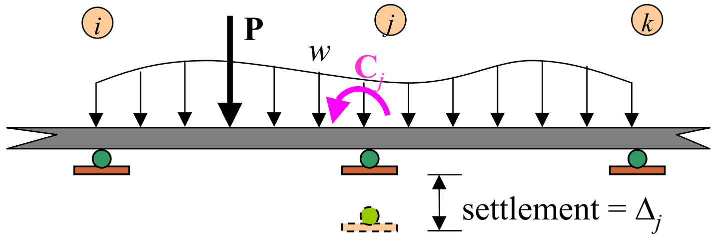
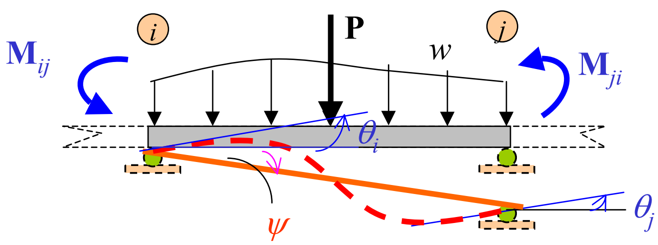
DISPLACEMENT METHOD OF ANALYSIS: SLOPE DEFLECTION EQUATIONS

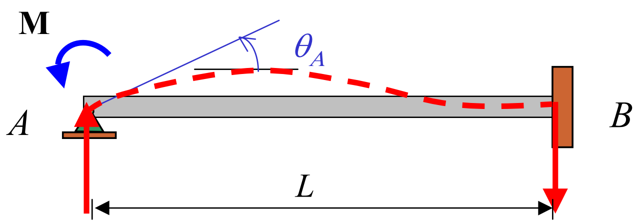
- General Case
- Stiffness Coefficients
- Stiffness Coefficients Derivation
- Fixed-End Moments
- Pin-Supported End Span
- Typical Problems
- Analysis of Beams
- Analysis of Frames: No Sidesway
- Analysis of Frames: Sidesway

Slope – Deflection Equations

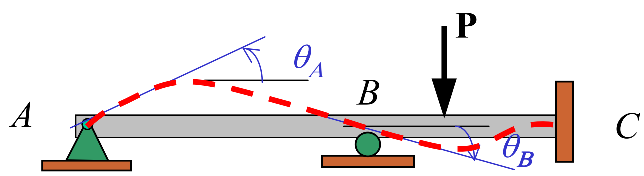




Degrees of Freedom

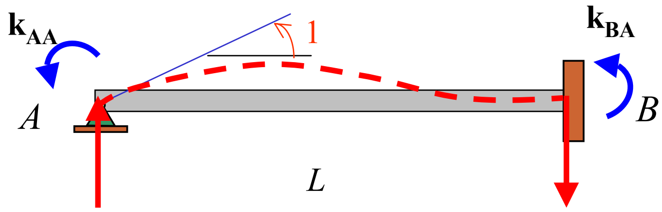


1 DOF: θ_A



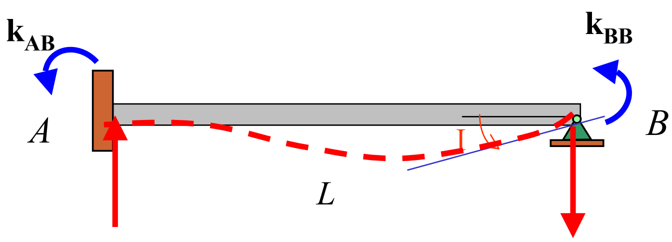
2 DOF: θ_A, θ_B

Stiffness



$$k_{AA} = \frac{4EI}{L}$$

$$k_{BA} = \frac{2EI}{L}$$

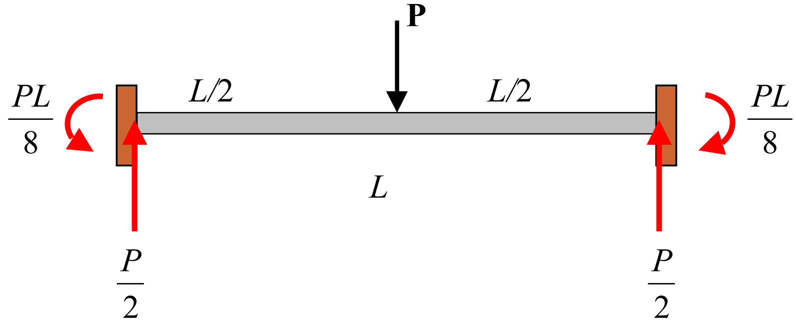


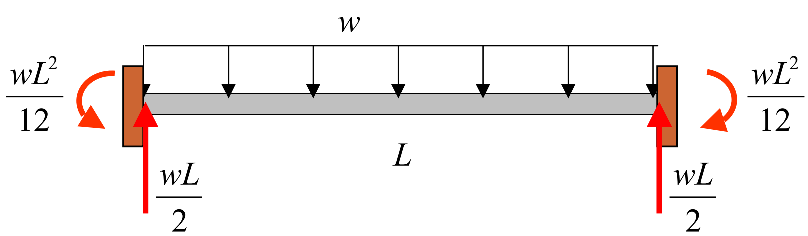
$$k_{BB} = \frac{4EI}{L}$$

$$k_{AB} = \frac{2EI}{L}$$

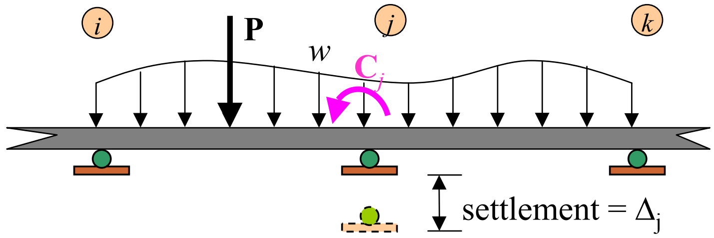
Fixed-End Forces

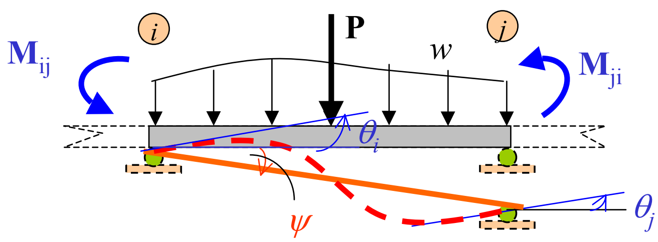
- **Fixed-End Moments: Loads**

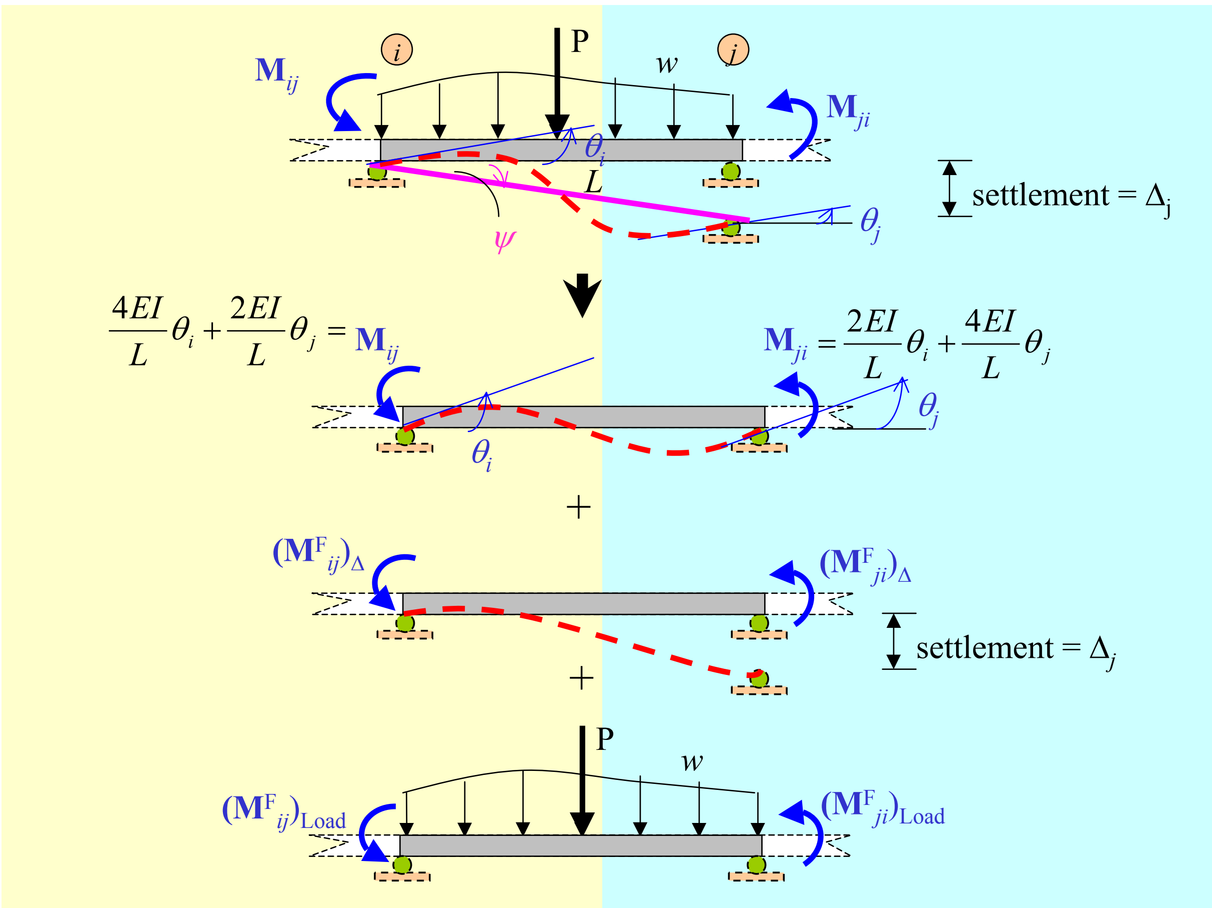




General Case

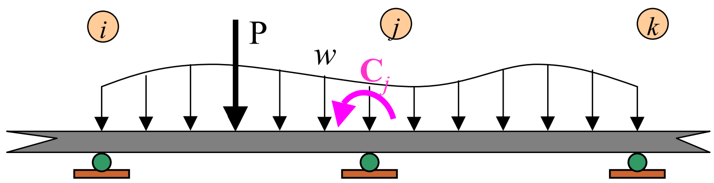


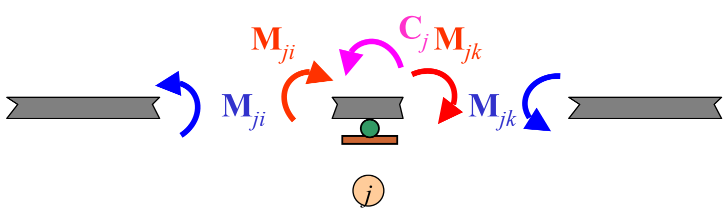




$$M_{ij} = (\frac{4EI}{L})\theta_i + (\frac{2EI}{L})\theta_j + (M^F{}_{ij})_\Delta + (M^F{}_{ij})_{Load}\ ,\ M_{ji} = (\frac{2EI}{L})\theta_i + (\frac{4EI}{L})\theta_j + (M^F{}_{ji})_\Delta + (M^F{}_{ji})_{Load}$$

