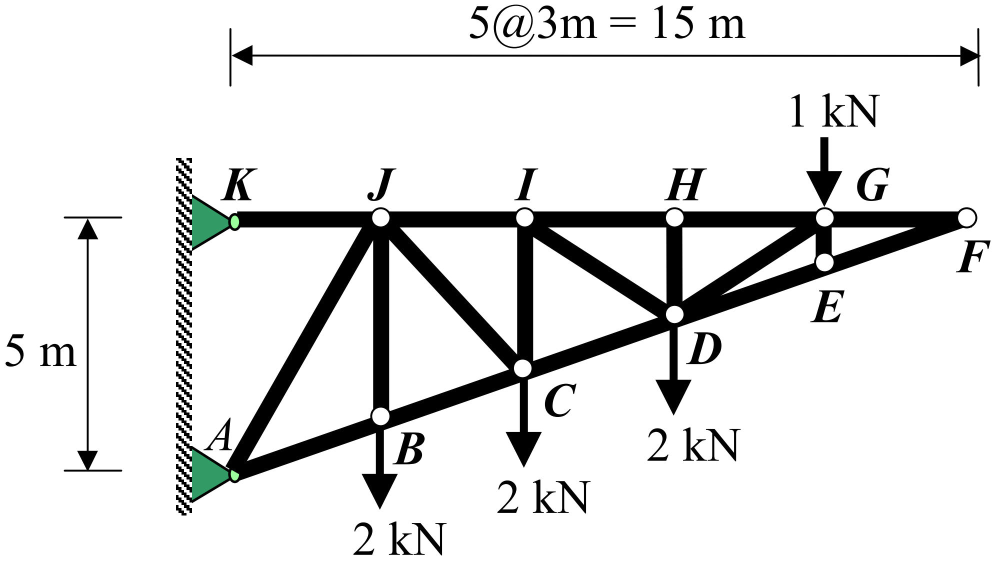
$$+\nearrow \Sigma F_y = 0: \qquad F_{HB} = 0$$

Joint G

$$+\uparrow \Sigma F_y = 0: \qquad F_{GA} = 0$$

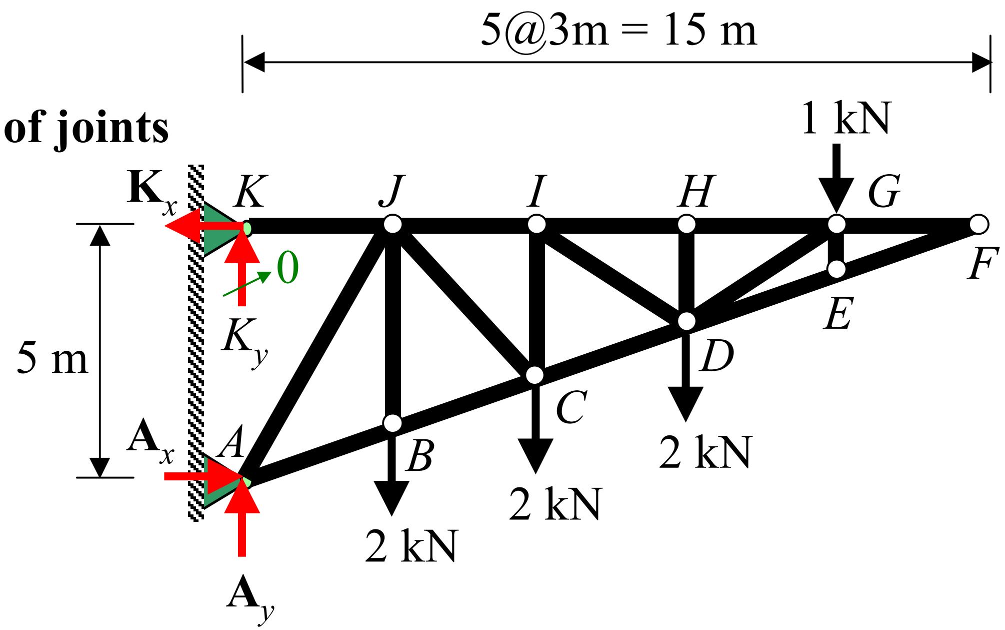
Example 3-5

- Determine all the member forces
- Identify zero-force members



SOLUTION

Use method of joints



$$\overset{4}{\cancel{r}} + \overset{18}{\cancel{b}} = \overset{2(11)}{\cancel{2j}}, \longrightarrow$$

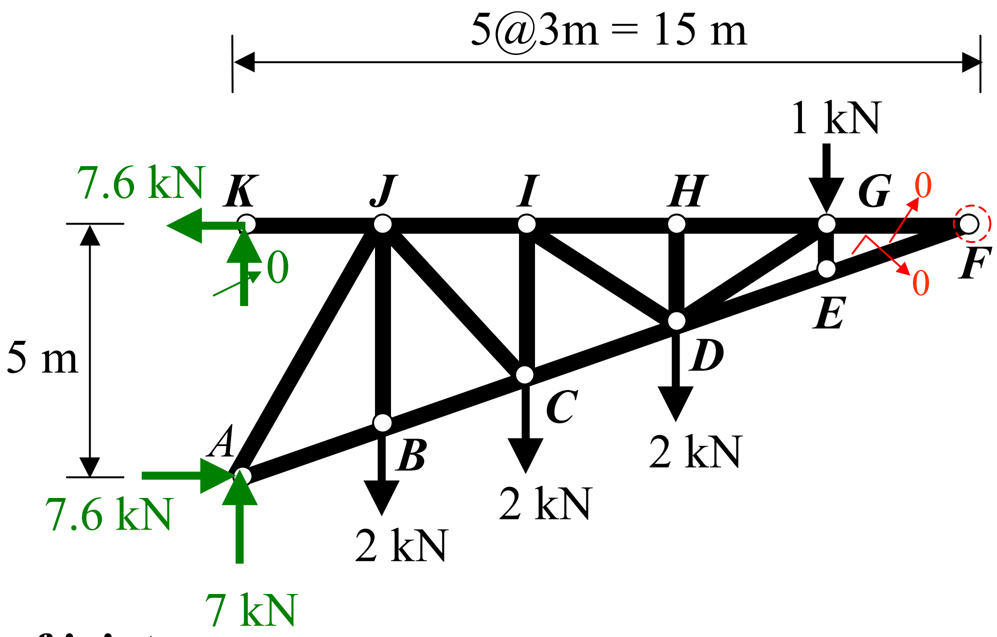
- Determinate
- Stable

$$+\circlearrowright \Sigma M_A = 0: \quad K_x(5) - 2(3) - 2(6) - 2(9) - 1(12) = 0$$

$$K_x = 7.6 \text{ kN}, \leftarrow$$

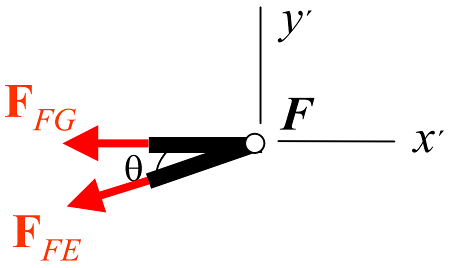
$$\xrightarrow{+} \Sigma F_x = 0: \quad -7.6 + A_x = 0, \quad A_x = 7.6 \text{ kN}, \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 2 - 2 - 2 - 1 = 0, \quad A_y = 7 \text{ kN}, \uparrow$$



Use method of joint

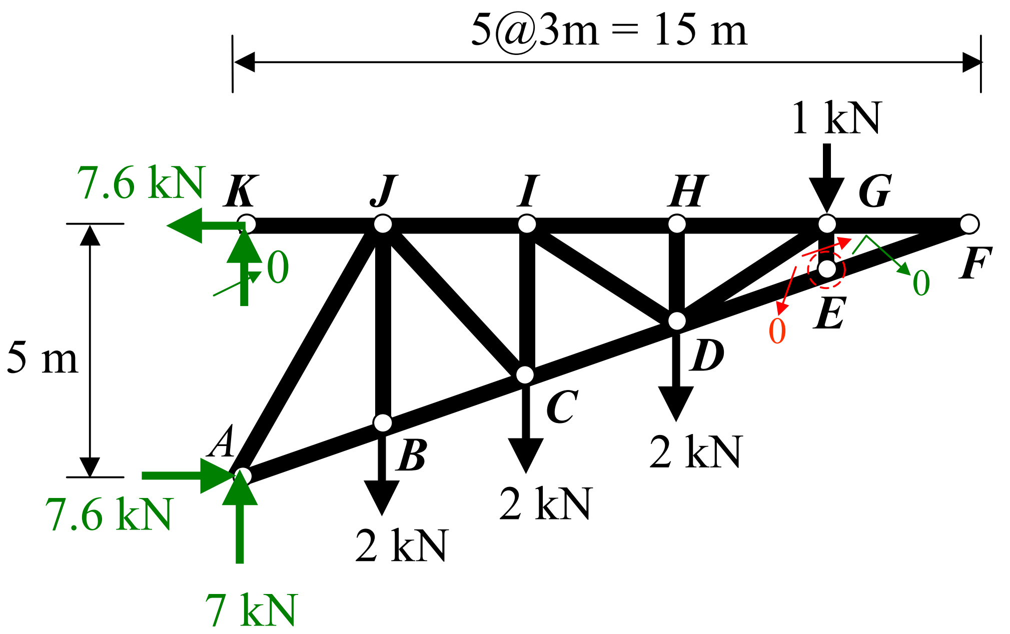
- ***Joint F***



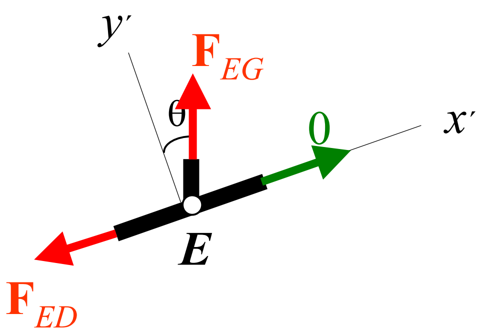
$$+\uparrow \Sigma F_{y'} = 0: \qquad F_{FE}\sin\theta = 0$$

$$F_{FE} = 0$$

$$\overset{+}{\rightarrow} \Sigma F_{x'} = 0: \qquad F_{FG} = 0$$



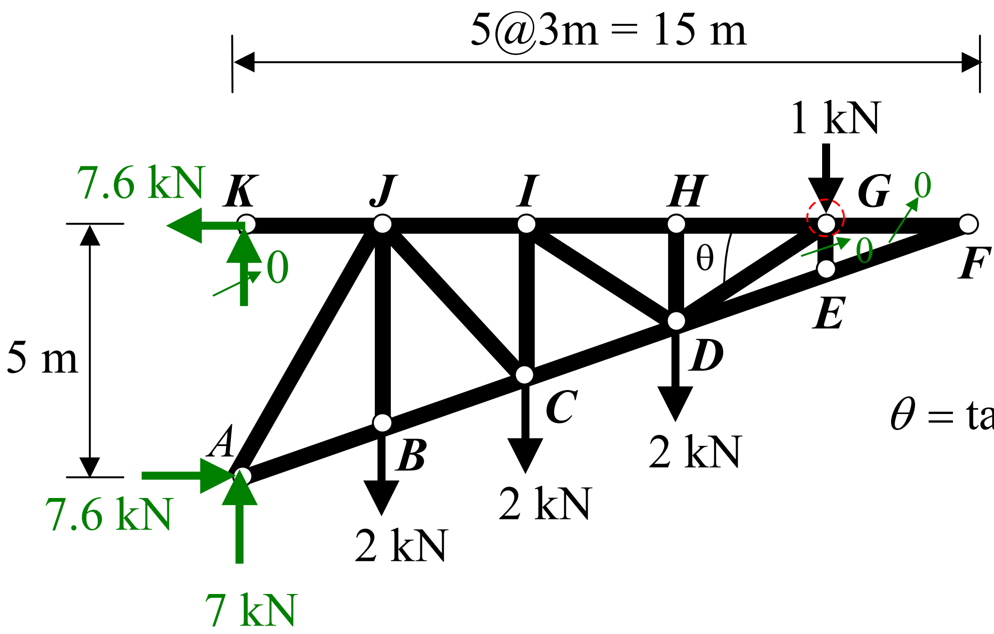
- ***Joint E***



$$+\uparrow \Sigma F_{y'} = 0: \qquad F_{EG}\cos\theta = 0$$

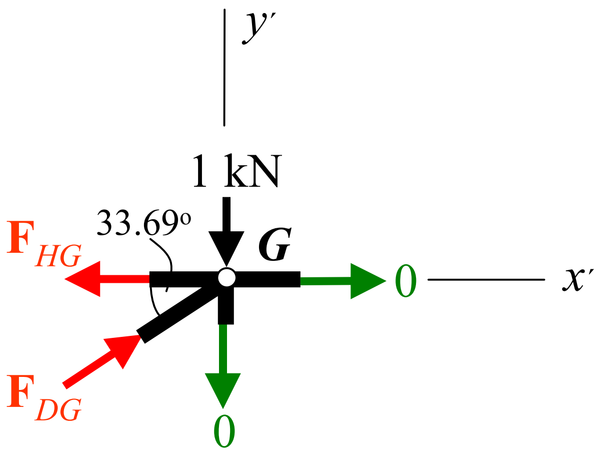
$$F_{EG} = 0$$

$$\xrightarrow{+} \Sigma F_{x'} = 0: \qquad -F_{ED} = 0$$



$$\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^o$$

- ***Joint G***

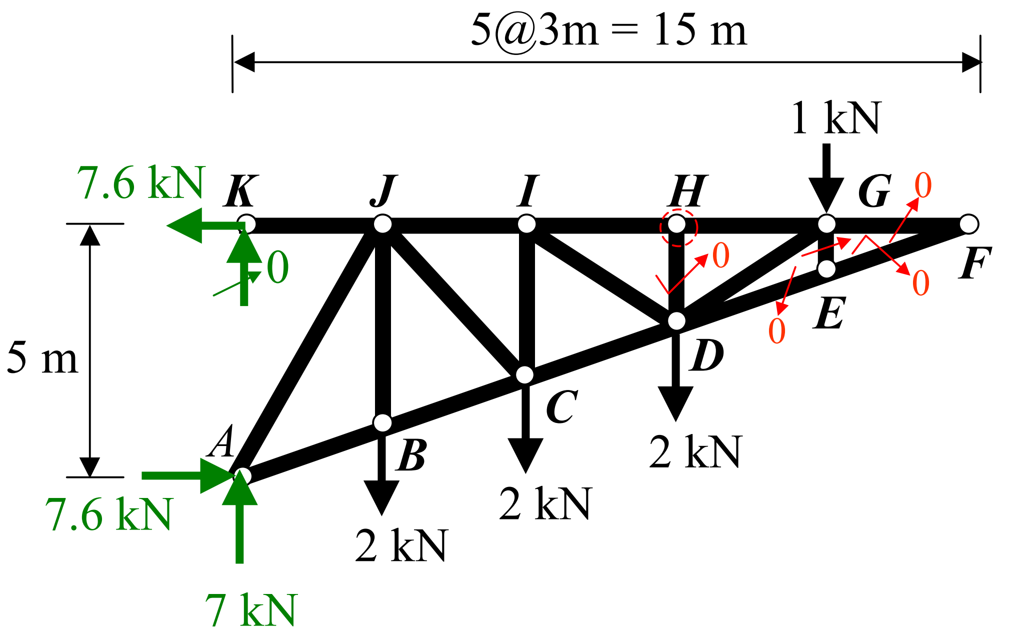


$$+\uparrow \Sigma F_{y'} = 0: \qquad F_{DG} \sin 33.69^o - 1 = 0$$

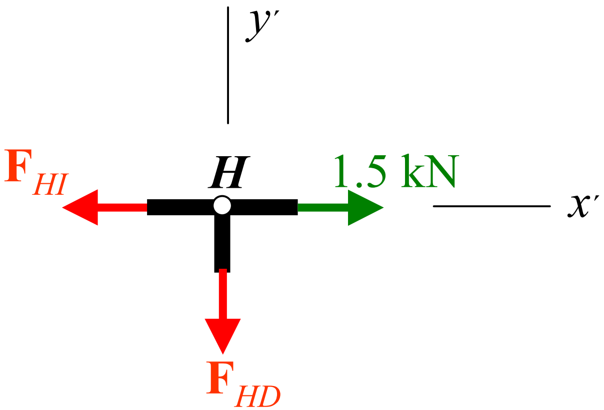
$$F_{DG} = 1.803 \text{ kN (C)}$$

$$\overset{+}{\rightarrow} \Sigma F_{x'} = 0: \qquad -F_{HG} + 1.803 \cos 33.69 = 0$$

$$F_{HG} = 1.5 \text{ kN (T)}$$



• ***Joint H***

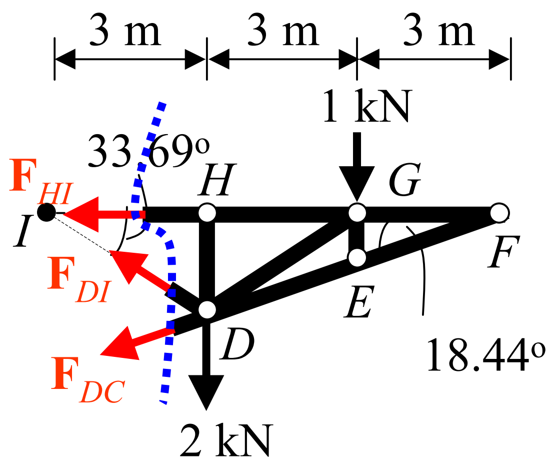


$$+\uparrow \Sigma F_{y'} = 0: \qquad F_{HD} = 0$$

$$\xrightarrow{+} \Sigma F_{x'} = 0: \qquad -F_{HI} + 1.5 = 0$$

$$F_{HI} = 1.5 \text{ kN (T)}$$

Use method of sections



$$\circlearrowright + \; \Sigma M_D = 0: \qquad F_{HI}(2) - 1(3) = 0$$

$$F_{HI} = 1.5 \text{ kN (T)}$$

$$\circlearrowright + \; \Sigma M_F = 0: \qquad -F_{DI}\sin 33.69(9) + 1(3) + 2(6) = 0$$

