## ENCE 353 Final Exam, Open Notes and Open Book

Name:

Exam Format and Grading. The exam will be 2 hrs plus five minutes to read the questions. Only attempt four questions. Cross out the question that you did not attempt in the table below. No extra credit will be given for attempting five questions - we will simply grade and count the first four questions.

Partial credit will be given for partially correct answers, so please show all your working.

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 10 |  |
| Total | 40 |  |

## Question 1: 10 points

Consider the two-span beam structure shown in Figure 1.


Figure 1: Front elevation view of a cantilevered beam structure.
[1a] (5 pts) Use the Muller-Breslau Principle to compute the influence line diagram for the vertical reaction at B .
[1b] (5 pts) Now suppose that span B-C carries a uniform load of $w_{o} / L \mathrm{~N} / \mathrm{m}$. Using your influence line diagram from Part [1a], compute the vertical reaction at B.

## Question 2: 10 points

Consider the cantilevered beam structure shown in Figure 2.


Figure 2: Front elevation view of a cantilevered beam structure.

Notice that segments A-B and B-C have cross-sectional properties EI and 2EI, respectively.
[2a] (5 pts) Draw and label a diagram showing how the rotation at A is related to the beam deflections at points B and C.
[2b] (5 pts) Use the method of moment-area to compute the vertical deflection of the beam at point C.

## Question 3: 10 points

Consider the S-shaped beam structure shown in Figure 3.


Figure 3: S-shaped beam structure.
[3a] (5 pts) By taking moments, compute the support reactions at A and B. Compute and draw the bending moment diagram.
[3b] (5 pts) Use the principle of virtual displacements to compute the vertical reaction at Point B.

## Question 4: 10 points

Consider the cantilevered beam structure shown in Figure 4.


Figure 4: Front elevation view of a cantilevered beam structure.
[4a] (5 pts) Use the principle of virtual forces to compute the vertical deflection at point C.
[4b] (5 pts) Use the principle of virtual forces to compute the rotation at point C.

## Question 5: 10 points

Consider the supported cantilevered beam structure shown in Figure 4.


Figure 5: Front elevation view of a supported cantilevered beam structure.

Use the principle of virtual forces to compute the two-by-two flexibility matrix connecting displacements at points B and C to applied loads $P_{b}$ and $P_{c}$, i.e.,

$$
\left[\begin{array}{l}
\triangle_{b}  \tag{1}\\
\triangle_{c}
\end{array}\right]=\left[\begin{array}{ll}
f_{11} & f_{12} \\
f_{21} & f_{22}
\end{array}\right]\left[\begin{array}{l}
P_{b} \\
P_{c}
\end{array}\right] .
$$

Question 5 continued ...