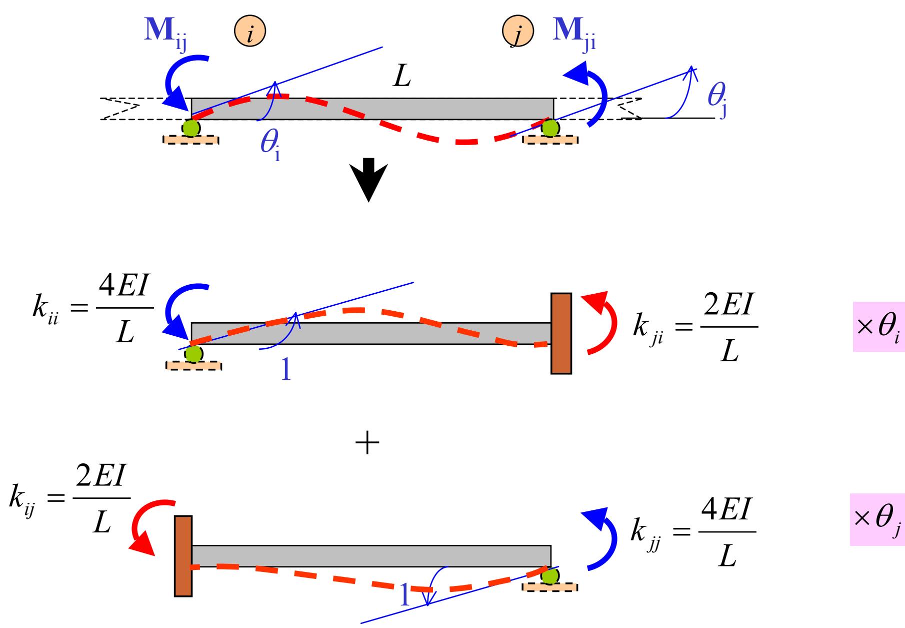
Equilibrium Equations





$$+\circlearrowright\ \Sigma M_j = 0: \quad -M_{ji} - M_{jk} + C_j = 0$$

Stiffness Coefficients



Matrix Formulation

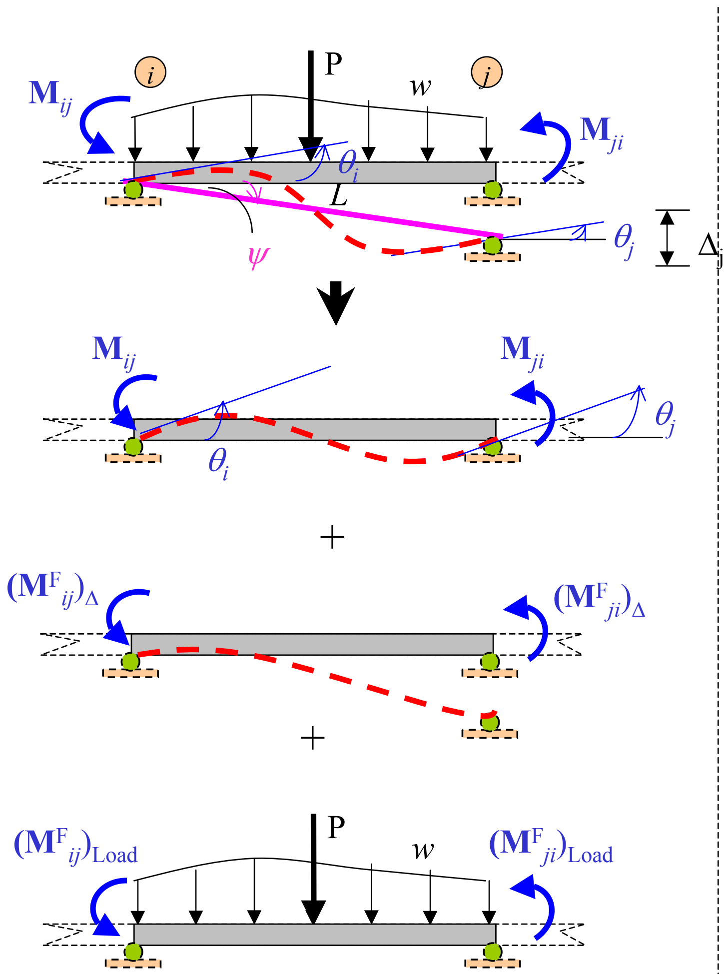
$$M_{ij} = (\frac{4EI}{L})\theta_i + (\frac{2EI}{L})\theta_j + (M^F{}_{ij})$$

$$M_{ji} = (\frac{2EI}{L})\theta_i + (\frac{4EI}{L})\theta_j + (M^F{}_{ji})$$

$$\begin{bmatrix} M_{ij} \\ M_{ji} \end{bmatrix} = \begin{bmatrix} (4EI/L) & (2EI/L) \\ (2EI/L) & (4EI/L) \end{bmatrix} \begin{bmatrix} \theta_{iI} \\ \theta_j \end{bmatrix} + \begin{bmatrix} M_{ij}{}^F \\ M_{ji}{}^F \end{bmatrix}$$

$$\boxed{[k] = \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix}}$$

Stiffness Matrix



$$[M] = [K][\theta] + [FEM]$$

$$([M] - [FEM]) = [K][\theta]$$

$$[\theta] = [K]^{-1}[M] - [FEM]$$

Stiffness matrix

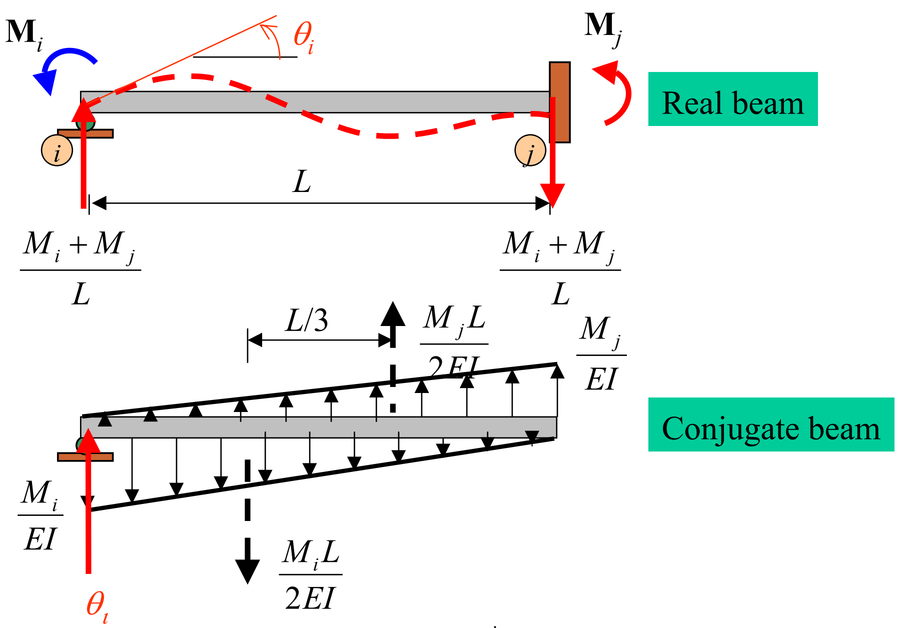
Fixed-end moment matrix

$$\boxed{[D] = [K]^{-1}([Q] - [\mathrm{FEM}])}$$

Displacement matrix

Force matrix

Stiffness Coefficients Derivation: Fixed-End Support



$$+\circlearrowright \ \Sigma M'_i = 0: \quad -\left(\frac{M_i L}{2EI}\right)\left(\frac{L}{3}\right) + \left(\frac{M_j L}{2EI}\right)\left(\frac{2L}{3}\right) = 0$$

$$M_i = 2M_j \quad ---(1)$$

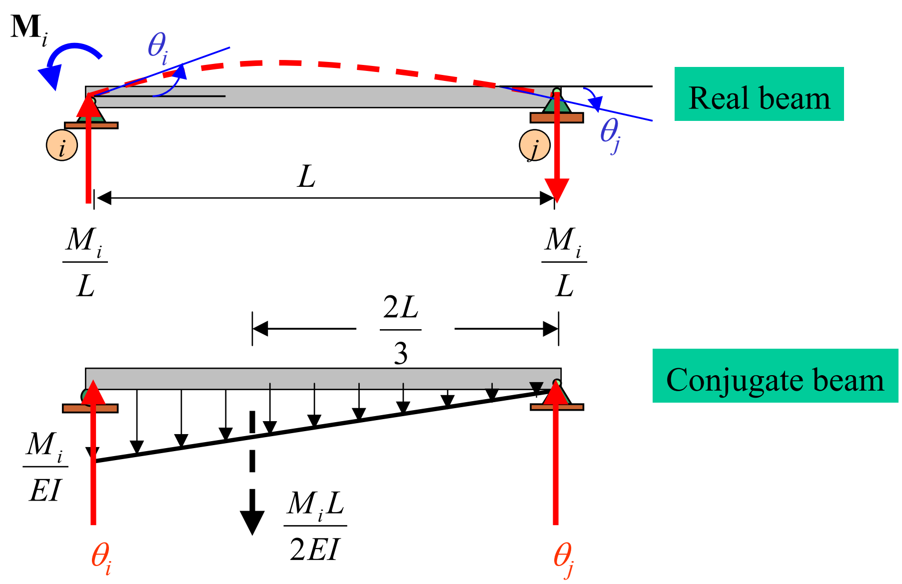
$$+\uparrow \quad \Sigma F_y = 0: \quad \theta_i - \left(\frac{M_i L}{2EI}\right) + \left(\frac{M_j L}{2EI}\right) = 0 \quad ---(2)$$