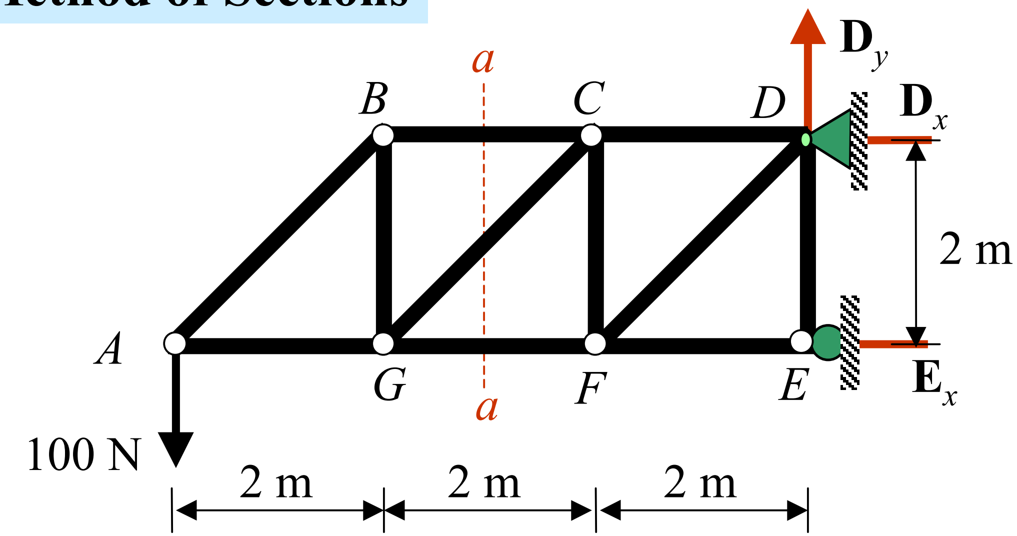
$$F_{DI} = 3 \text{ kN (T)}$$

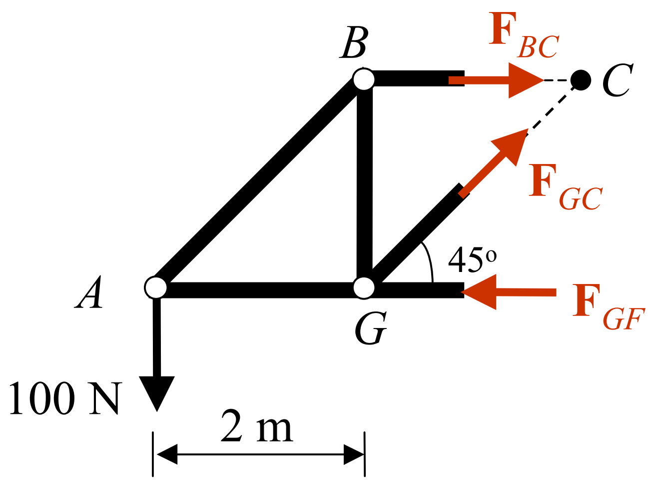
$$\circlearrowright + \; \Sigma M_I = 0: \qquad -F_{DC}\sin 18.44(9) - 1(6) - 2(3) = 0$$

$$F_{DC} = \text{-}4.25 \text{ kN (C)}$$

Check : $+\uparrow \; \Sigma F_y = 0: \qquad \cancelto{3}{F_{DI}} \sin 33.69 - \cancelto{-4.25}{F_{DC}} \sin 18.44 - 2 - 1 = 0 \qquad O.K.$

The Method of Sections





$$\circlearrowright + \ \Sigma M_G = 0:$$

$$100(2) - F_{BC}(2) = 0$$

$$F_{BC} = 100 \text{ N (T)}$$

$$+\uparrow \ \Sigma F_y = 0:$$

$$-100 + F_{GC}\sin 45^{\circ} = 0$$

$$F_{GC} = 141.42 \text{ N (T)}$$

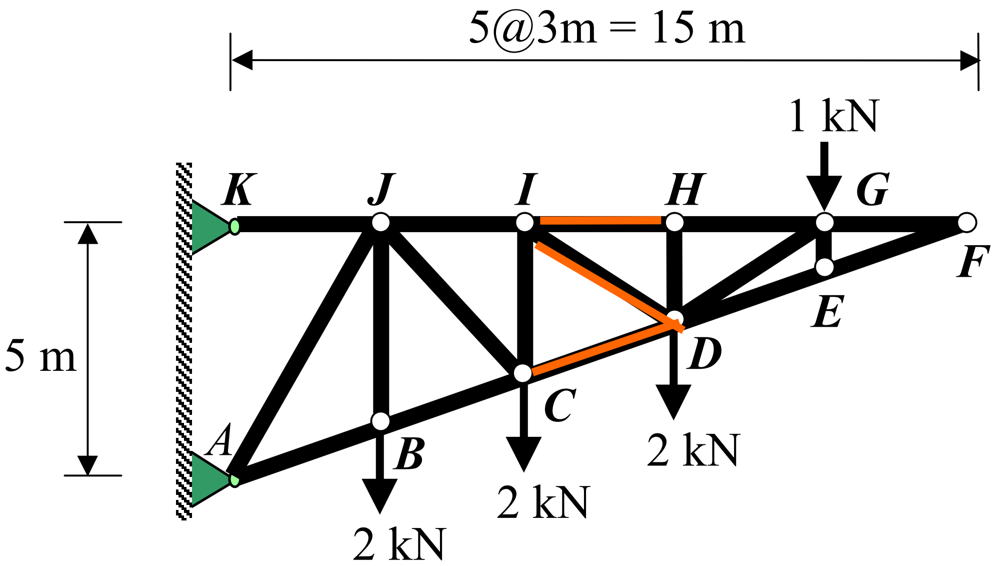
$$\circlearrowright + \ \Sigma M_C = 0:$$

$$100(4) - F_{GF}(2) = 0$$

$$F_{GF} = 200 \text{ N (C)}$$

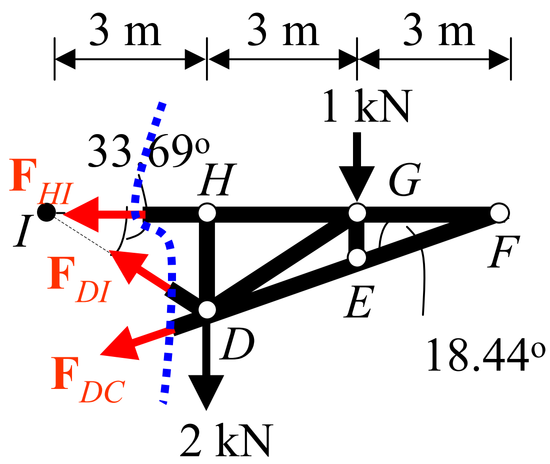
Example 3-6

- Determine member force *CD*, ID, and *IH*



SOLUTION

Use method of sections



+1.50E+00

-4.22E+00

3.00E+00

$\circlearrowright + \ \Sigma M_D = 0: \qquad F_{HI}(2) - 1(3) = 0$

$$F_{HI} = 1.5 \text{ kN (T)}$$

$\circlearrowright + \ \Sigma M_F = 0: \qquad -F_{DI}\sin 33.69(9) + 1(3) + 2(6) = 0$

$$F_{DI} = 3 \text{ kN (T)}$$

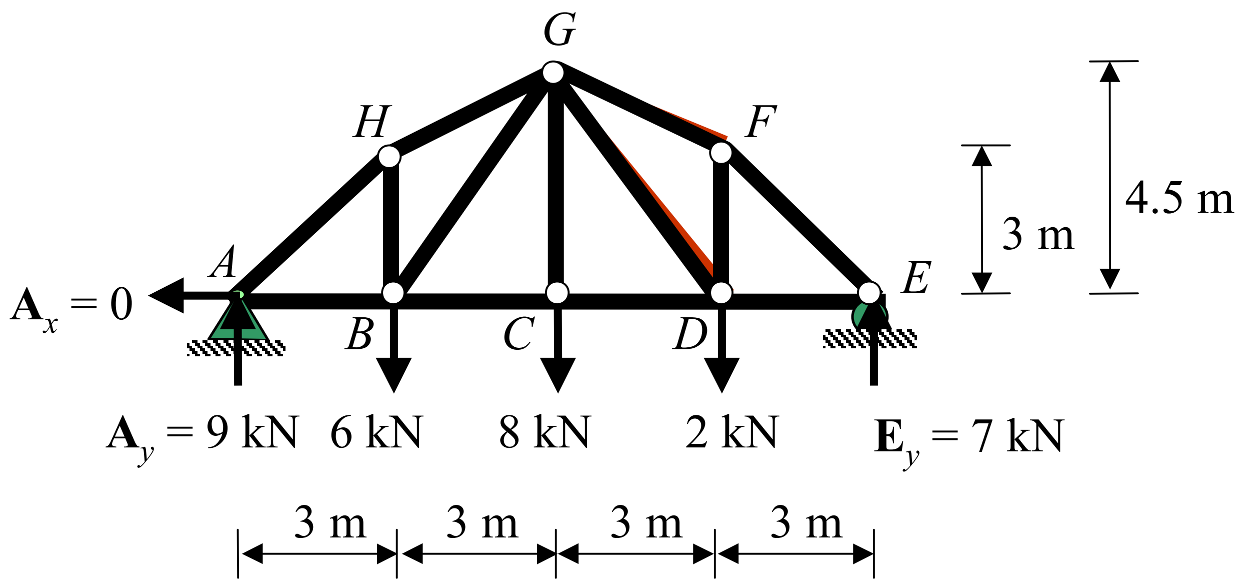
$\circlearrowright + \ \Sigma M_I = 0: \qquad -F_{DC}\sin 18.44(9) - 1(6) - 2(3) = 0$

$$F_{DC} = \text{-}4.25 \text{ kN (C)}$$

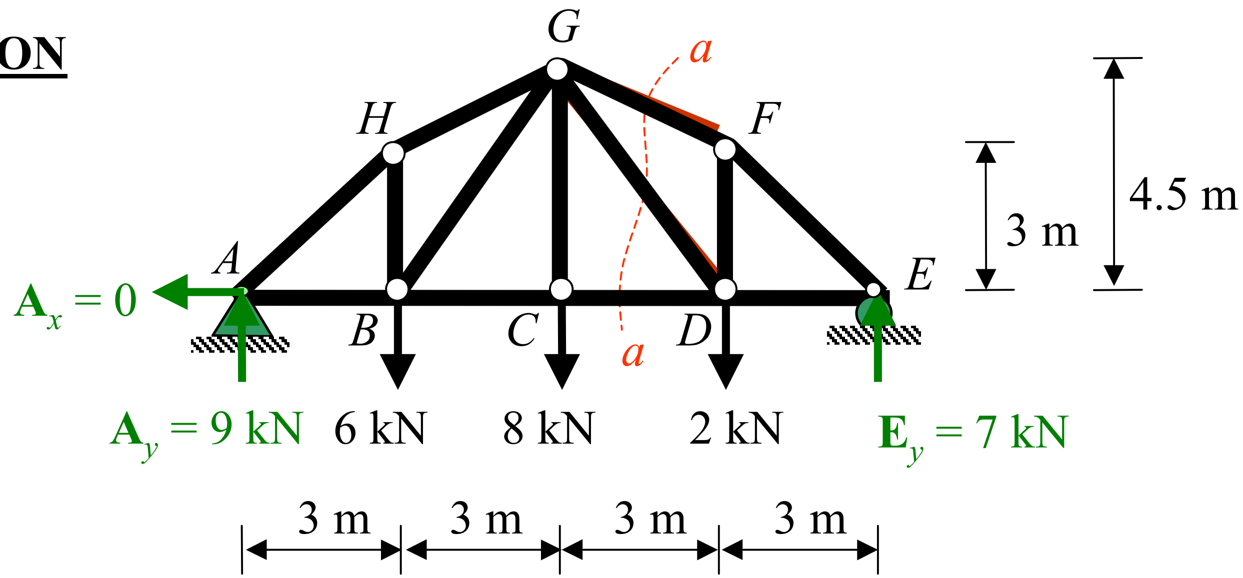
Check : $+\uparrow \Sigma F_y = 0: \qquad \cancelto{3}{F_{DI}}\sin 33.69 - \cancelto{-4.25}{F_{DC}}\sin 18.44 - 2 - 1 = 0$ O.K.

Example 3-7

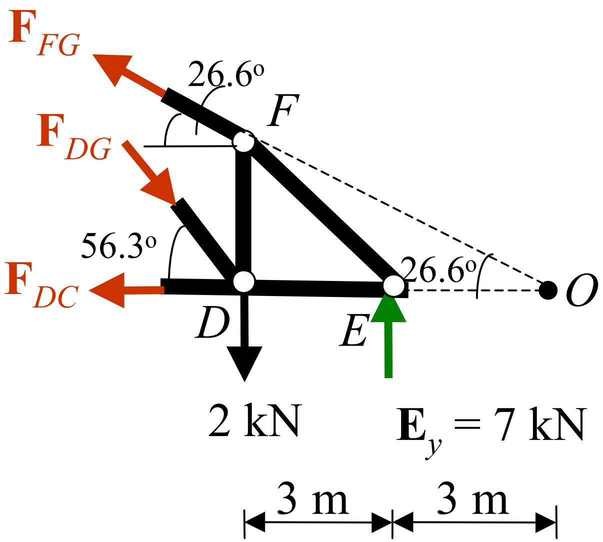
Determine the force in members *GF* and *GD* of the truss shown in the figure below. State whether the members are in tension or compression. The reactions at the supports have been calculated.



SOLUTION



Section a-a



$$+\circlearrowright \Sigma M_D = 0:$$

$$F_{FG}\sin 26.6^\circ(3.6) + 7(3) = 0,$$

$$F_{FG} = -17.83 \text{ kN (C)}$$

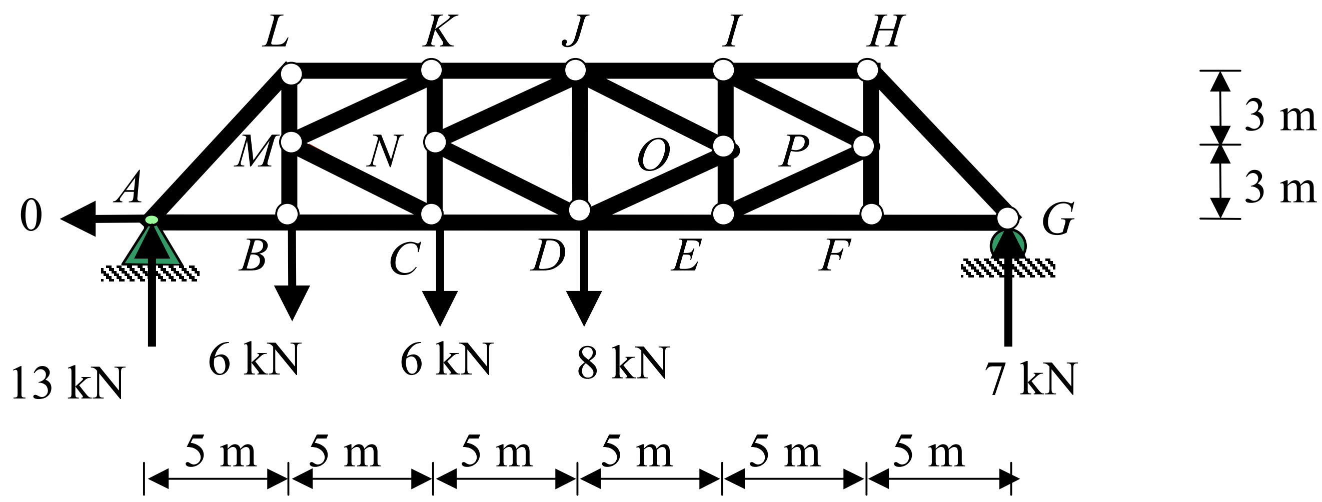
$$+\circlearrowright \Sigma M_O = 0:$$

$$-7(3) + 2(6) + F_{DG}\sin 56.3^\circ(6) = 0,$$

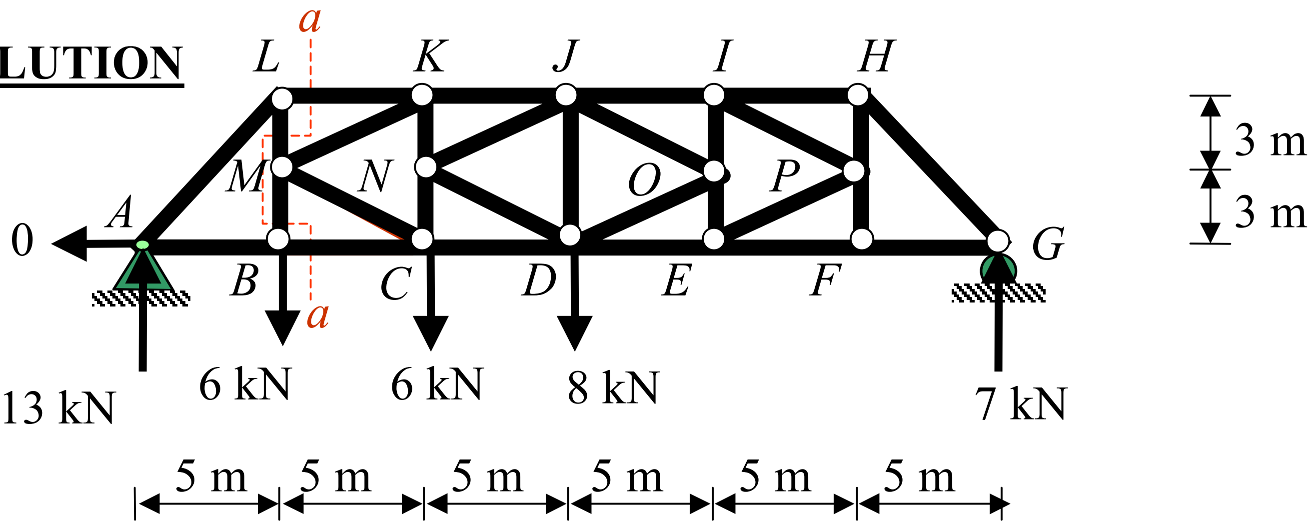
$$F_{DG} = 1.80 \text{ kN (C)}$$

Example 3-8

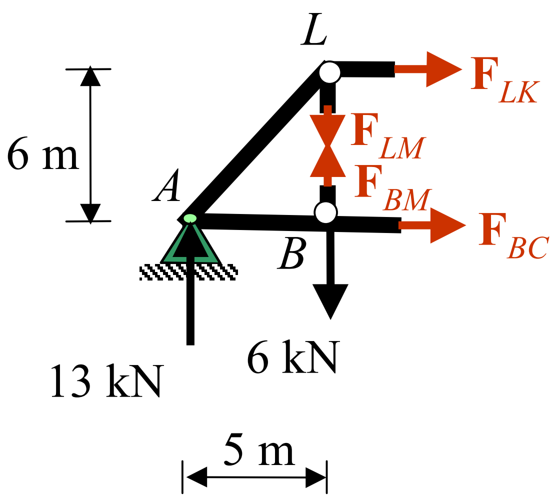
Determine the force in members *BC* and MC of the K-truss shown in the figure below. State whether the members are in tension or compression. The reactions at the supports have been calculated.



SOLUTION



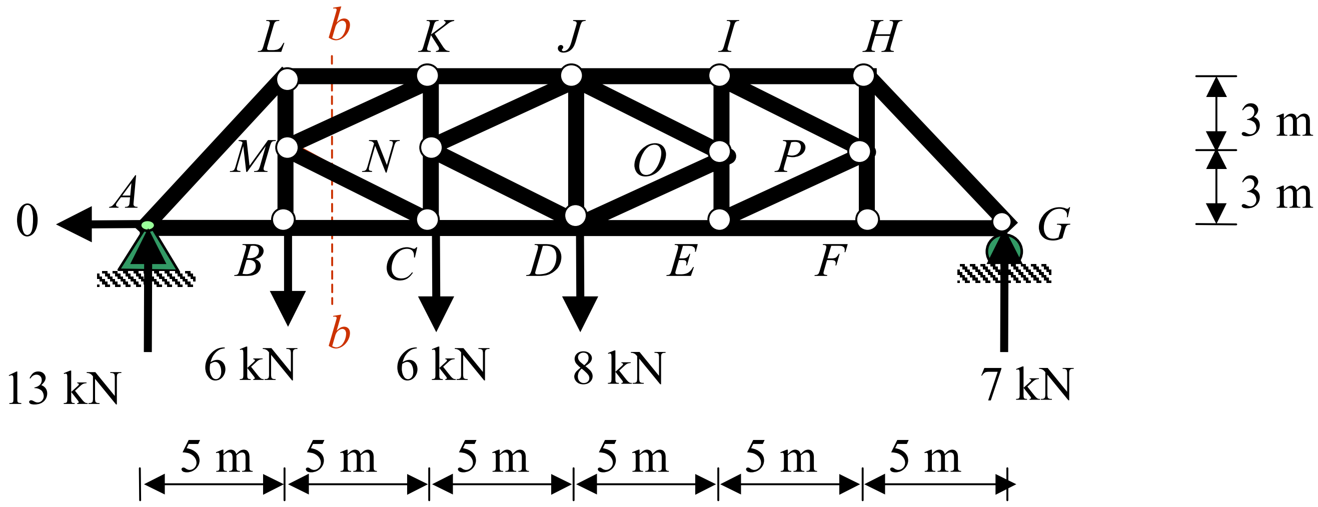
Section a-a

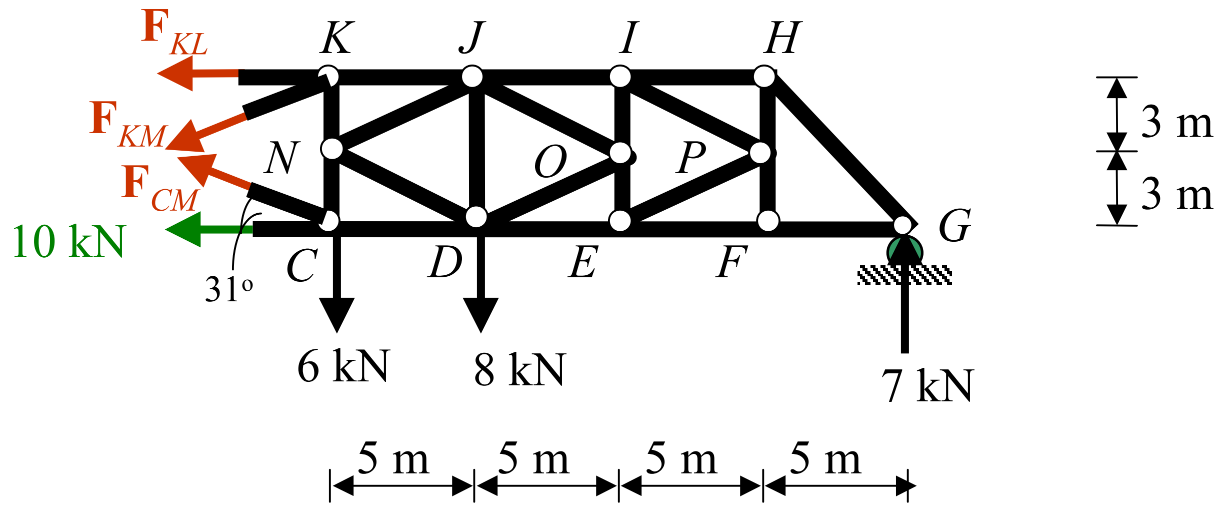


$+\circlearrowright \Sigma M_L = 0:$

$F_{BC}(6) - 13(5) = 0,$

$F_{BC} = 10 \text{ kN (T)}$





$+\circlearrowright \Sigma M_K = 0$: $\quad -F_{CM}\cos 31^{\circ}(6) - 10(6) - 8(5) + 7(20) = 0$

$F_{CM} = 7.77$ kN (T)

Compound Trusses

Procedure for Analysis

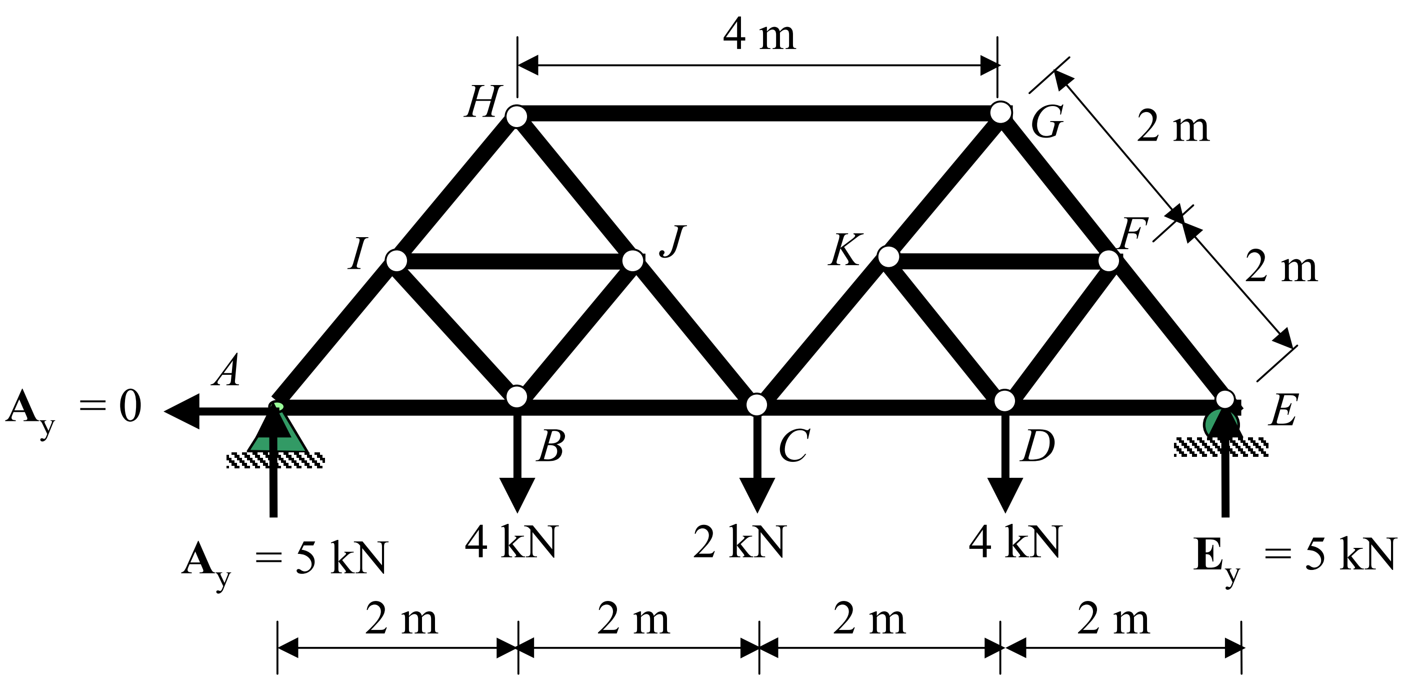
Step 1. Identify the simple trusses

Step 2. Obtain external loading

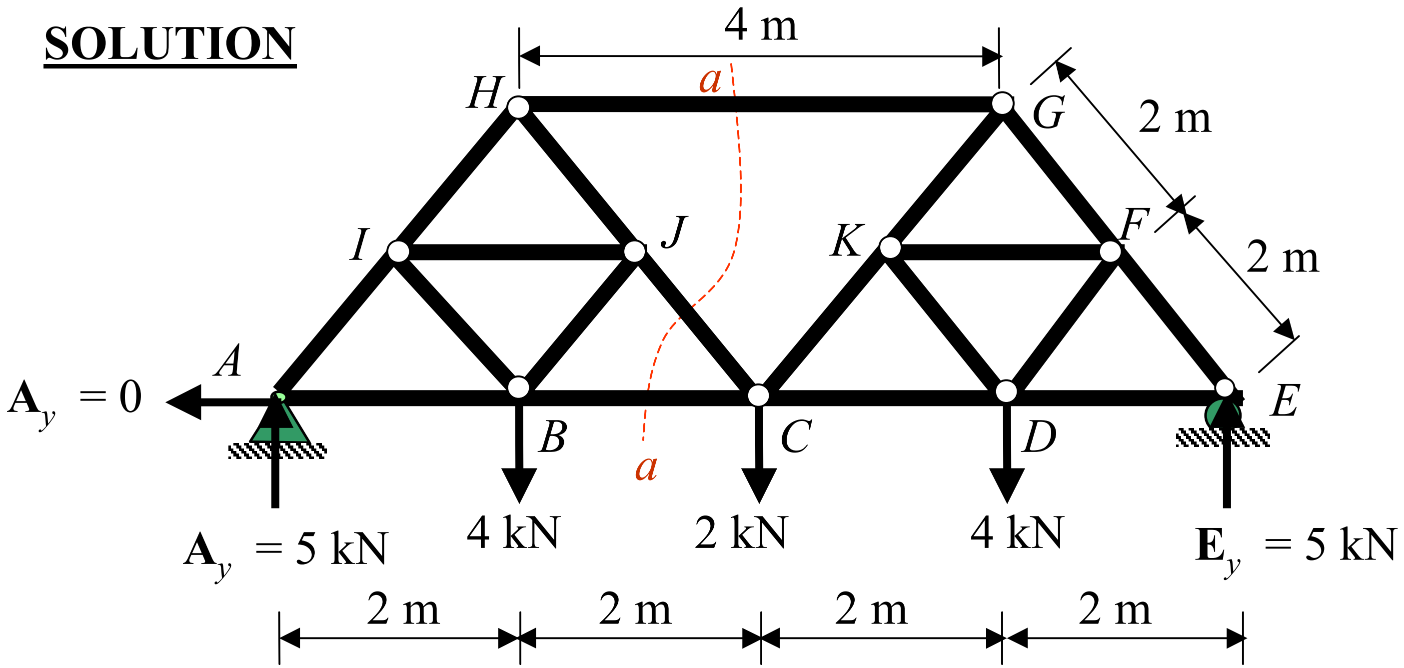
Step 3. Solve for simple trusses separately

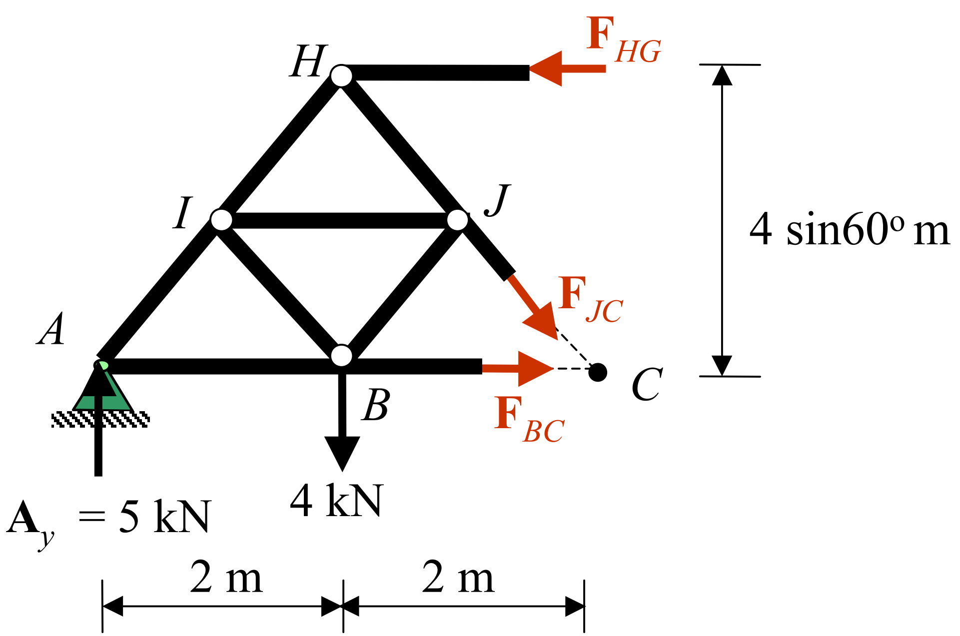
Example 3-9

Indicate how to analyze the compound truss shown in the figure below. The reactions at the supports have been calculated.