From (1) *and* (2);

$$M_i = \left(\frac{4EI}{L}\right)\theta_i$$

$$M_j = \left(\frac{2EI}{L}\right)\theta_i$$

Stiffness Coefficients Derivation: Pinned-End Support



$$+\circlearrowright \quad \Sigma M'_j = 0: \quad \left(\frac{M_i L}{2EI}\right)\left(\frac{2L}{3}\right) - \theta_i L = 0$$

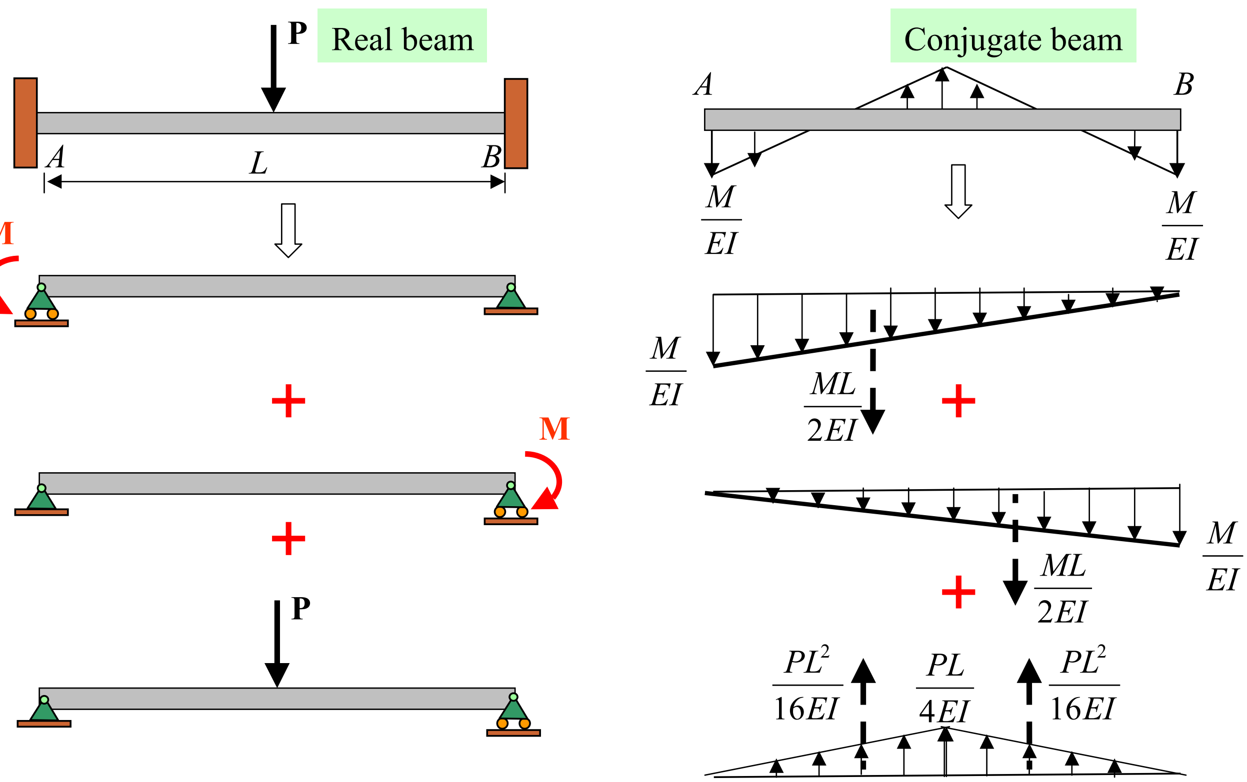
$$\theta_i = \left(\frac{M_i L}{3EI}\right)$$

$$+\uparrow \quad \Sigma F_y = 0: \quad \left(\frac{M_i L}{3EI}\right) - \left(\frac{M_i L}{2EI}\right) + \theta_j = 0$$

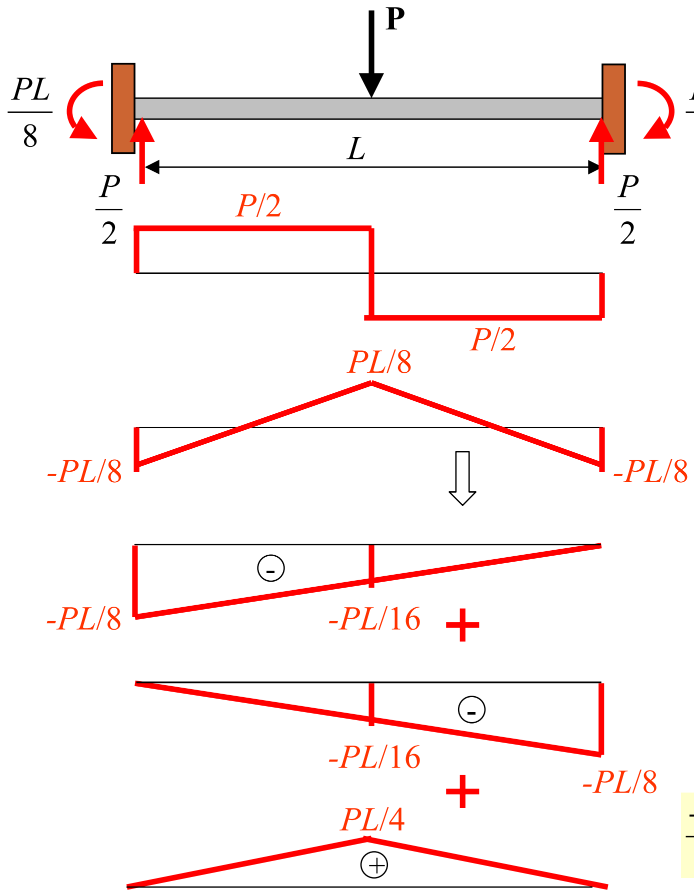
$$\theta_j = \left(\frac{-M_i L}{6EI}\right)$$

$$\theta_i = 1 = \left(\frac{M_i L}{3EI}\right) \rightarrow \quad M_i = \frac{3EI}{L}$$

Fixed end moment : Point Load

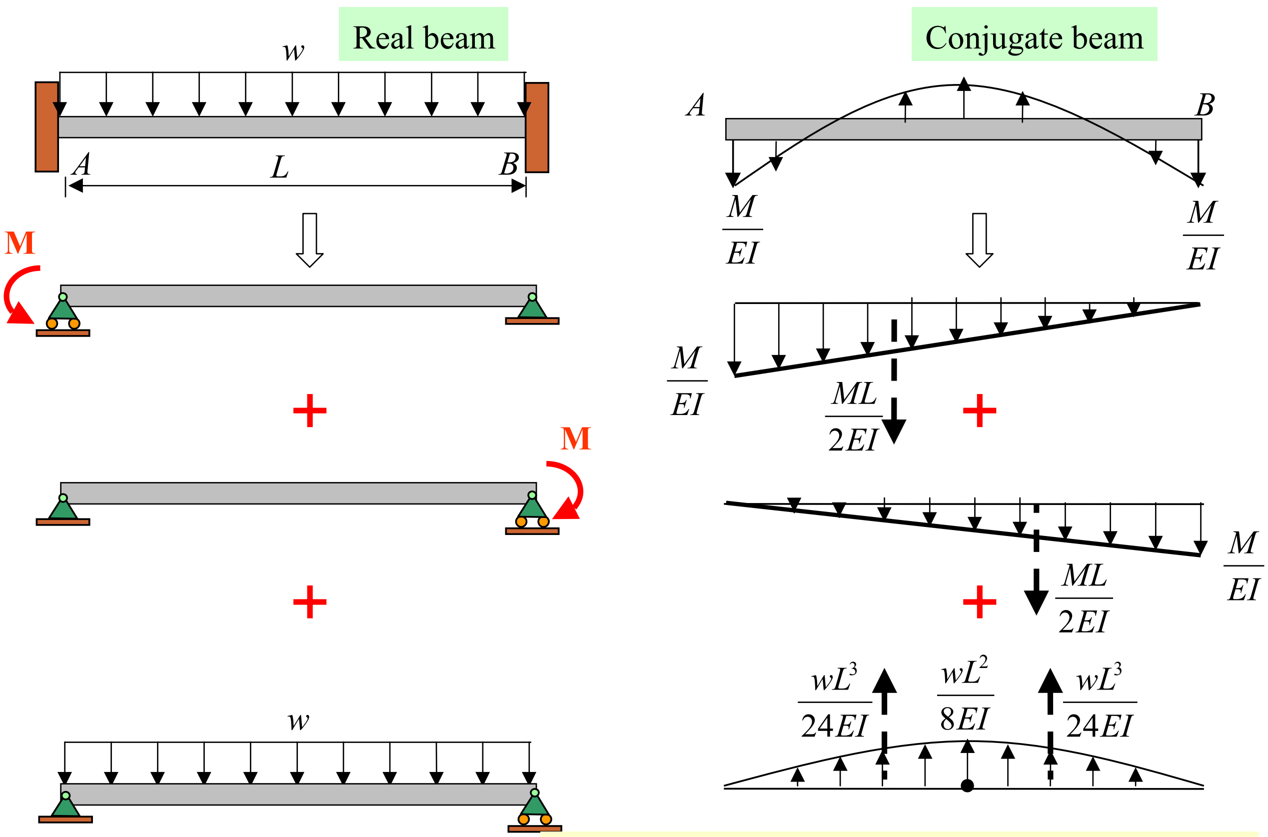


$$+\uparrow \quad \Sigma F_y = 0: \quad -\frac{ML}{2EI} - \frac{ML}{2EI} + \frac{2PL^2}{16EI} = 0, \quad M = \frac{PL}{8}$$