SOLUTION

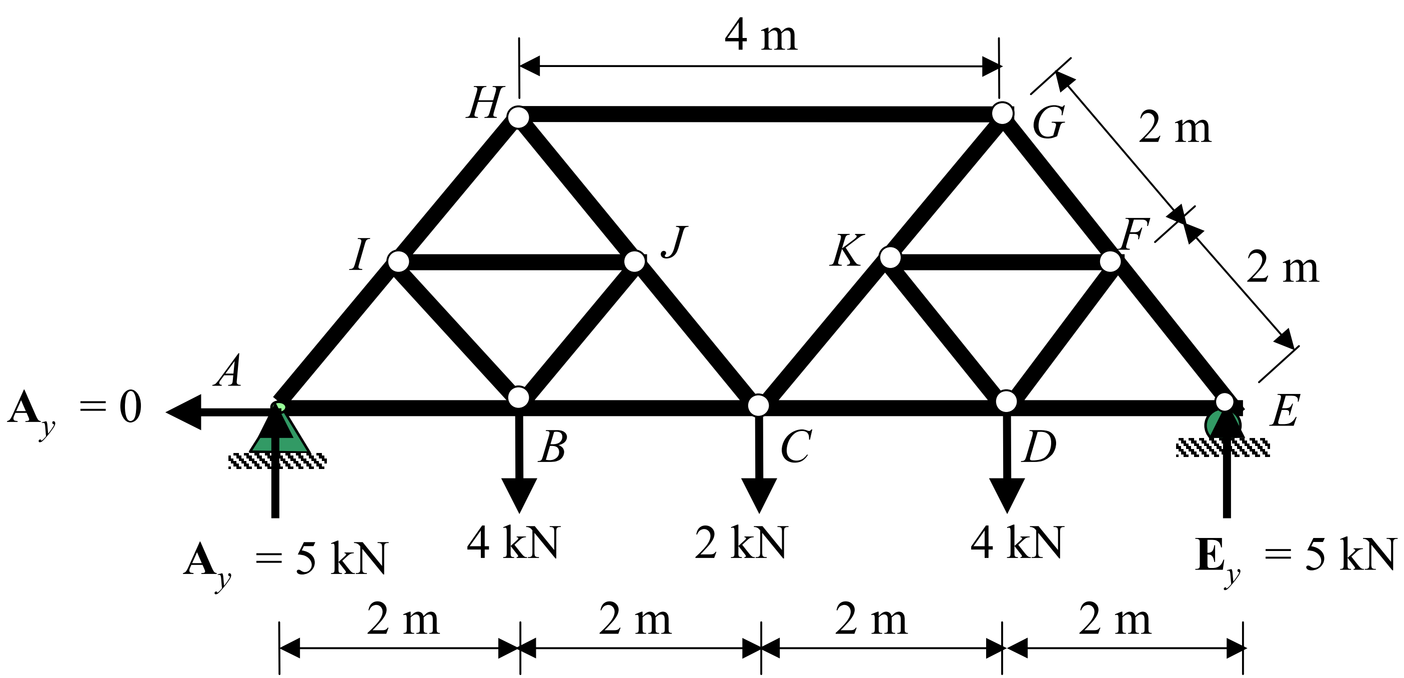


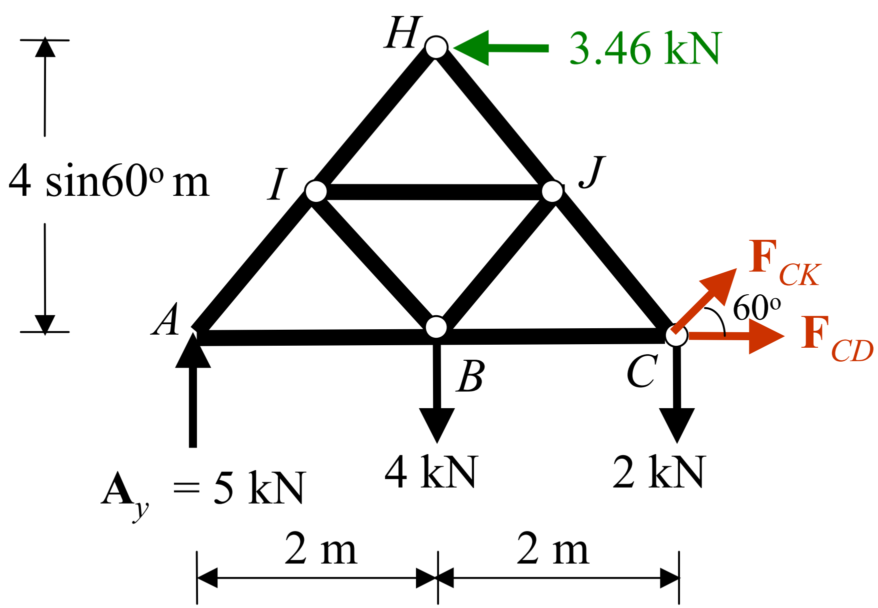


$+\circlearrowright \Sigma M_C = 0:$

$$-5(4) + 4(2) + F_{HG}(4\sin 60^{\circ}) = 0$$

$$F_{HG} = 3.46 \text{ kN (C)}$$





$\circlearrowright + \; \Sigma M_A = 0:$

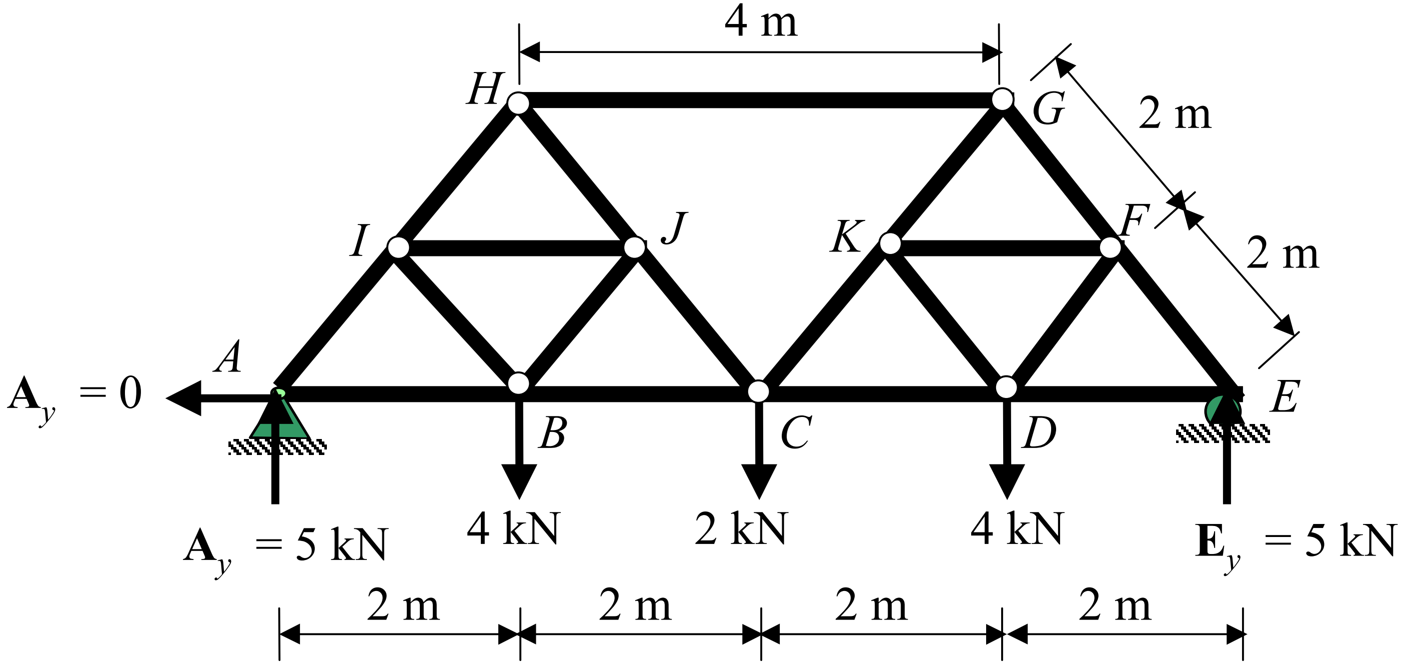
$3.46(4\sin 60^{\circ}) + F_{CK}\sin 60^{\circ}(4) - 4(2) - 2(4) = 0$

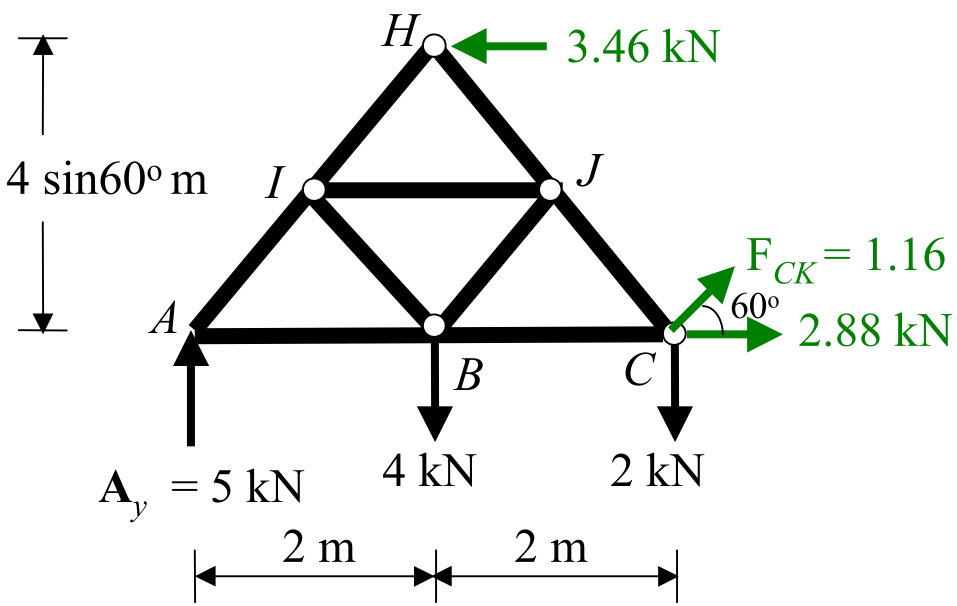
$F_{CK} = 1.16$ kN (T)

$\xrightarrow{+} \; \Sigma F_x = 0:$

$-3.46 + 1.16\cos 60^{\circ} + F_{CD} = 0$

$F_{CK} = 2.88$ kN (T)





Using the method of joints.