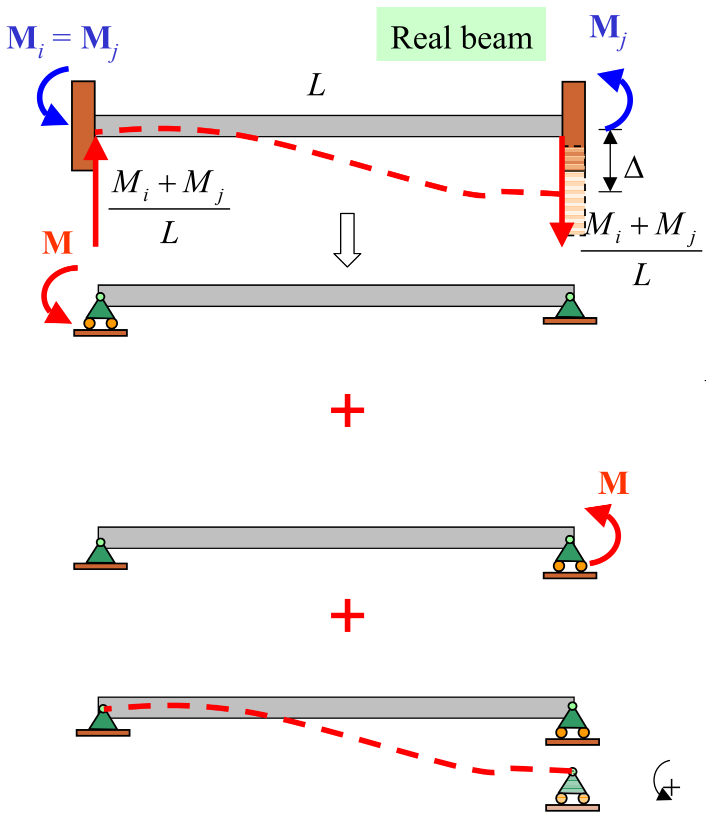
$$\frac{-PL}{16}+\frac{-PL}{16}+\frac{PL}{4}=\frac{PL}{8}$$

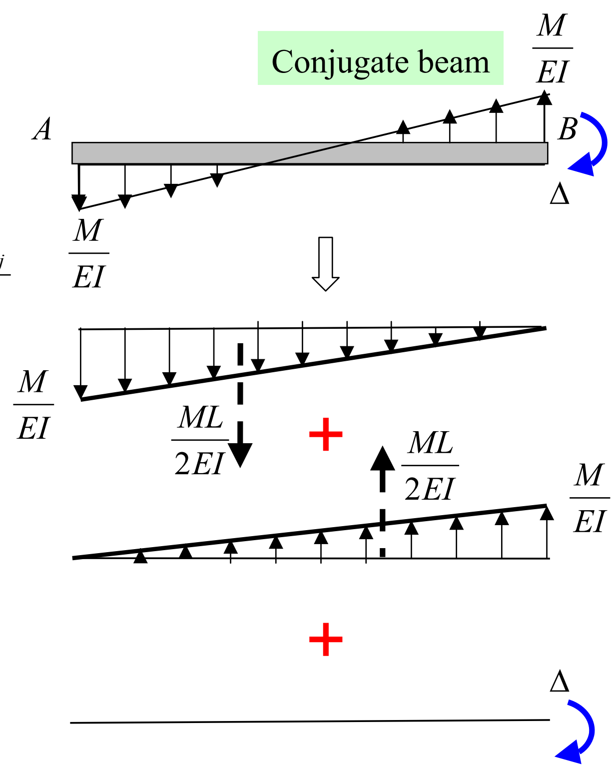
▶ **Uniform load**



$$+\uparrow \quad \Sigma F_y = 0: \quad -\frac{ML}{2EI} - \frac{ML}{2EI} + \frac{2wL^3}{24EI} = 0, \quad M = \frac{wL^2}{12}$$

▶ Settlements

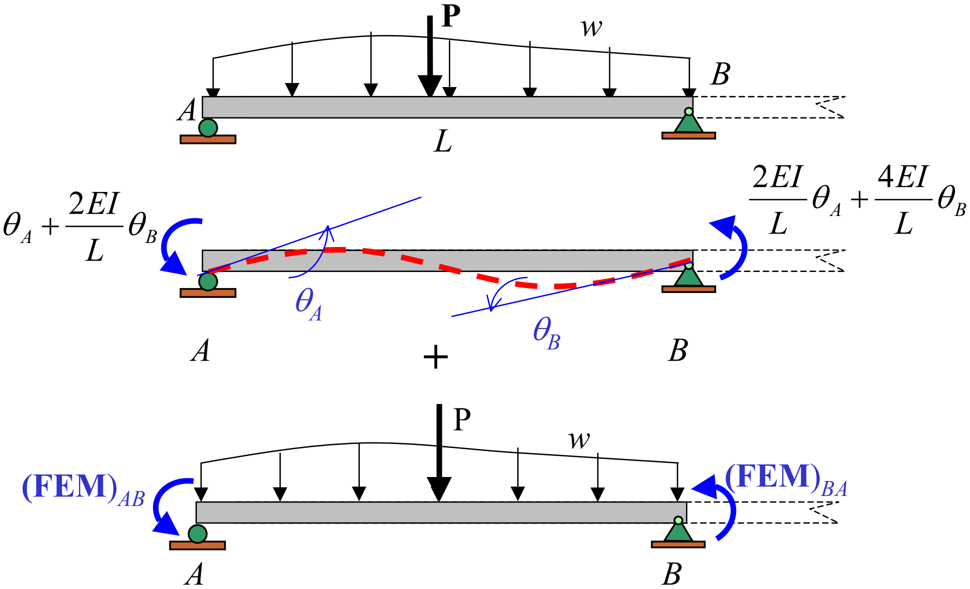




$$+\Sigma M_B = 0: \quad -\Delta - \left(\frac{ML}{2EI}\right)\left(\frac{L}{3}\right) + \left(\frac{ML}{2EI}\right)\left(\frac{2L}{3}\right) = 0,$$

$$M = \frac{6EI\Delta}{L^2}$$

Pin-Supported End Span: Simple Case



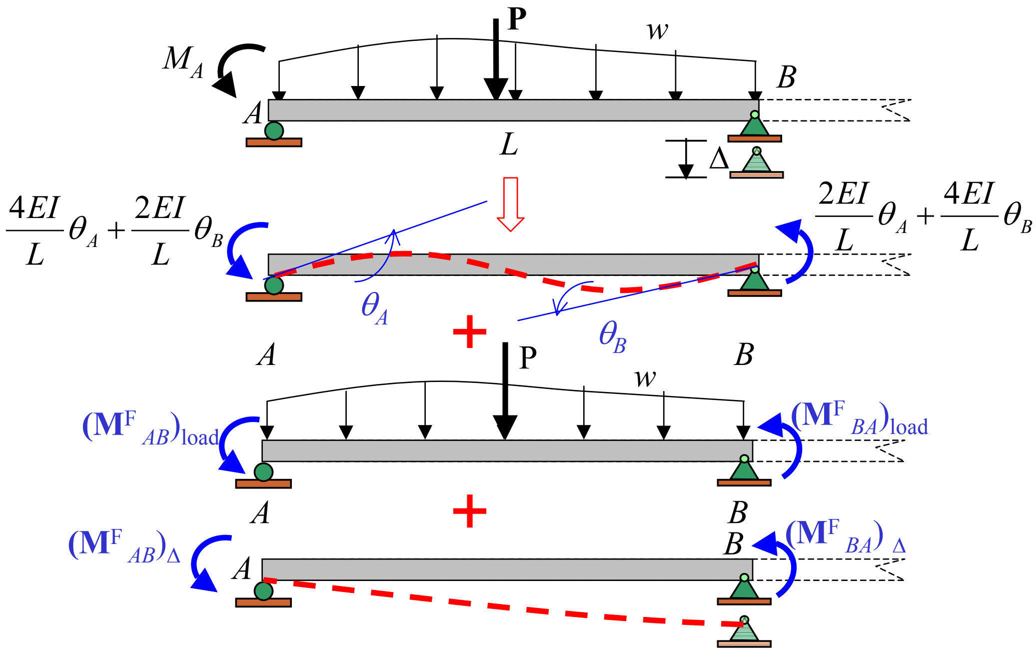
$$M_{AB} = 0 = (4EI/L)\theta_A + (2EI/L)\theta_B + (FEM)_{AB} \quad ---(1)$$

$$M_{BA} = 0 = (2EI/L)\theta_A + (4EI/L)\theta_B + (FEM)_{BA} \quad ---(2)$$

$$2(2)-(1): \quad 2M_{BA} = (6EI/L)\theta_B + 2(FEM)_{BA} - (FEM)_{BA}$$

$$M_{BA} = (3EI/L)\theta_B + (FEM)_{BA} - \frac{(FEM)_{BA}}{2}$$

Pin-Supported End Span: With End Couple and Settlement



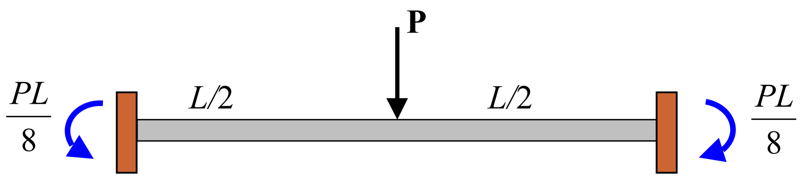
$$M_{AB} = M_A = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B + (M^F_{AB})_{load} + (M^F_{AB})_\Delta \quad ---(1)$$

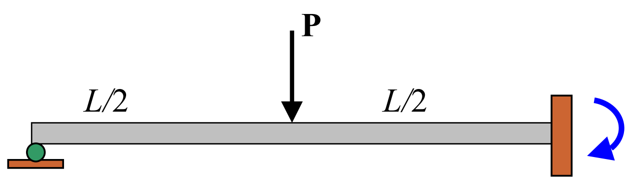
$$M_{BA} = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B + (M^F_{BA})_{load} + (M^F_{BA})_\Delta \quad ---(2)$$

$$E\lim inate\ \theta_A\ by\quad \frac{2(2)-(1)}{2}: \quad M_{BA} = \frac{3EI}{L}\theta_B + [(M^F_{BA})_{load} - \frac{1}{2}(M^F_{AB})_{load}] + \frac{1}{2}(M^F_{BA})_\Delta + \frac{M_A}{2}$$

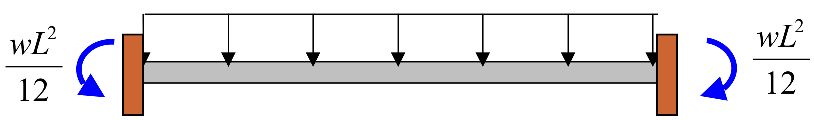
Fixed-End Moments

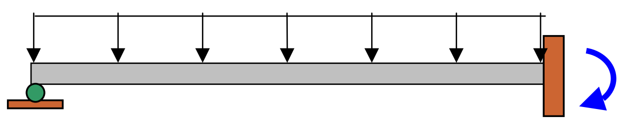
- **Fixed-End Moments: Loads**





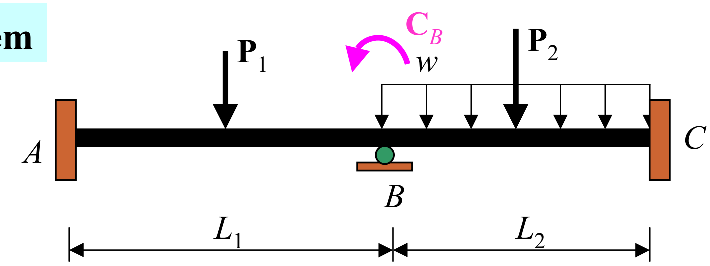
$$\frac{PL}{8} + (\frac{1}{2})[-(-\frac{PL}{8})] = \frac{3PL}{16}$$

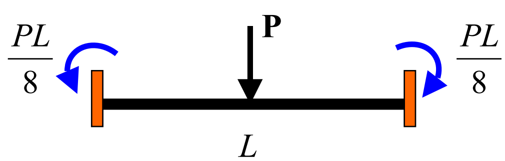


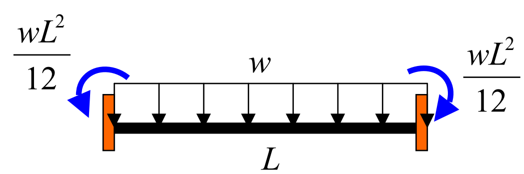


$$\frac{wL^2}{12} + (\frac{1}{2})[-(-\frac{wL^2}{12})] = \frac{wL^2}{8}$$

Typical Problem





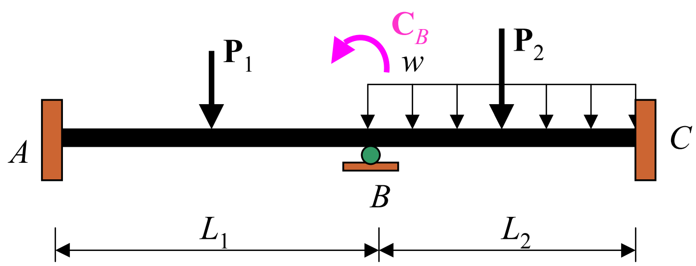


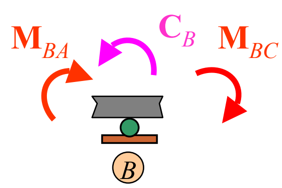
$$M_{AB} = \frac{4EI}{L_1}\cancelto{0}{\theta_A} + \frac{2EI}{L_1}\theta_B + 0 + \frac{P_1L_1}{8}$$

$$M_{BA} = \frac{2EI}{L_1}\cancelto{0}{\theta_A} + \frac{4EI}{L_1}\theta_B + 0 - \frac{P_1L_1}{8}$$

$$M_{BC} = \frac{4EI}{L_2}\theta_B + \frac{2EI}{L_2}\cancelto{0}{\theta_C} + 0 + \frac{P_2L_2}{8} + \frac{wL_2^{\ 2}}{12}$$

$$M_{CB} = \frac{2EI}{L_2}\theta_B + \frac{4EI}{L_2}\cancelto{0}{\theta_C} + 0 + \frac{-P_2L_2}{8} - \frac{wL_2^{\ 2}}{12}$$

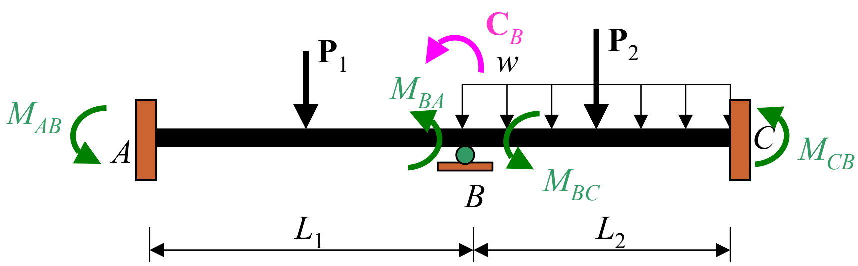




$$M_{BA} = \frac{2EI}{L_1}\theta_A + \frac{4EI}{L_1}\theta_B + 0 - \frac{P_1 L_1}{8}$$

$$M_{BC} = \frac{4EI}{L_2}\theta_B + \frac{2EI}{L_2}\theta_C + 0 + \frac{P_2 L_2}{8} + \frac{wL_2^{\ 2}}{12}$$

$$+ \quad \Sigma M_B = 0: \quad C_B - M_{BA} - M_{BC} = 0 \quad \rightarrow \quad Solve\ for\ \theta_B$$



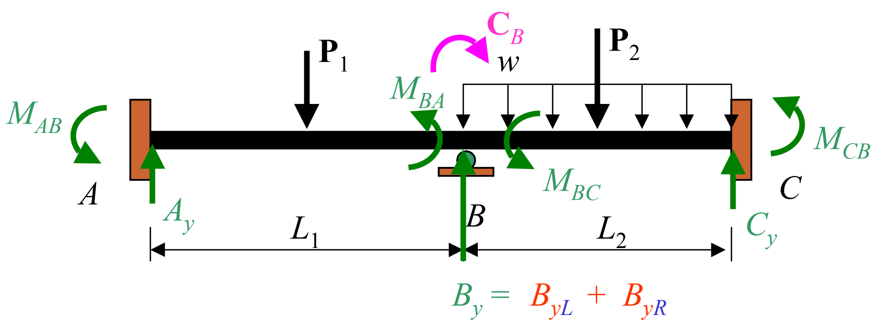
Substitute θ_B in M_{AB}, M_{BA}, M_{BC}, M_{CB}

$$M_{AB} = \frac{4EI}{L_1}\cancelto{0}{\theta_A} + \frac{2EI}{L_1}\theta_B + 0 + \frac{P_1L_1}{8}$$

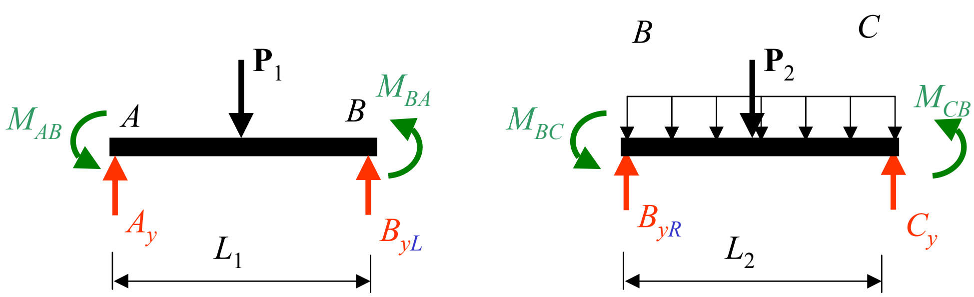
$$M_{BA} = \frac{2EI}{L_1}\cancelto{0}{\theta_A} + \frac{4EI}{L_1}\theta_B + 0 - \frac{P_1L_1}{8}$$

$$M_{BC} = \frac{4EI}{L_2}\theta_B + \frac{2EI}{L_2}\cancelto{0}{\theta_C} + 0 + \frac{P_2L_2}{8} + \frac{wL_2^{\;2}}{12}$$

$$M_{CB} = \frac{2EI}{L_2}\theta_B + \frac{4EI}{L_2}\cancelto{0}{\theta_C} + 0 + \frac{-P_2L_2}{8} - \frac{wL_2^{\;2}}{12}$$



$\mathbf{C}_B$
$\mathbf{P}_1$
$\mathbf{P}_2$
w
M_{AB}
M_{BA}
M_{BC}
M_{CB}
A
B
C
A_y
C_y
L_1
L_2
$B_y = B_{yL} + B_{yR}$

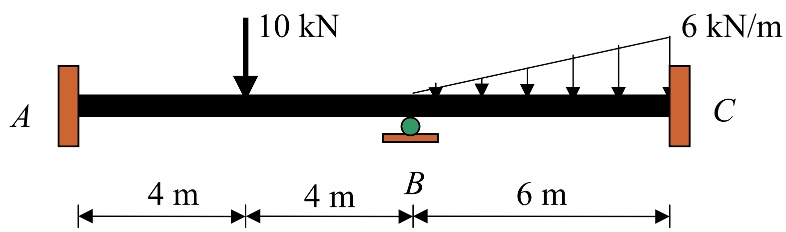


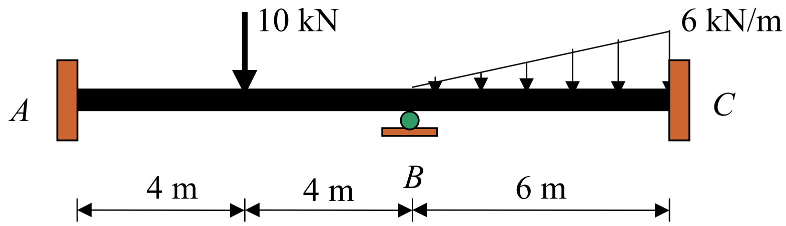
$\mathbf{P}_1$
M_{AB}
A
B
M_{BA}
A_y
B_{yL}
L_1
B
C
$\mathbf{P}_2$
M_{BC}
M_{CB}
B_{yR}
C_y
L_2

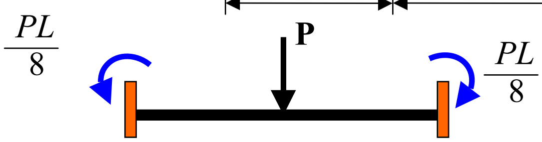
Example of Beams

Example 1

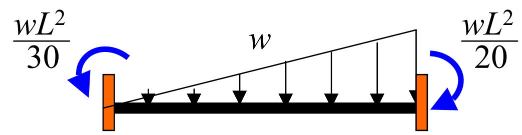
Draw the **quantitative shear , bending moment** diagrams and **qualitative deflected curve** for the beam shown. *EI* is constant.