Joint A : Determine F_{AB} and F_{AI}

Joint H : Determine F_{HI} and F_{HJ}

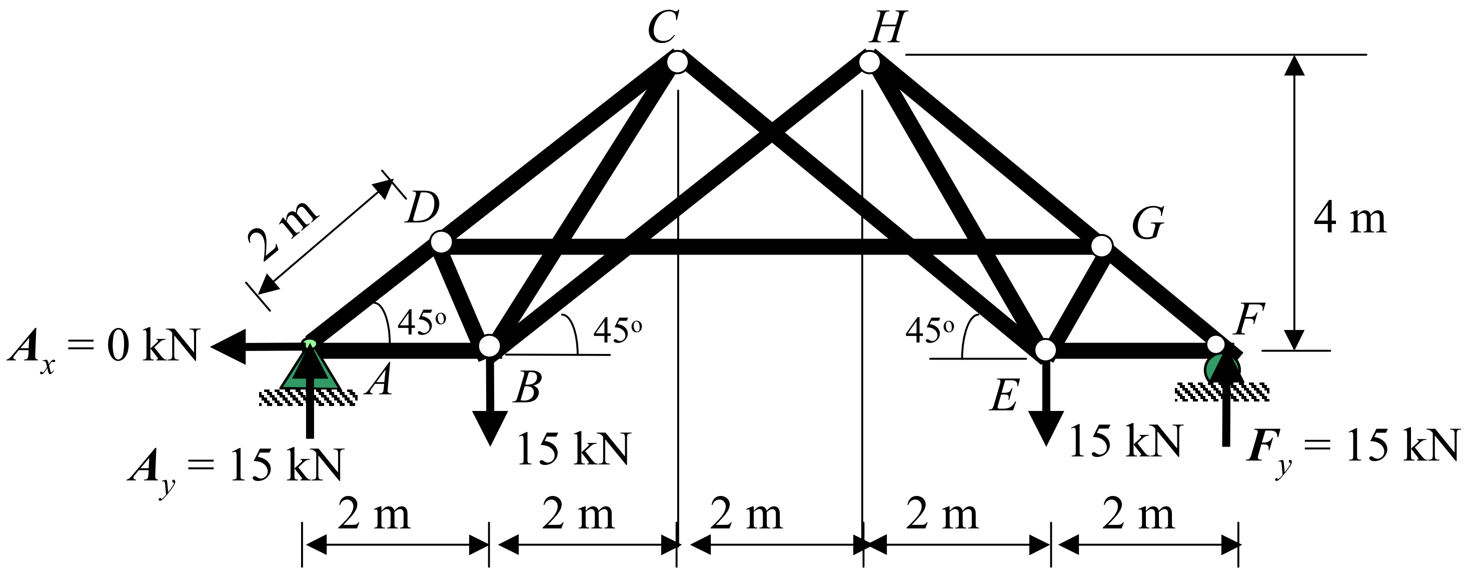
Joint I : Determine F_{IJ} and F_{IB}

Joint B : Determine F_{BC} and F_{BJ}

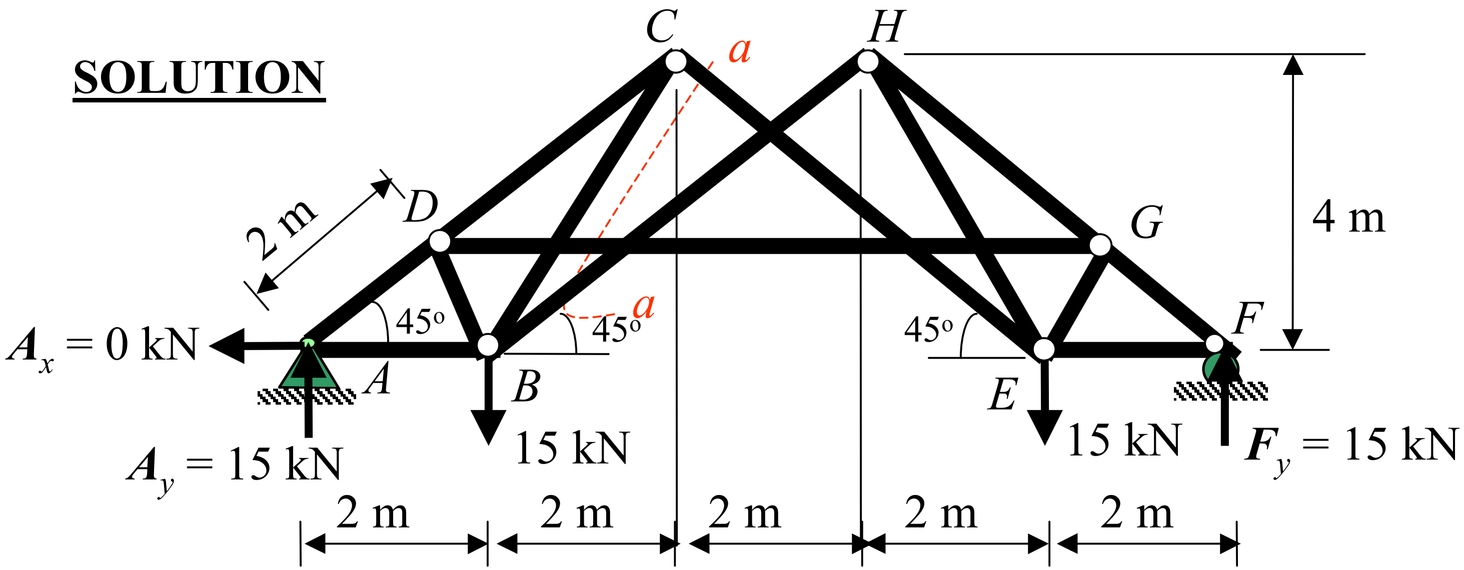
Joint J : Determine F_{JC}

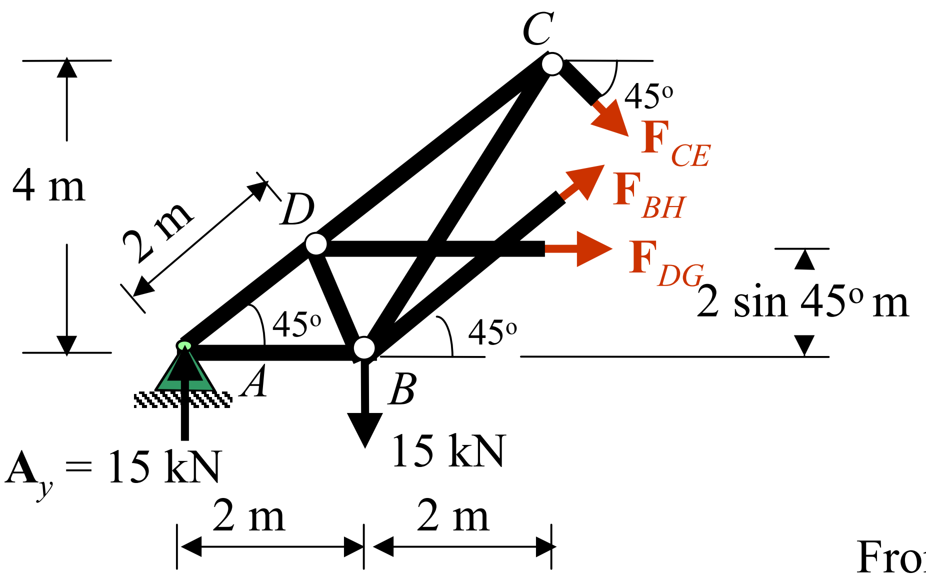
Example 3-10

Indicate how to analyze the compound truss shown in the fugure below. The reactions at the supports have been calculated.



SOLUTION





$+\circlearrowright \Sigma M_B = 0:$

$-15(2) - F_{DG}(2\sin 45^\circ) - F_{CE}\cos 45^\circ(4)$
$- F_{CE}\sin 45^\circ(2) = 0$ -----(1)

$+\uparrow \Sigma F_y = 0:$

$15 - 15 + F_{BH}\sin 45^\circ - F_{CE}\sin 45^\circ = 0$

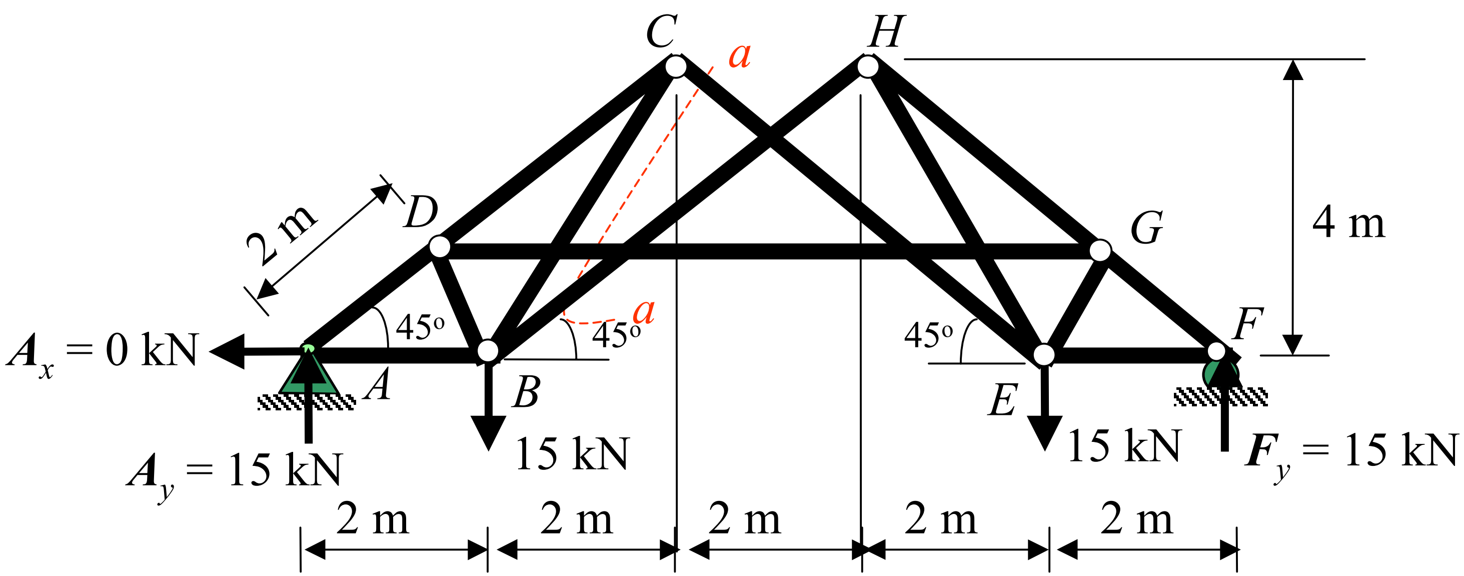
$F_{BH} = F_{CE}$-----(2)

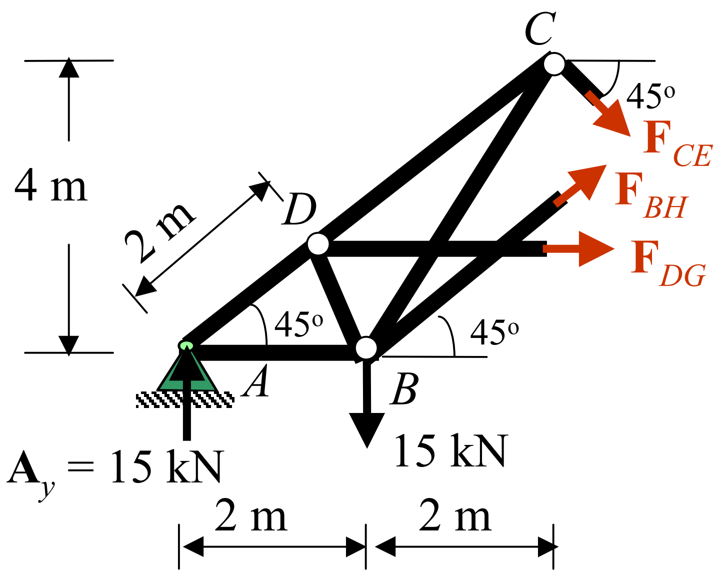
$\overset{+}{\rightarrow} \Sigma F_x = 0:$

$F_{BH}\cos 45^\circ + F_{DG} + F_{CE}\cos 45^\circ = 0$ -----(3)

From eq.(1)-(3): $F_{BH} = F_{CE} = -13.38$ kN (C)

$F_{DG} = 18.92$ kN (T)





From eq.(1)-(3): $F_{BH} = F_{CE} = -13.38$ kN (C)

$F_{DG} = 18.92$ kN (T)

Analysis of each connected simple truss can now be performed using the method of joints.