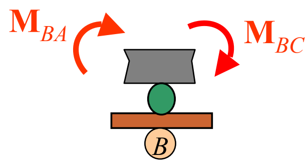






FEM





$[M] = [K][Q] + [\mathrm{FEM}]$

$$M_{AB} = \frac{4EI}{8}\cancelto{0}{\theta_A} + \frac{2EI}{8}\theta_B + \frac{(10)(8)}{8}$$

$$M_{BA} = \frac{2EI}{8}\cancelto{0}{\theta_A} + \frac{4EI}{8}\theta_B - \frac{(10)(8)}{8}$$

$$M_{BC} = \frac{4EI}{6}\theta_B + \frac{2EI}{6}\cancelto{0}{\theta_C} + \frac{(6)(6^2)}{30}$$

$$M_{CB} = \frac{2EI}{6}\theta_B + \frac{4EI}{6}\cancelto{0}{\theta_C} - \frac{(6)(6)^2}{20}$$

$$\circlearrowleft + \quad \Sigma M_B = 0: \quad -M_{BA} - M_{BC} = 0$$

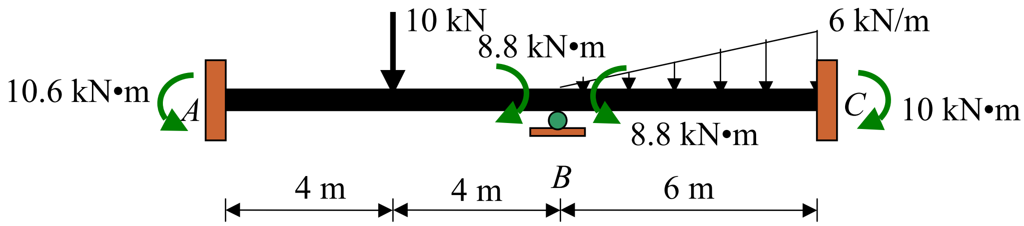
$$\left(\frac{4EI}{8} + \frac{4EI}{6}\right)\theta_B - 10 + \frac{(6)(6^2)}{30} = 0$$

$$\theta_B = \frac{2.4}{EI}$$

Substitute θ_B in the moment equations:

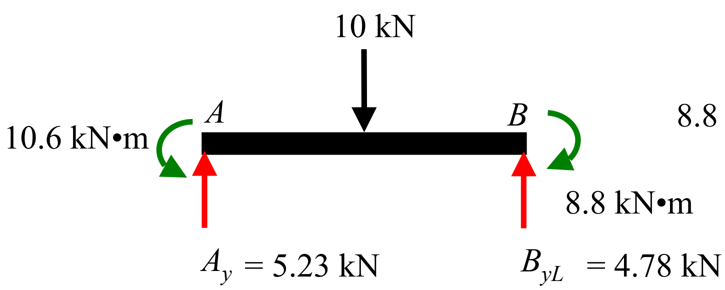
M_{AB} = 10.6 kN•m, M_{BC} = 8.8 kN•m

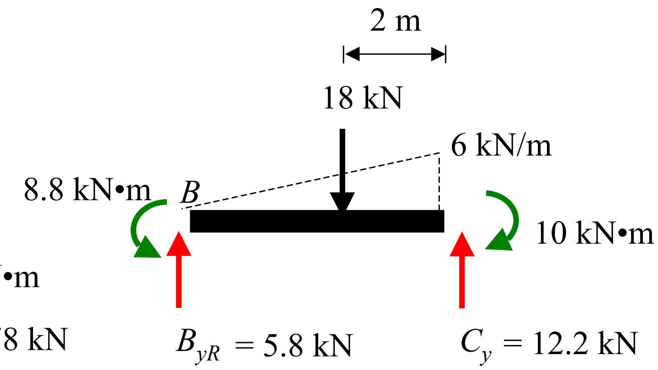
M_{BA} = - 8.8 kN•m, M_{CB} = -10 kN•m

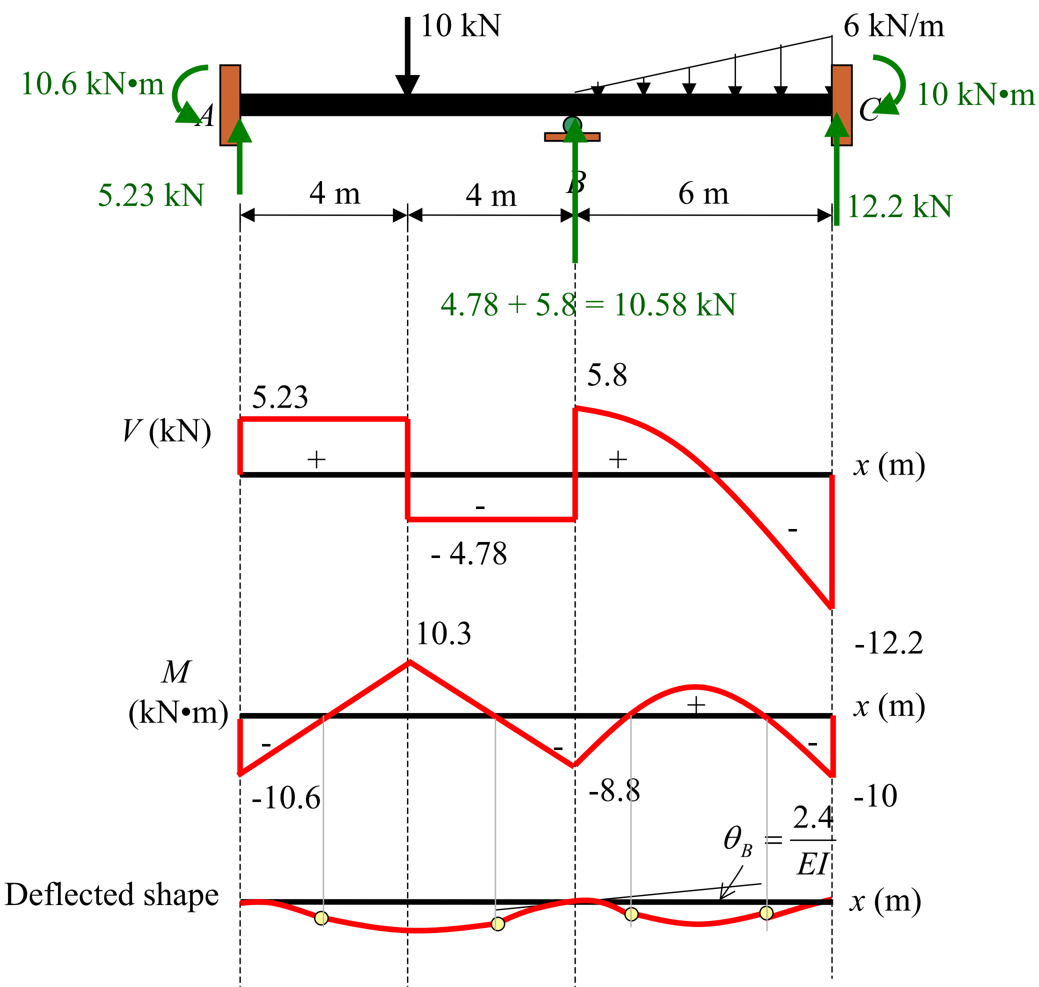


M_{AB} = 10.6 kN•m, M_{BC} = 8.8 kN•m

M_{BA} = - 8.8 kN•m, M_{CB}= -10 kN•m

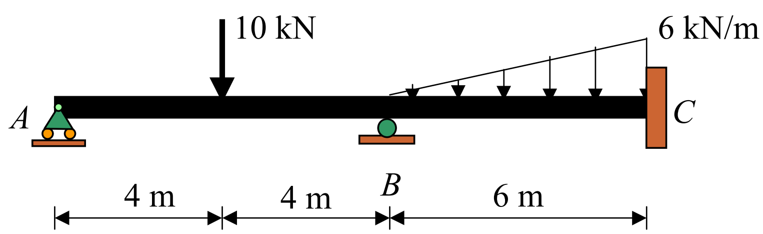


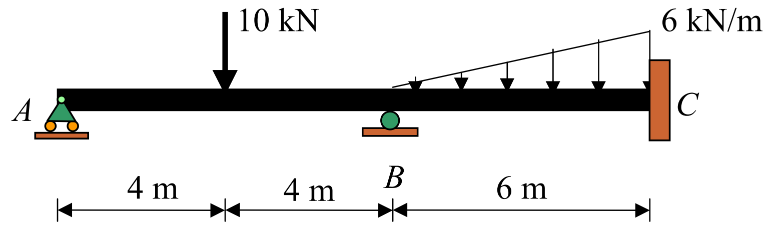


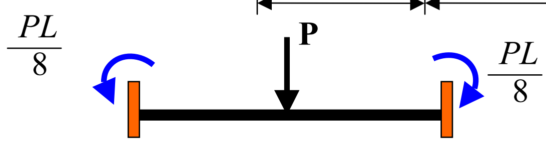

10 kN
6 kN/m
10.6 kN•m
A
10 kN•m
C
5.23 kN
4 m
4 m
B
6 m
12.2 kN
4.78 + 5.8 = 10.58 kN
5.8
5.23
V (kN)
+
+
x (m)
-
-
- 4.78
10.3
-12.2
M
(kN•m)
+
x (m)
-
-
-
-10.6
-8.8
-10
$\theta_B = \frac{2.4}{EI}$
Deflected shape
x (m)

Example 2

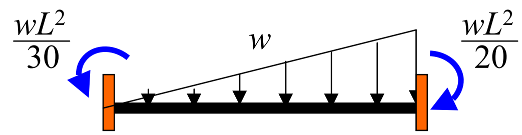
Draw the **quantitative shear , bending moment** diagrams and **qualitative deflected curve** for the beam shown. *EI* is constant.







FEM



$\boxed{[M] = [K][Q] + [\text{FEM}]}$

$$\cancelto{0}{M_{AB}} = \frac{4EI}{8}\theta_A + \frac{2EI}{8}\theta_B + \cancelto{10}{\frac{(10)(8)}{8}} \quad ---(1)$$

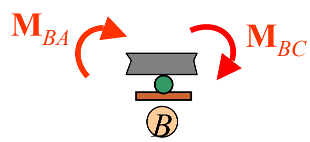
$$M_{BA} = \frac{2EI}{8}\theta_A + \frac{4EI}{8}\theta_B - \cancelto{10}{\frac{(10)(8)}{8}} \quad ---(2)$$

$$M_{BC} = \frac{4EI}{6}\theta_B + \frac{2EI}{6}\cancelto{0}{\theta_C} + \frac{(6)(6^2)}{30} \quad ---(3)$$

$$M_{CB} = \frac{2EI}{6}\theta_B + \frac{4EI}{6}\cancelto{0}{\theta_C} - \frac{(6)(6)^2}{20} \quad ---(4)$$

$$2(2)-(1): \quad 2M_{BA} = \frac{6EI}{8}\theta_B - 30$$

$$M_{BA} = \frac{3EI}{8}\theta_B - 15 \quad ---(5)$$



$$M_{BC} = \frac{4EI}{6}\theta_B + \frac{(6)(6^2)}{30} \quad ---(3)$$

$$M_{CB} = \frac{2EI}{6}\theta_B - \frac{(6)(6)^2}{20} \quad ---(4)$$

$$M_{BA} = \frac{3EI}{8}\theta_B - 15 \quad ---(5)$$

$$+ \quad \Sigma M_B = 0: \quad -M_{BA} - M_{BC} = 0$$

$$(\frac{3EI}{8} + \frac{4EI}{6})\theta_B - 15 + \frac{(6)(6^2)}{30} = 0 \quad ---(6)$$

$$\theta_B = \frac{7.488}{EI}$$

$$Substitute\ \theta_B\ in\ (1): \quad 0 = \frac{4EI}{8}\theta_A + \frac{2EI}{8}\theta_B - 10$$

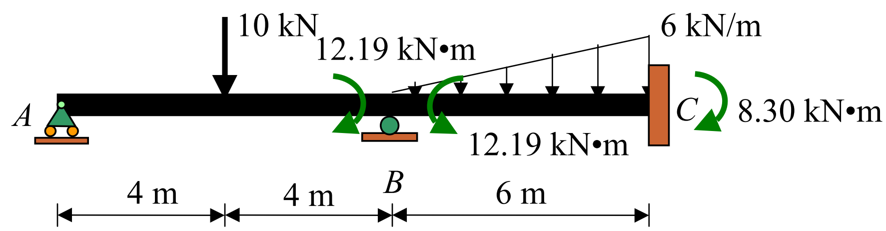
$$\theta_A = \frac{-23.74}{EI}$$

Substitute θ_A and θ_B in (5), (3) and (4):

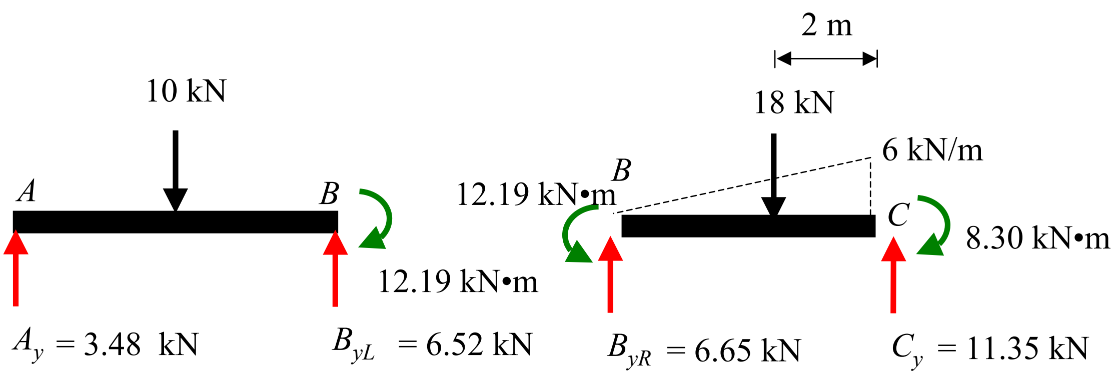
M_{BA} = - 12.19 kN•m

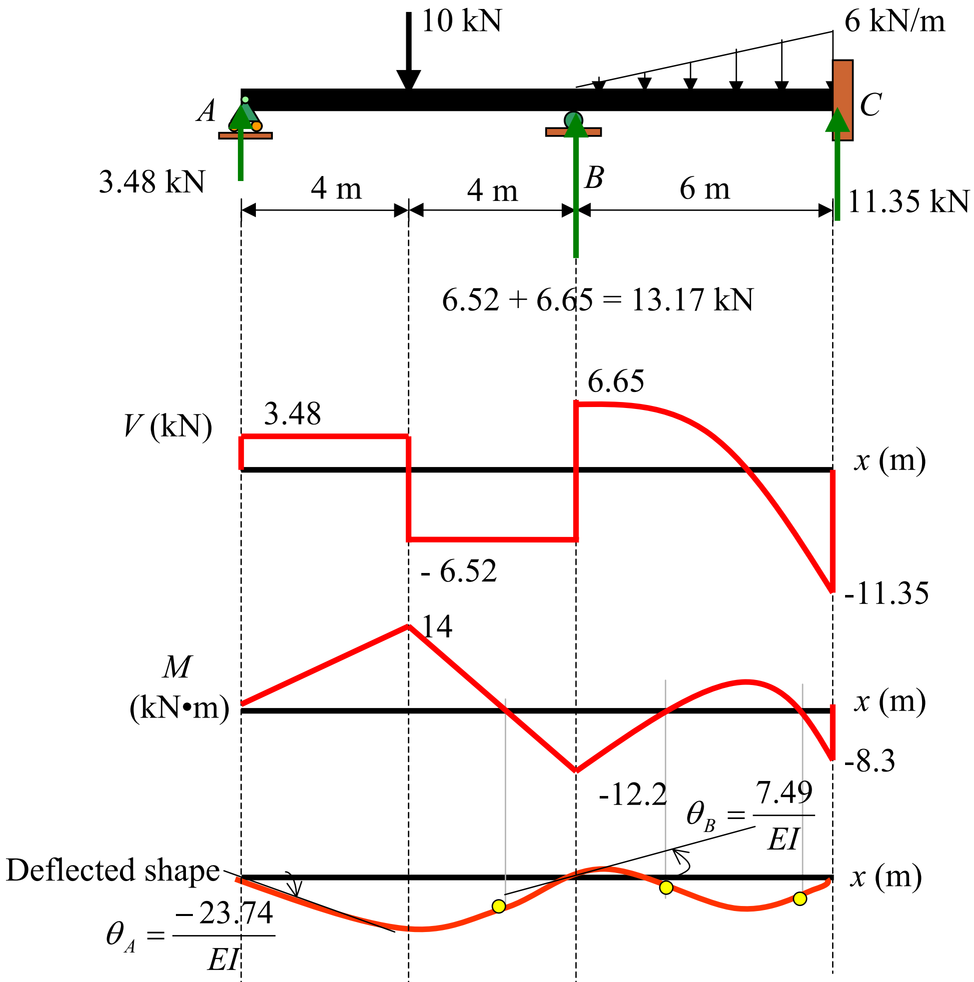
M_{BC} = 12.19 kN•m

M_{CB} = - 8.30 kN•m



M_{BA} = - 12.19 kN•m, M_{BC} = 12.19 kN•m, M_{CB} = - 8.30 kN•m

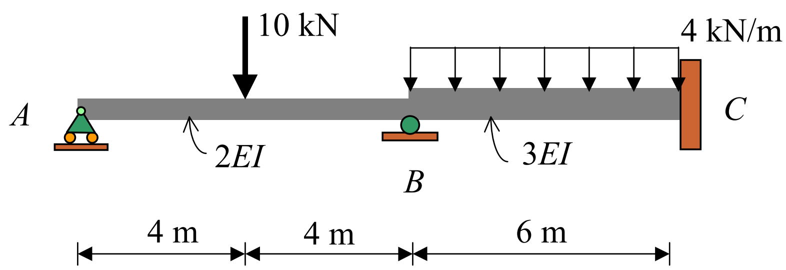


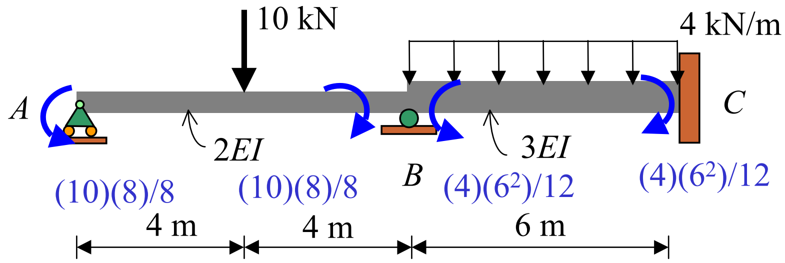


10 kN
6 kN/m
A
C
B
3.48 kN
4 m
4 m
6 m
11.35 kN
6.52 + 6.65 = 13.17 kN
V (kN)
3.48
6.65
x (m)
- 6.52
-11.35
14
M
(kN•m)
x (m)
-8.3
-12.2
$\theta_B = \frac{7.49}{EI}$
Deflected shape
x (m)
$\theta_A = \frac{-23.74}{EI}$

<u>Example 3</u>

Draw the **quantitative shear , bending moment** diagrams and **qualitative deflected curve** for the beam shown. *EI* is constant.