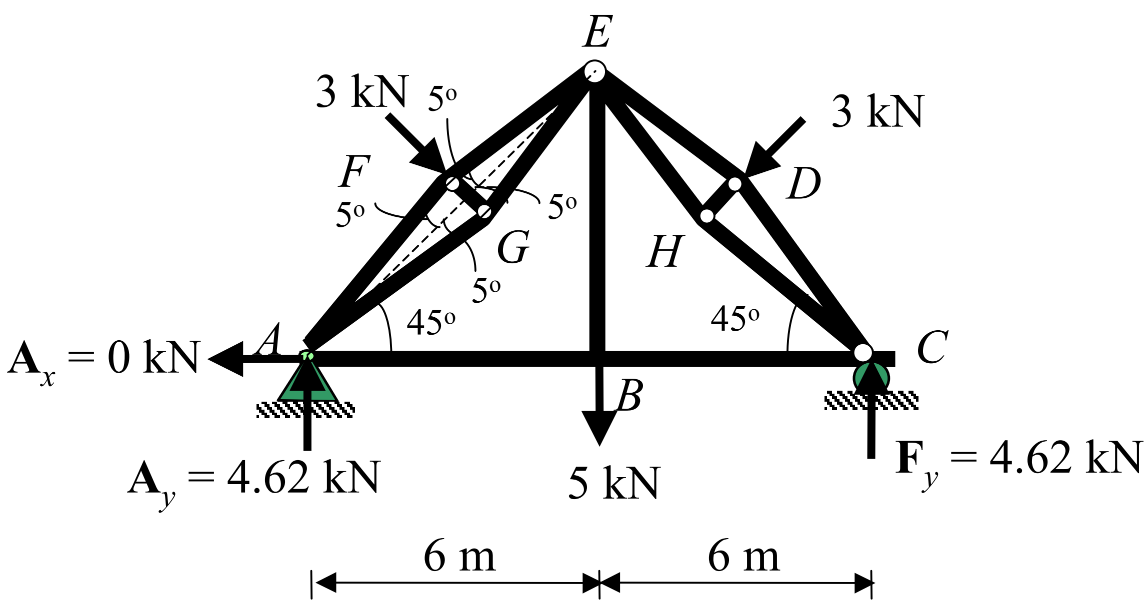
Joint A : Determine F_{AB} and F_{AD}

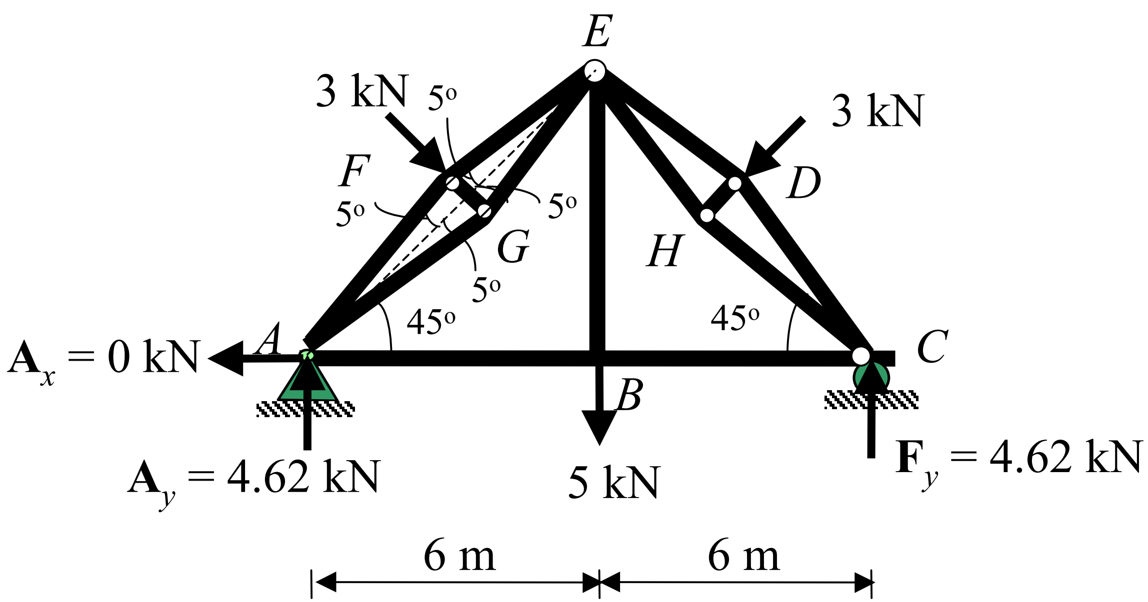
Joint D : Determine F_{DC} and F_{DB}

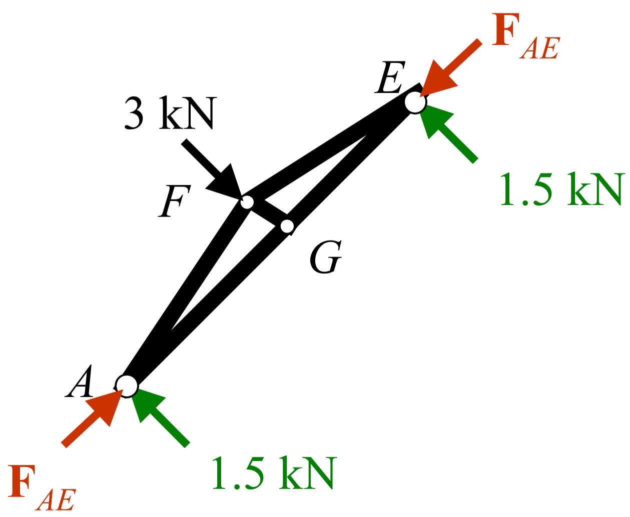
Joint C : Determine F_{CB}

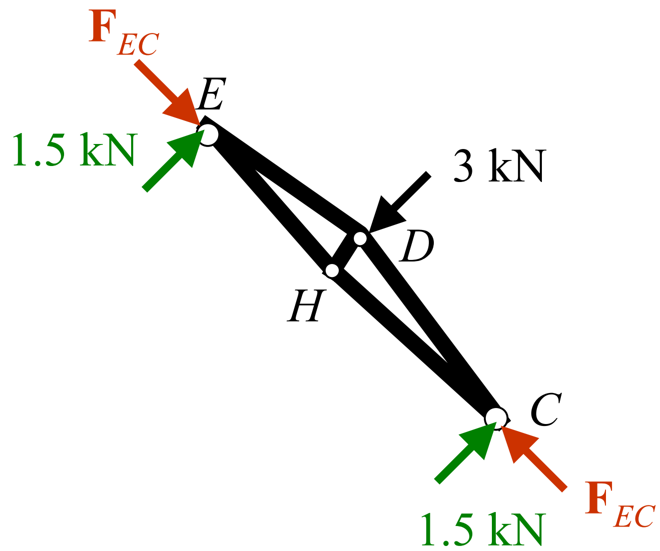
Example 3-11

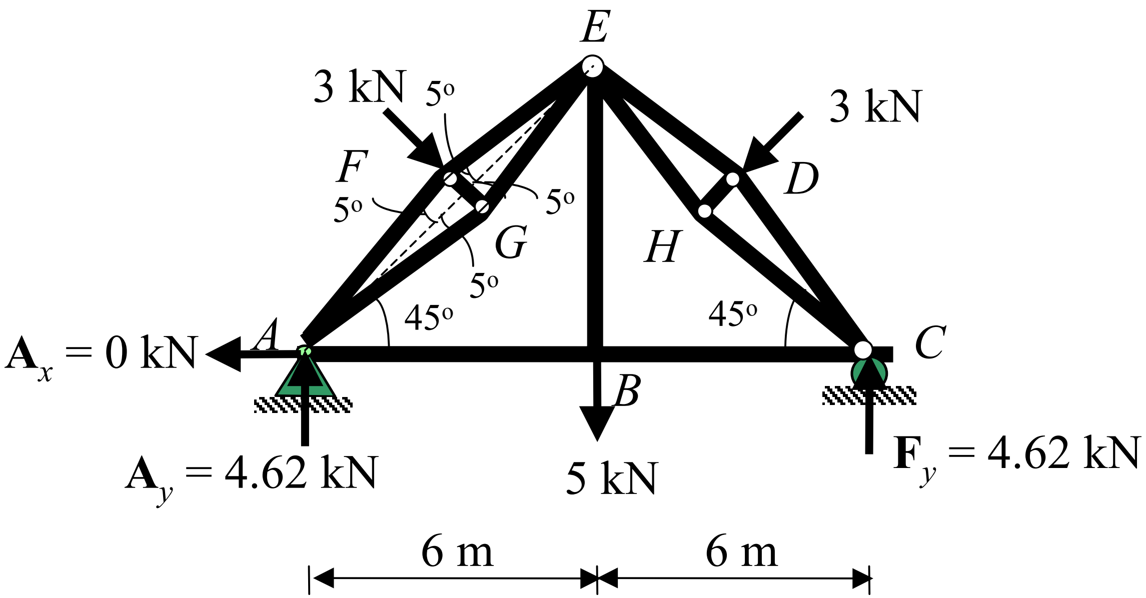
Indicate how to analyze the symmetrical compound truss shown in the figure below. The reactions at the supports have been calculated.

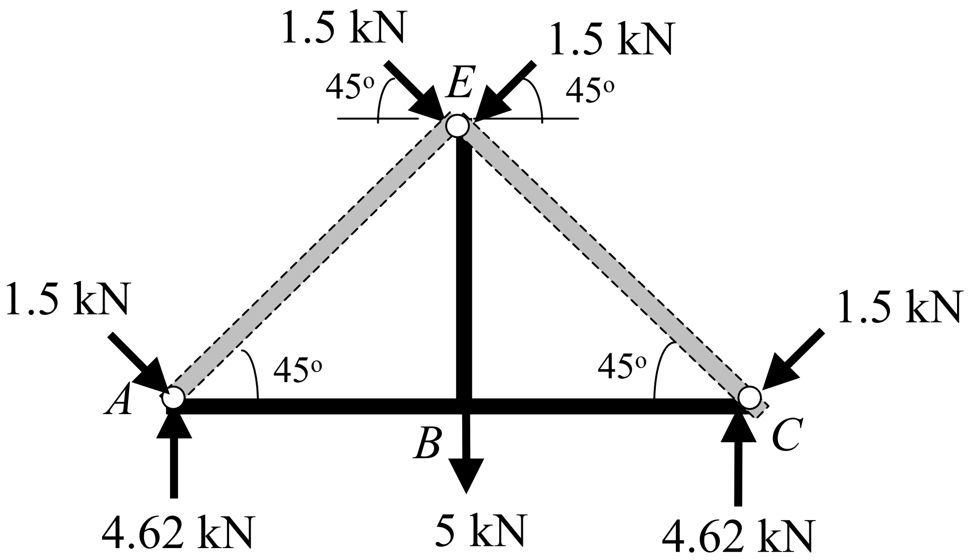


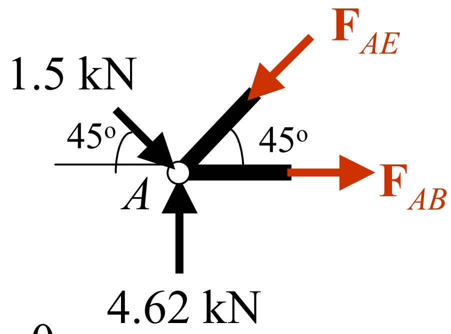

E
3 kN
5°
3 kN
F
D
5°
5°
G
H
5°
45°
45°
A_x = 0 kN
A
C
B
A_y = 4.62 kN
5 kN
F_y = 4.62 kN
6 m
6 m


F_{AE}
E
3 kN
F
1.5 kN
G
A
F_{AE}
1.5 kN


F_{EC}
E
1.5 kN
3 kN
D
H
C
1.5 kN
F_{EC}







$+\uparrow \Sigma F_y = 0:$

$$4.62 - 1.5\sin 45^{\circ} - F_{AE}\sin 45^{\circ} = 0$$

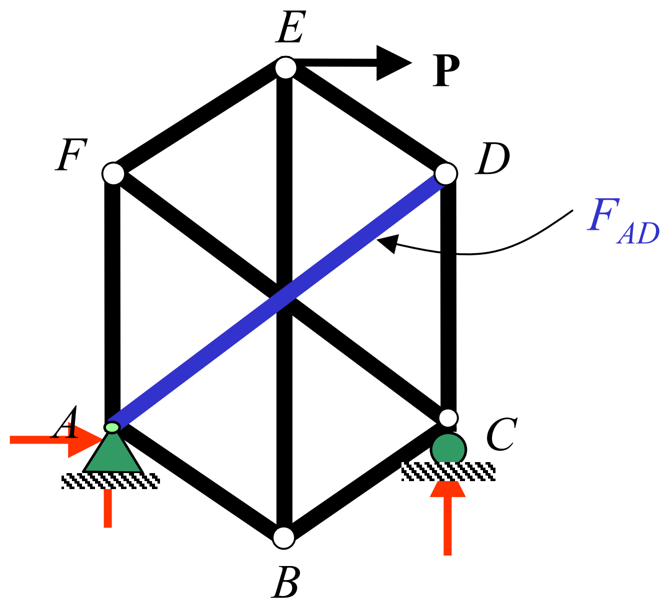
$$F_{AE} = 5.03 \text{ kN (C)}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0:$

$$1.5\cos 45^{\circ} - 5.03\cos 45^{\circ} + F_{AB} = 0$$

$$F_{AB} = 2.50 \text{ kN (T)}$$

Complex Trusses

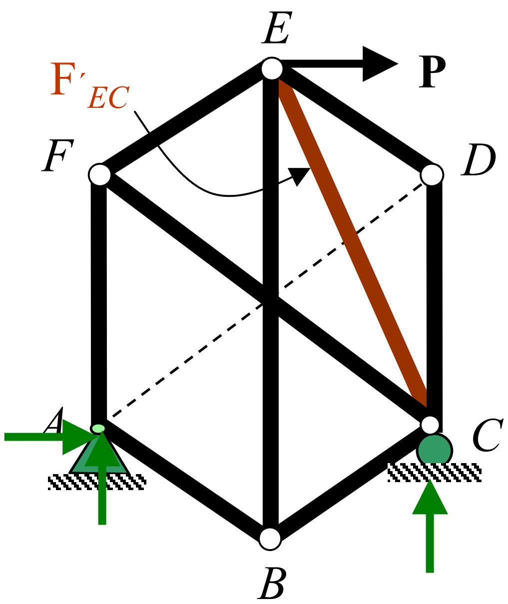


$$\overset{3}{r} + \overset{9}{b} = \overset{2(6)}{2j},$$

$\downarrow$

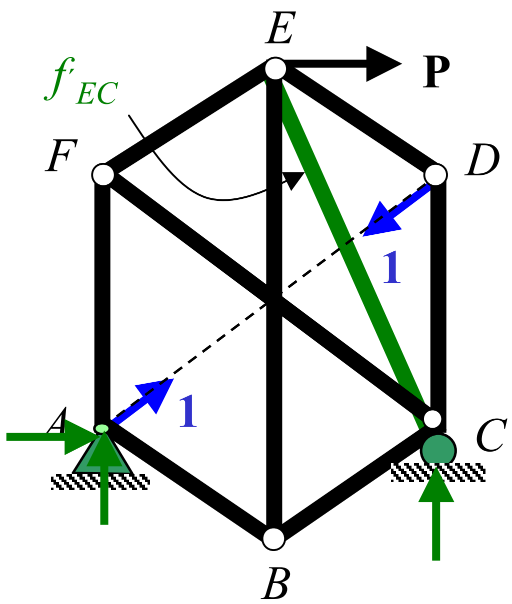
- Determinate
- Stable

=



+

X x



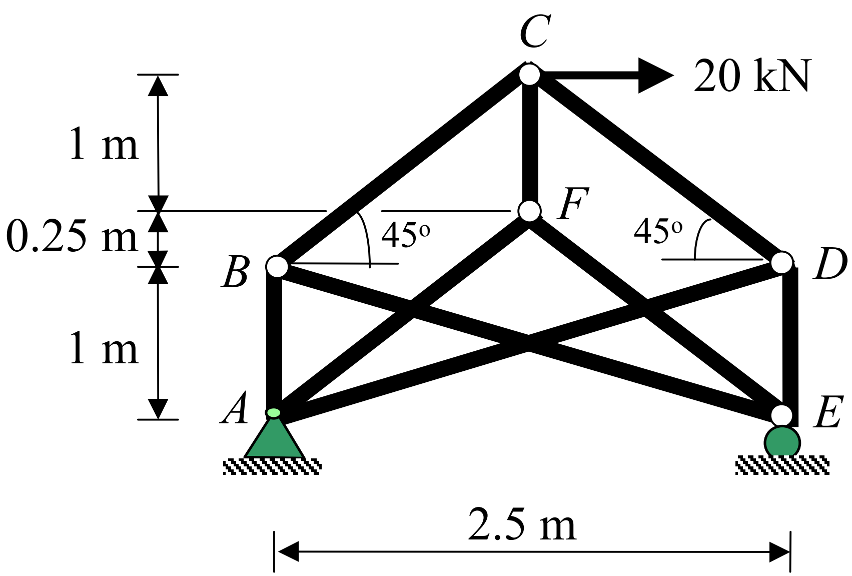
$$F'_{EC} + x\, f'_{EC} = 0$$

$$x = \frac{F'_{EC}}{f'_{EC}} = F_{AD}$$

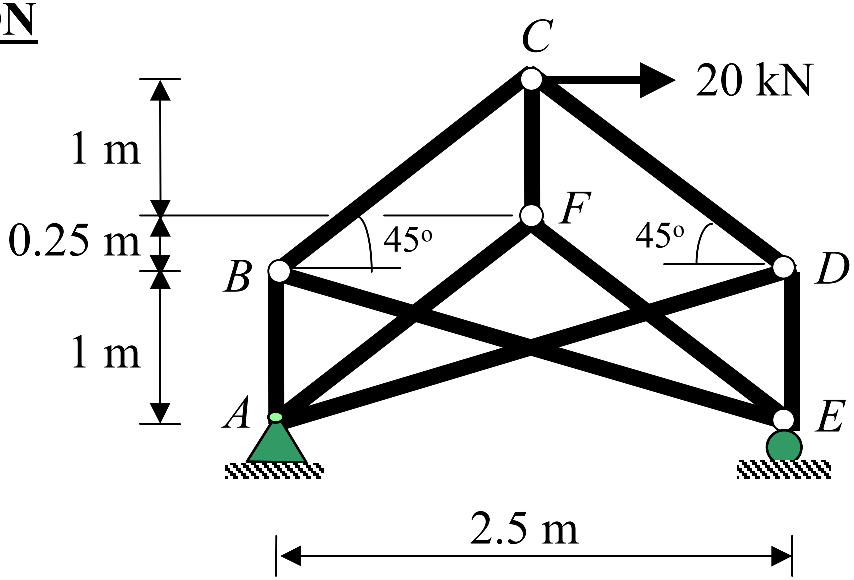
$$F_i = F'_i + x\, f'_i$$

Example 3-12

Determine the force in each member of the complex truss shown in the figure below. Assume joints *B*, *F*, and *D* are on the same horizontal line. State whether the members are in tension or compression.



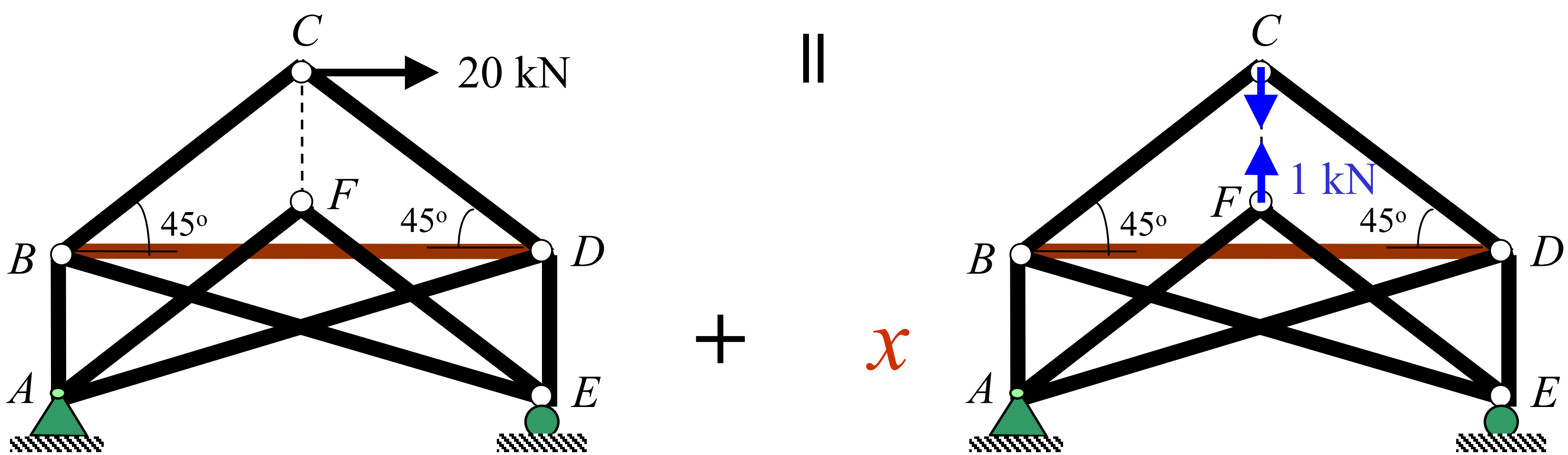
SOLUTION



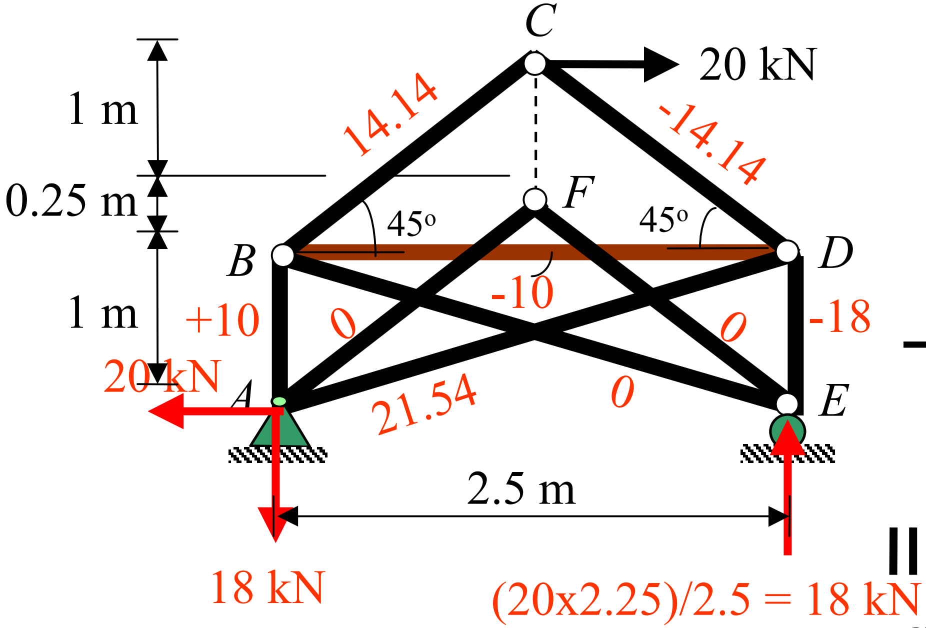
$$F'_{BD} + x\,f_{BD} = 0$$

$$x = \frac{F'_{BD}}{f_{BD}}$$

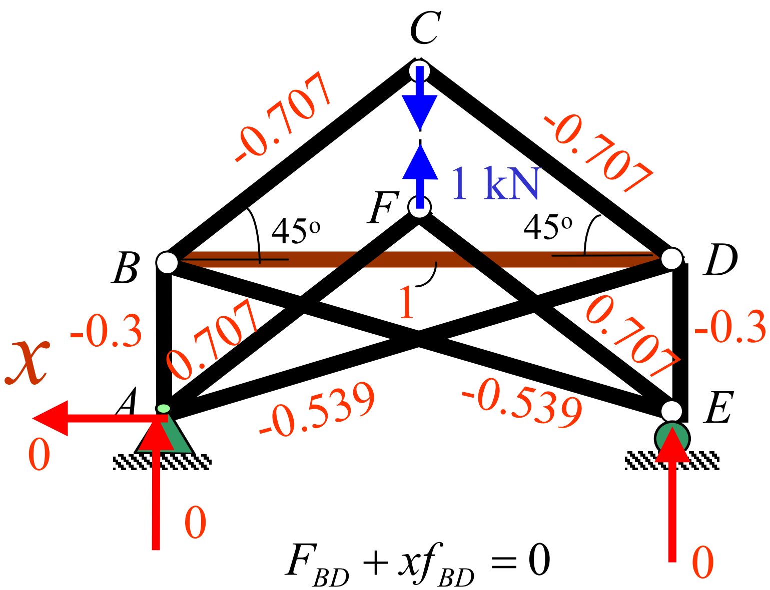
$$F_i = F'_i + x\,f_i$$



First determine reactions and next use the method of joint, start at join *C*, *F*, *E*, *D*, and *B*.



+ *x*

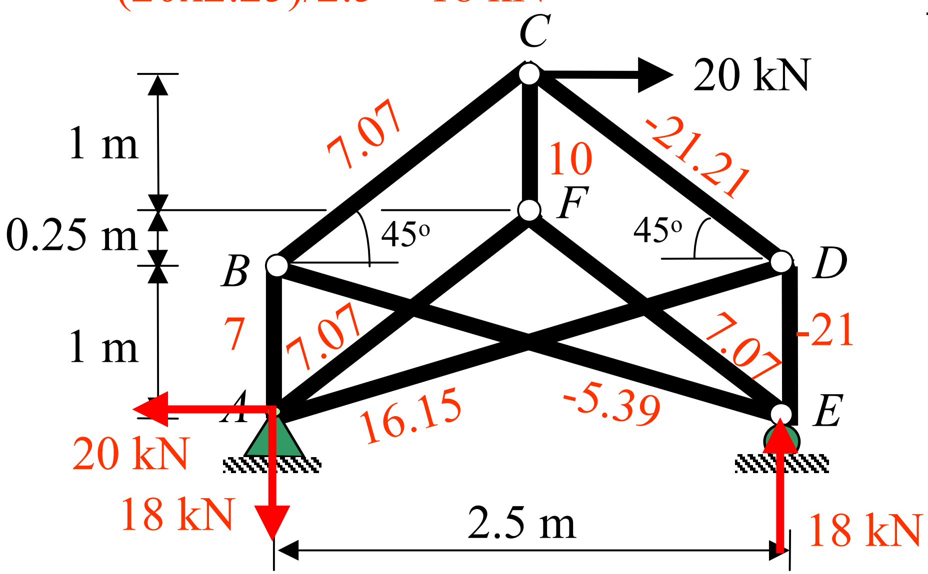


$$F_{BD} + xf_{BD} = 0$$

$$-10 + x(1) = 0$$

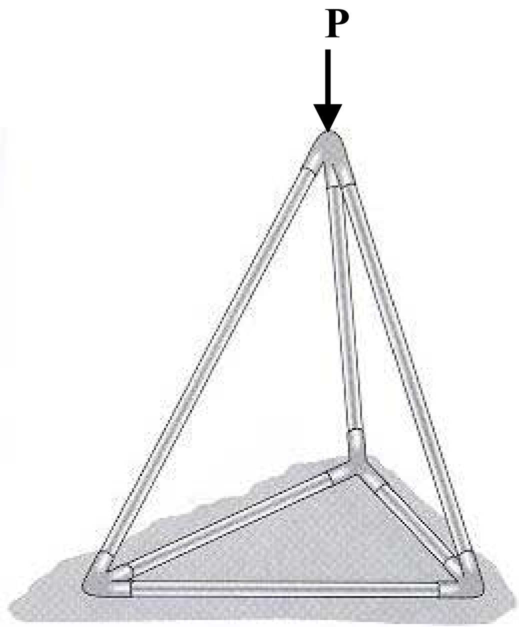
$$x = 10$$

=

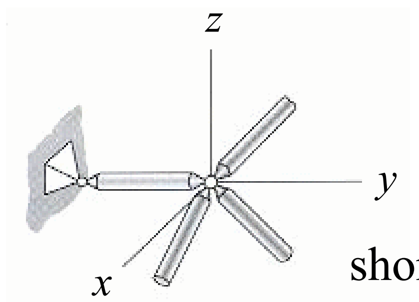
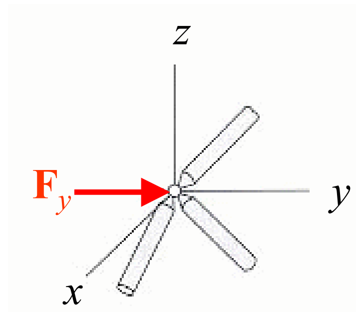
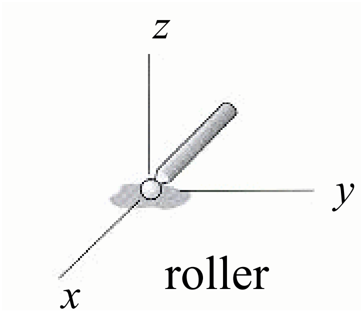
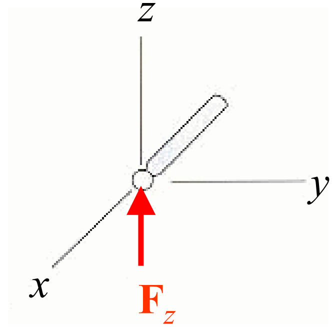
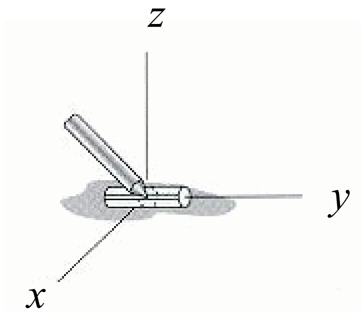
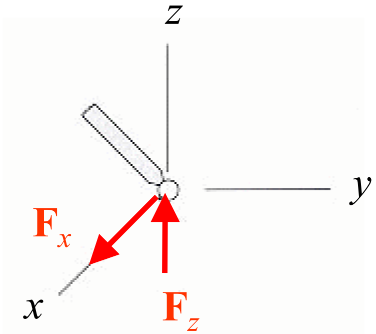
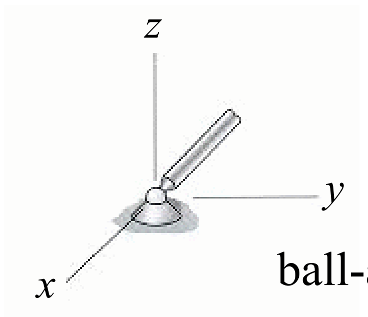
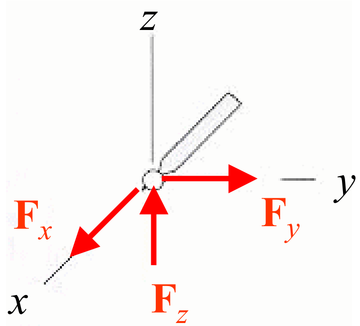


Space Trusses

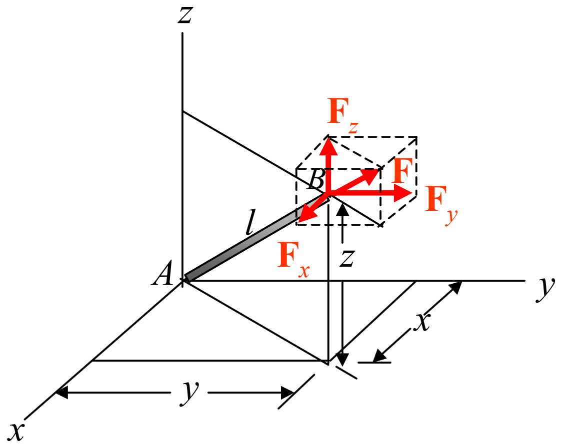
- **Determinacy and Stability**



$b + r < 3j$	unstable truss
$b + r = 3j$	statically determinate-check stability
$b + r \geqslant 3j$	statically determinate-check stability

 short link	
 roller	
 slotted roller constrained in a cylinder	
 ball-and -socket	

- ***x, y, z*, Force Components.**



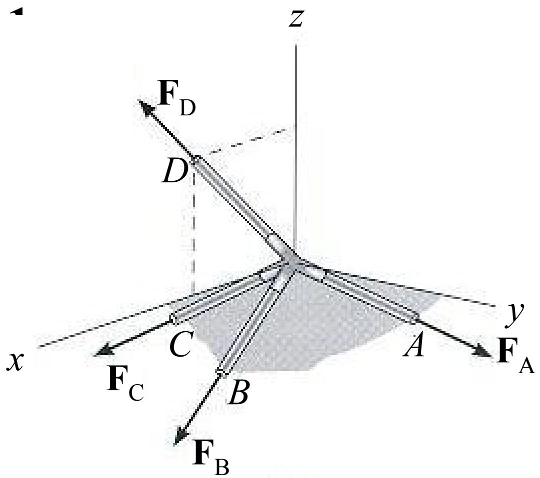
$$l = \sqrt{x^2 + y^2 + z^2}$$

$$F_x = F\left(\frac{x}{l}\right) \qquad F_y = F\left(\frac{y}{l}\right) \qquad F_z = F\left(\frac{z}{l}\right)$$

$$F = \sqrt{F_x^{\ 2} + F_y^{\ 2} + F_z^{\ 2}}$$

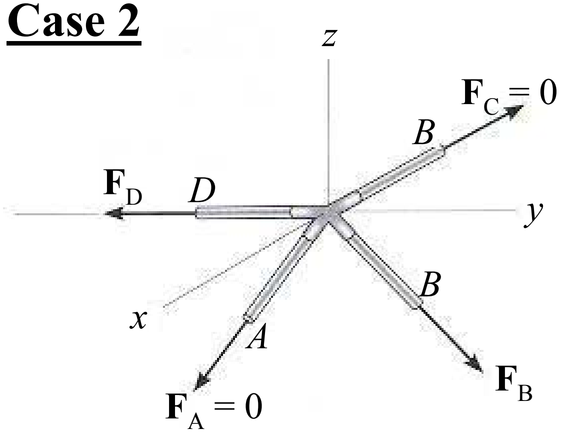
- **Zero-Force Members**

Case 1



$\Sigma F_z = 0$, $F_D = 0$

Case 2

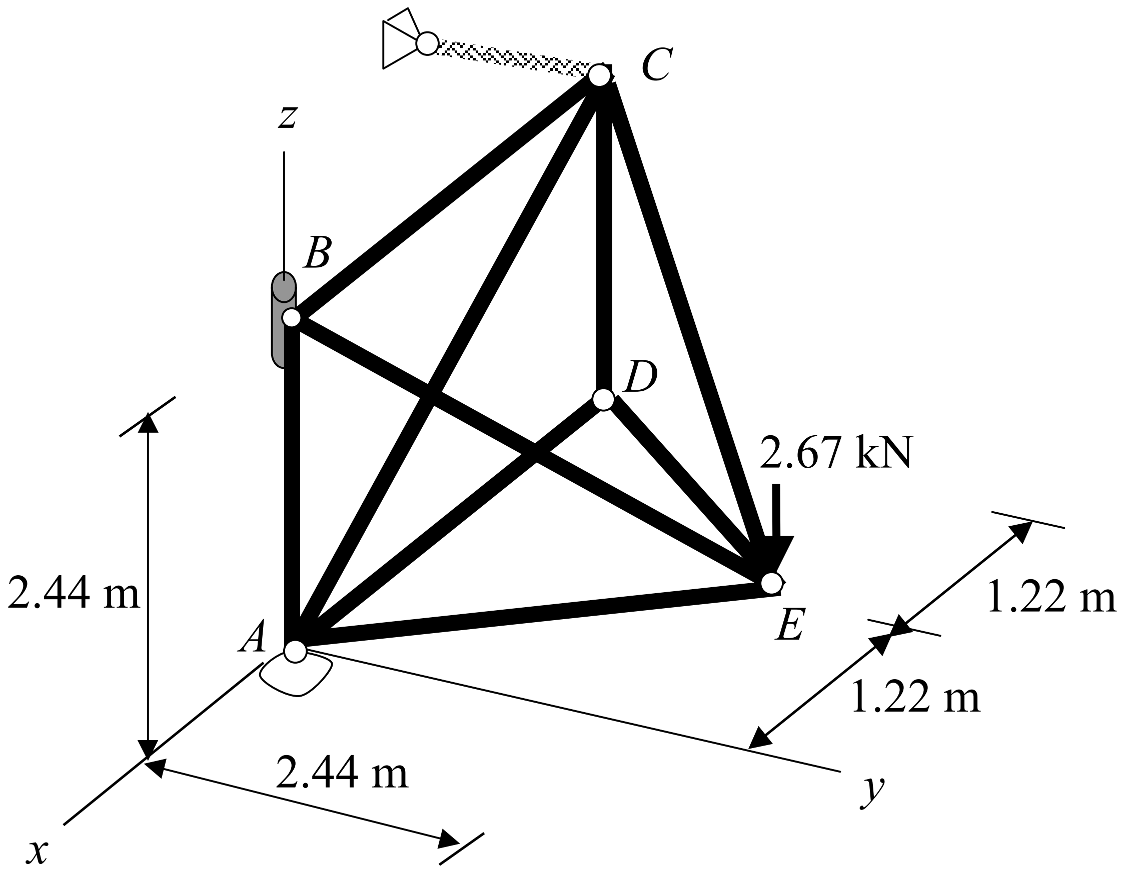


$\Sigma F_z = 0$, $F_B = 0$

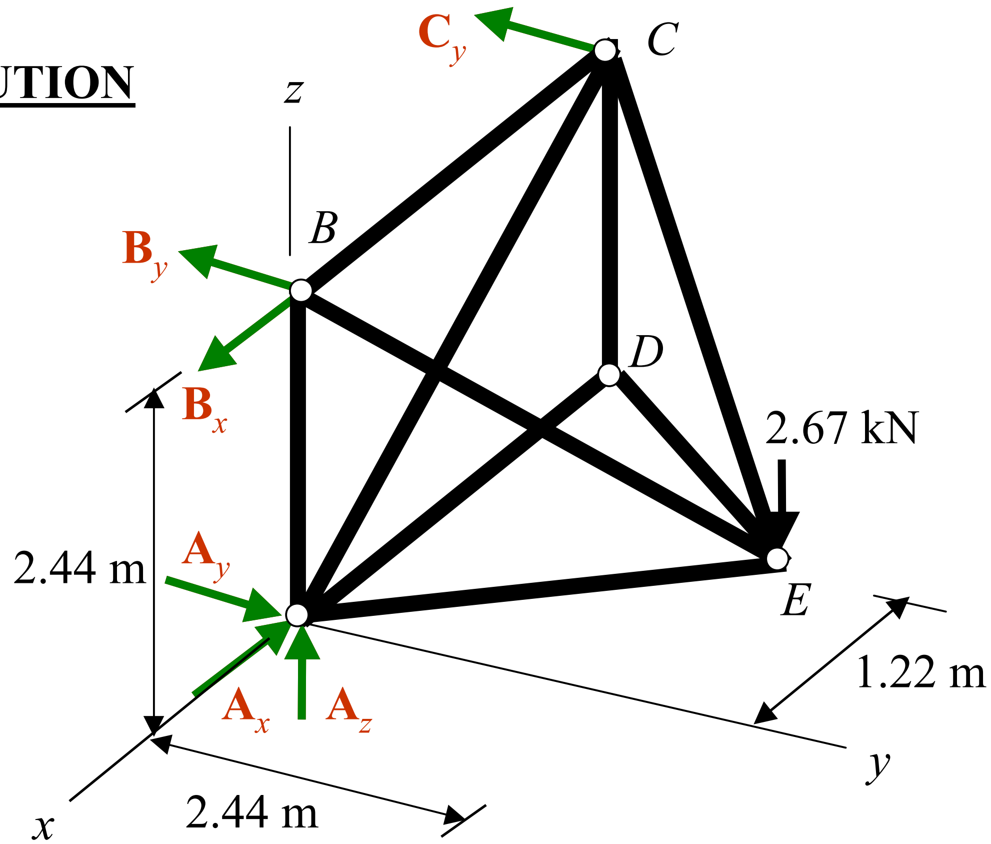
$\Sigma F_y = 0$, $F_D = 0$

Example 3-13

Determine the force in each member of the space truss shown in the figure below. The truss is supported by a ball-and-socket joint at *A*, a slotted roller joint at *B*, and a cable at *C*.

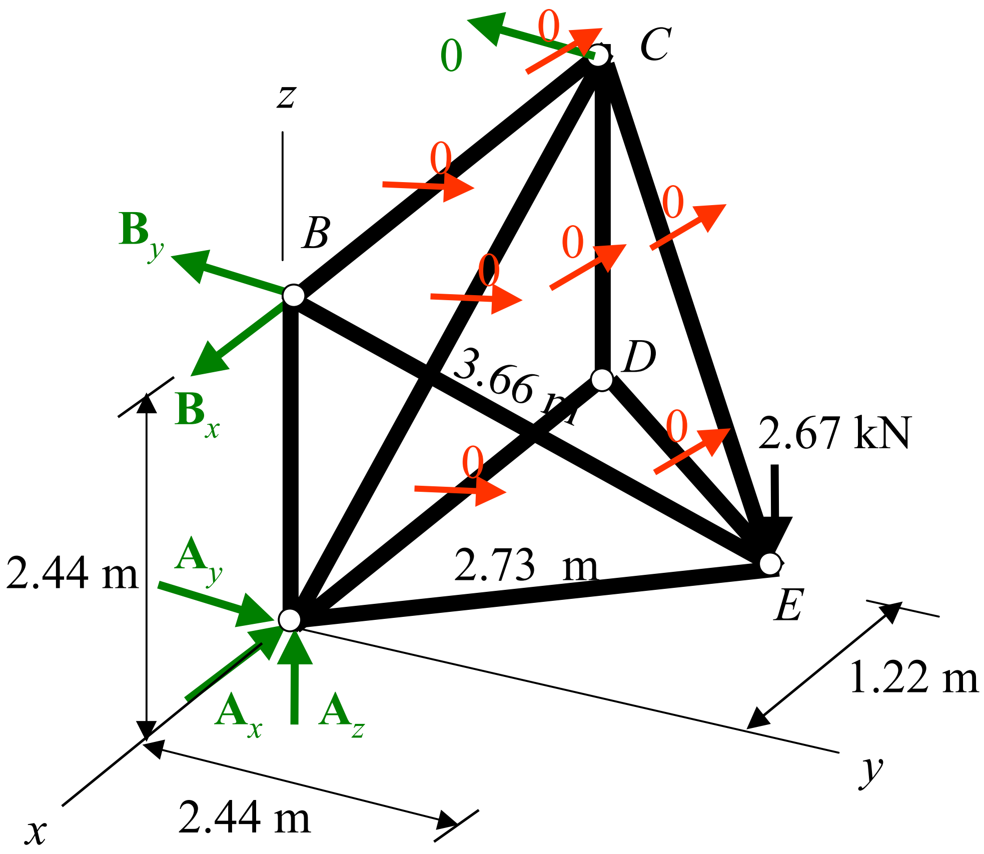


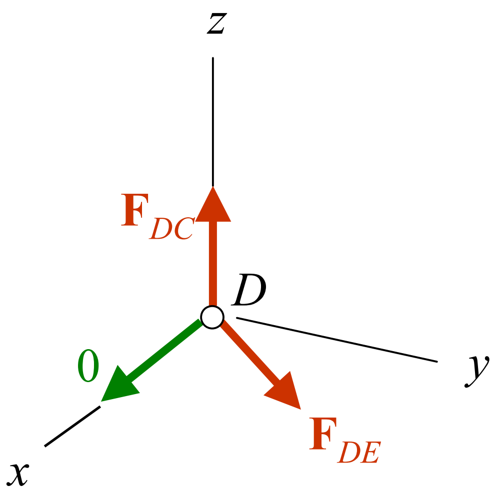
SOLUTION



The truss is statically determinate since $b + r = 3j$ or $9 + 6 = 3(5)$

$\Sigma M_y = 0$:	$-2.67(1.22) + B_x(2.44) = 0$	$B_x = 1.34$ kN
$\Sigma M_z = 0$:	$C_y = 0$ kN	
$\Sigma M_x = 0$:	$B_y(2.44) - 2.67(2.44) = 0$	$B_y = 2.67$ kN
$\Sigma F_x = 0$:	$-A_x + 1.34 = 0$	$A_x = 1.34$ kN
$\Sigma F_y = 0$:	$A_y - 2.67 = 0$	$A_y = 2.67$ kN
$\Sigma F_z = 0$:	$A_z - 2.67 = 0$	$A_z = 2.67$ kN



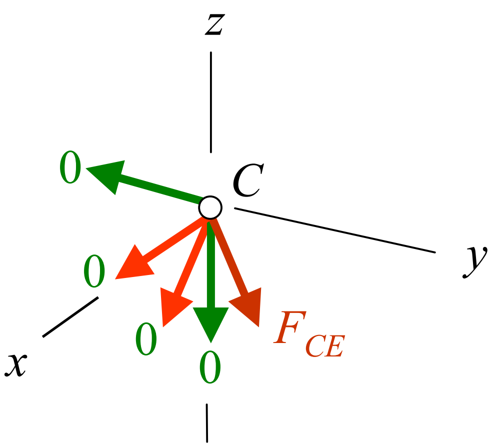


Joint *D*.

$\Sigma F_Z = 0$: $F_{DC} = 0$

$\Sigma F_Y = 0$: $F_{DE} = 0$

$\Sigma F_x = 0$: $F_{DA} = 0$

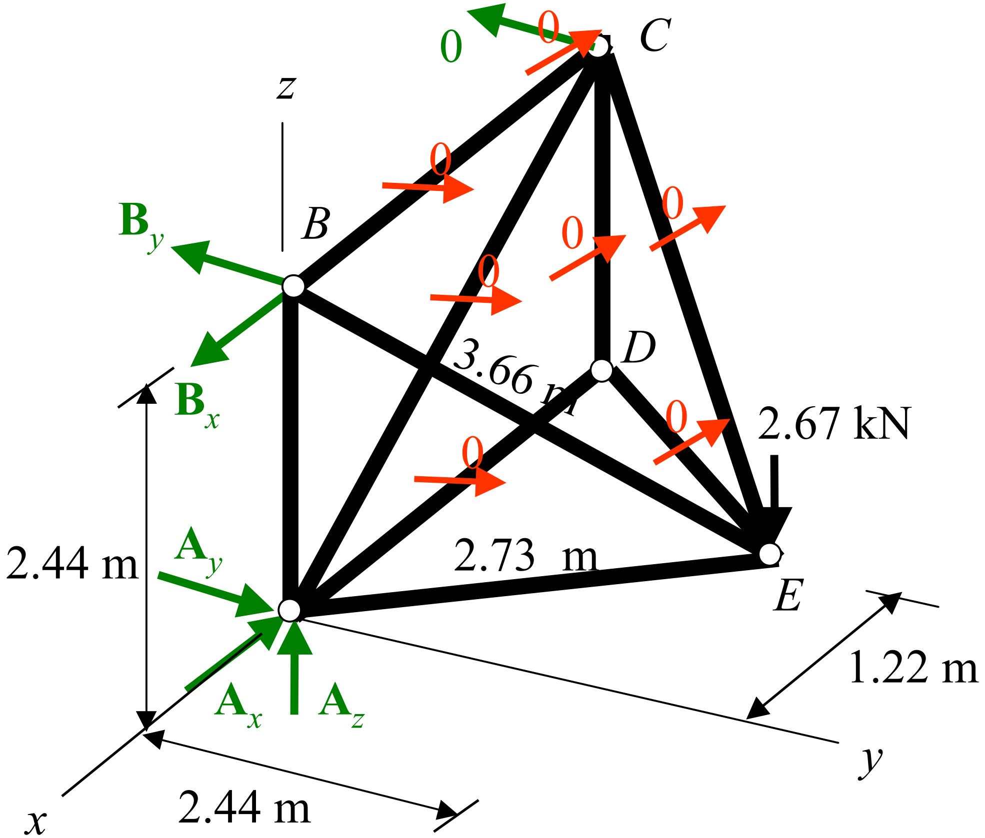


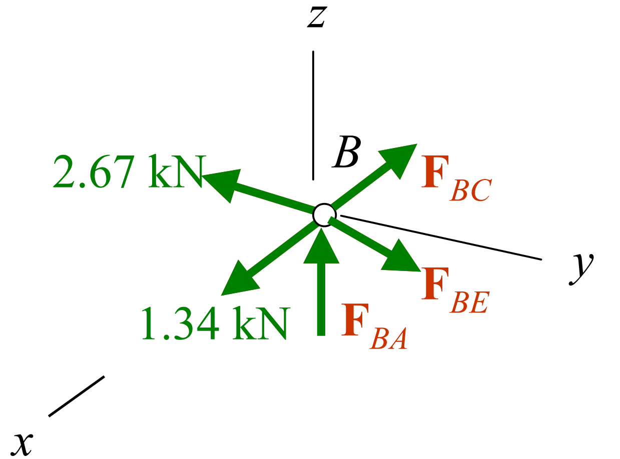
Joint *C*.

$\Sigma F_y = 0$: $F_{CE} = 0$

$\Sigma F_z = 0$: $F_{CA} = 0$

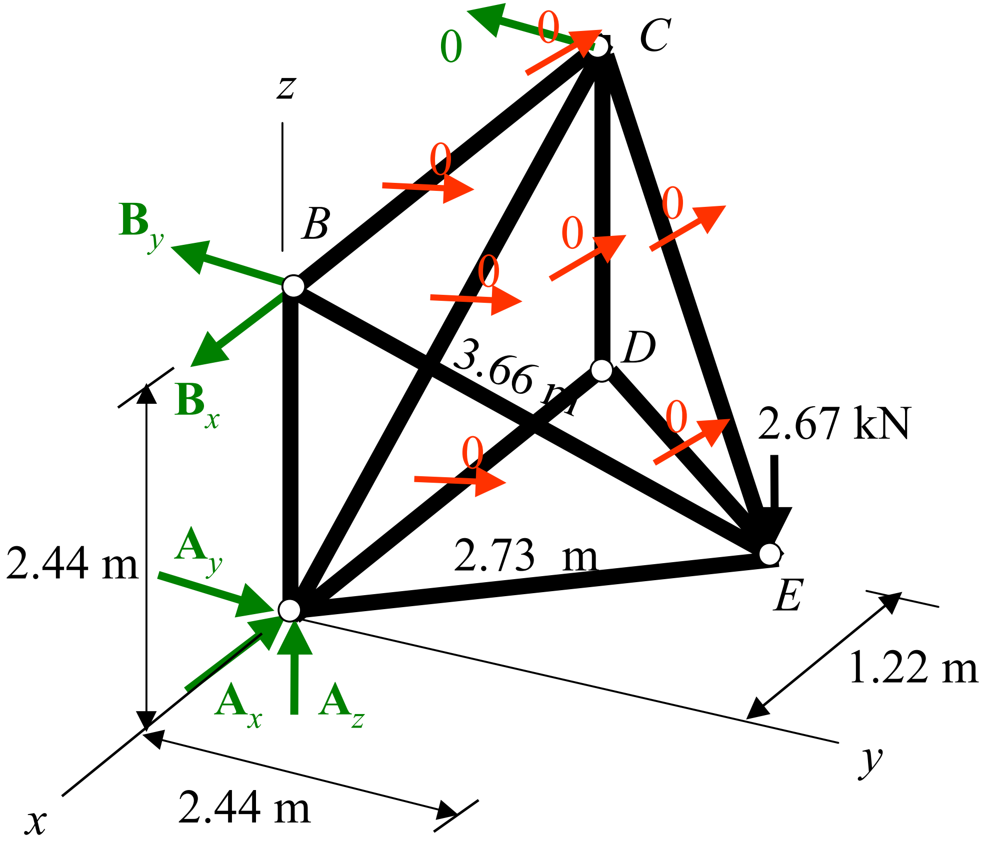
$\Sigma F_x = 0$: $F_{CB} = 0$

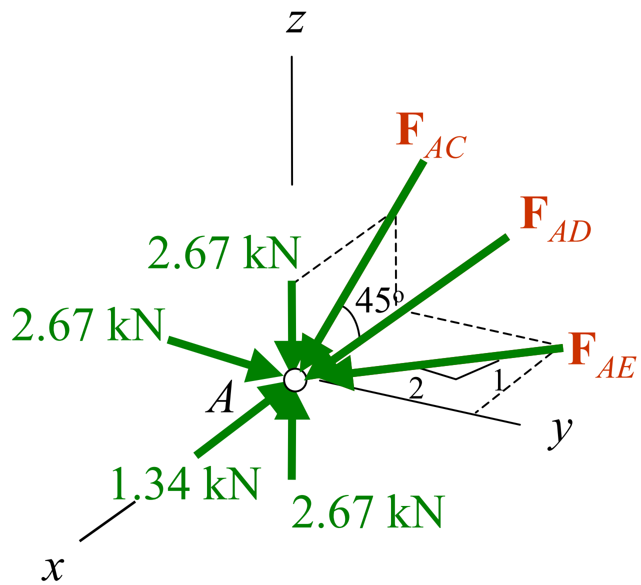




Joint *B*.

$\Sigma F_y = 0$:	$- 2.67 + F_{BE}(2.44/3.66) = 0$	$F_{BE} = 4$ kN (T)
$\Sigma F_x = 0$:	$1.34 - F_{BC} -4(1.22/3.66) = 0$	$F_{BC} = 0$
$\Sigma F_z = 0$:	$F_{BA} - 4(2.44/3.66) = 0$	$F_{BA} = 2.67$ kN (C)





Joint *A*.

$\Sigma F_z = 0$: $\quad 2.67 - 2.67 - F_{AC}\sin 45^\circ = 0 \qquad F_{AC} = 0,\ OK$

$\Sigma F_y = 0$: $\quad -F_{AE}\left(\frac{2}{\sqrt{5}}\right) + 2.67 = 0 \qquad F_{AE} = 2.99\text{ kN (C)}$

$\Sigma F_z = 0$: $\quad -1.34 + F_{AD} + 2.99\left(\frac{1}{\sqrt{5}}\right) = 0 \qquad F_{AD} = 0,\ OK$