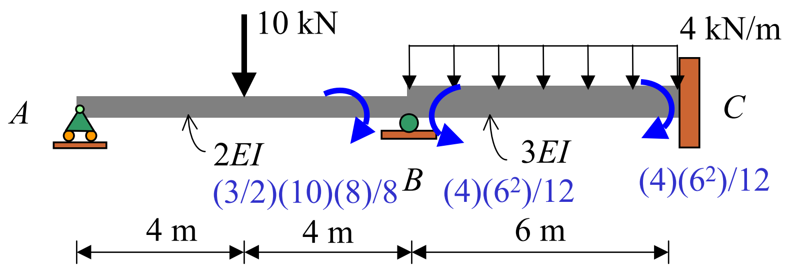


$$\cancelto{0}{M_{AB}} = \frac{4(2EI)}{8}\theta_A + \frac{2(2EI)}{8}\theta_B + \cancelto{10}{\frac{(10)(8)}{8}} \quad ---(1)$$

$$M_{BA} = \frac{2(2EI)}{8}\theta_A + \frac{4(2EI)}{8}\theta_B - \cancelto{10}{\frac{(10)(8)}{8}} \quad ---(2)$$

$$\frac{2(2)-(1)}{2}: \quad M_{BA} = \frac{3(2EI)}{8}\theta_B - \cancelto{15}{\frac{(3/2)(10)(8)}{8}} \quad ---(2a)$$

$$M_{BC} = \frac{4(3EI)}{6}\theta_B + \cancelto{12}{\frac{(4)(6^2)}{12}} \quad ---(3)$$



$$M_{BA} = \frac{3(2EI)}{8}\theta_B - \cancelto{15}{\frac{(3/2)(10)(8)}{8}} \quad ---(2a)$$

$$M_{BC} = \frac{4(3EI)}{6}\theta_B + \cancelto{12}{\frac{(4)(6^2)}{12}} \quad ---(3)$$

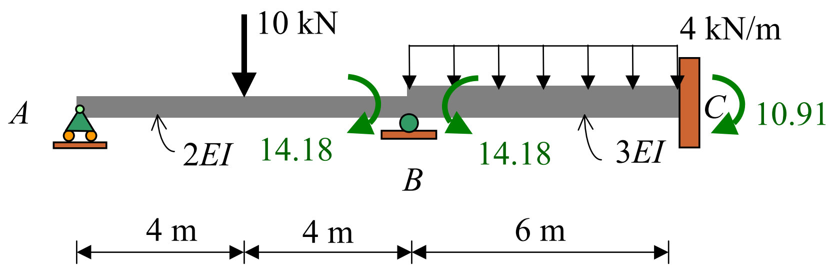
$$-M_{BA} - M_{BC} = 0: \quad 2.75EI\theta_B = -12 + 15 = 3$$

$$\theta_B = 1.091/EI$$

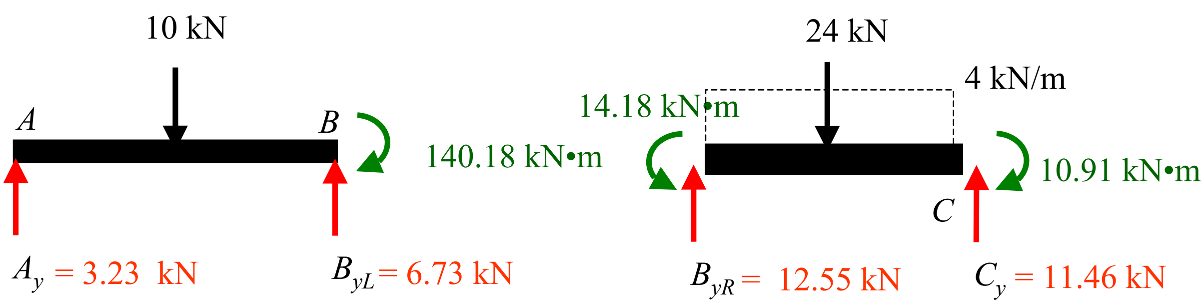
$$M_{BA} = \frac{3(2EI)}{8}\left(\frac{1.091}{EI}\right) - 15 = -14.18\ kN \bullet m$$

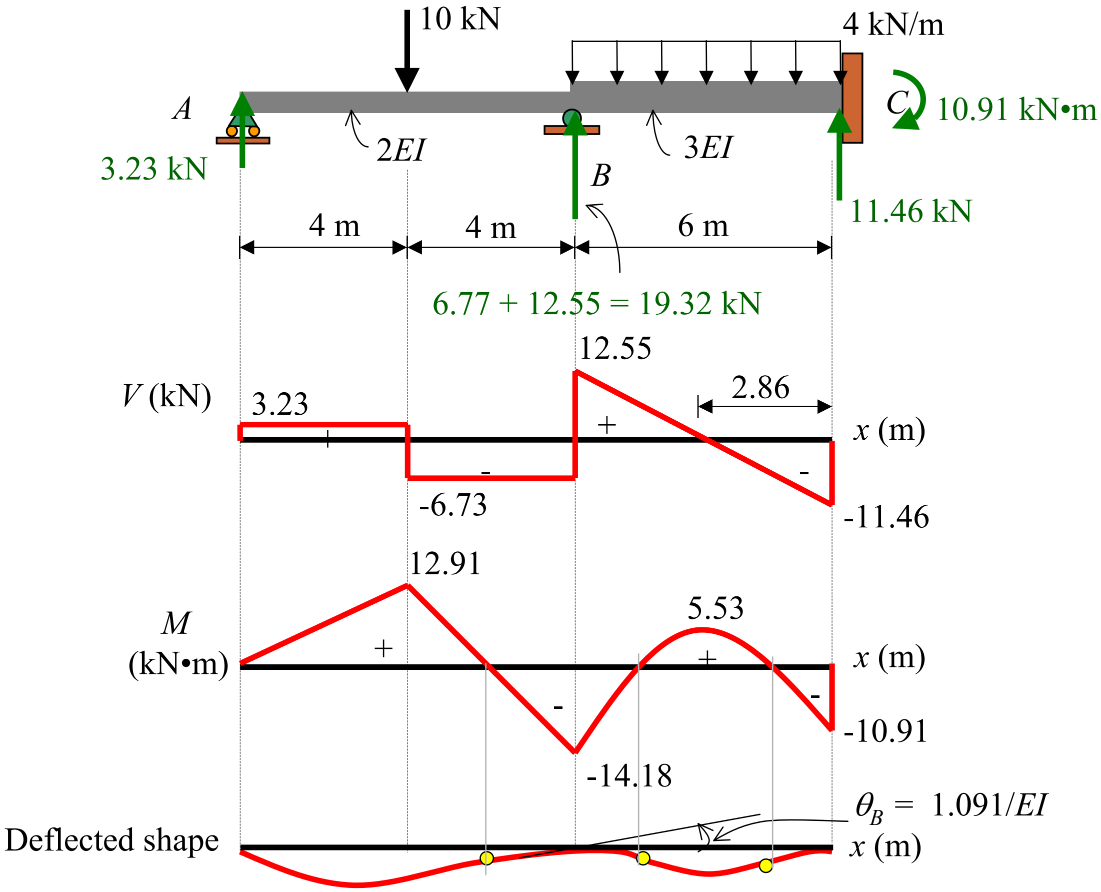
$$M_{BC} = \frac{4(3EI)}{6}\left(\frac{1.091}{EI}\right) - 12 = 14.18\ kN \bullet m$$

$$M_{CB} = \frac{2(3EI)}{6}\theta_B - 12 = -10.91\ kN \bullet m$$



M_{BA} = - 14.18 kN•m, M_{BC} = 14.18 kN•m, M_{CB} = -10.91 kN•m

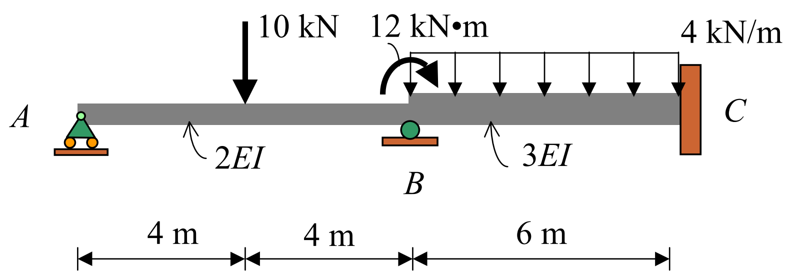


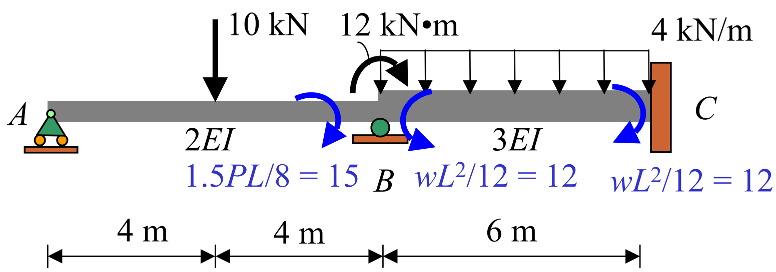


10 kN
4 kN/m
A
C
10.91 kN•m
2EI
3EI
3.23 kN
B
11.46 kN
4 m
4 m
6 m
6.77 + 12.55 = 19.32 kN
12.55
V (kN)
3.23
2.86
x (m)
-6.73
-11.46
12.91
5.53
M
(kN•m)
x (m)
-10.91
-14.18
θ_B = 1.091/EI
Deflected shape
x (m)

Example 4

Draw the **quantitative shear , bending moment** diagrams and **qualitative deflected curve** for the beam shown. *EI* is constant.

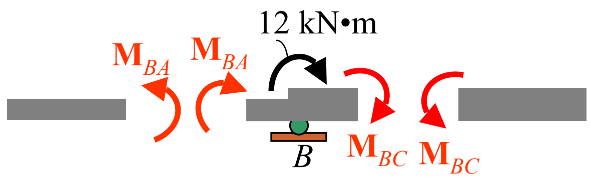




$$M_{BA} = \frac{3(2EI)}{8}\theta_B - 15 \quad ---(1)$$

$$M_{BC} = \frac{4(3EI)}{6}\theta_B + 12 \quad ---(2)$$

$$M_{CB} = \frac{2(3EI)}{6}\theta_B - 12 \quad ---(3)$$



$$Joint\ B: \quad -M_{BA} - M_{BC} - 12 = 0$$

$$-(0.75EI - 15) - (2EI\theta_B + 12) - 12 = 0$$

$$\theta_B = -\frac{3.273}{EI}$$

$$\underset{0}{M_{AB}} = \frac{4(2EI)}{8}\theta_A + \frac{2(3EI)}{8}\underset{-3.273/EI}{\theta_B} + \frac{(10)(8)}{8}$$

$$\theta_A = -\frac{7.21}{EI}$$

$$M_{BA} = 0.75EI(-\frac{3.273}{EI}) - 15 = -17.45\ kN \bullet m$$

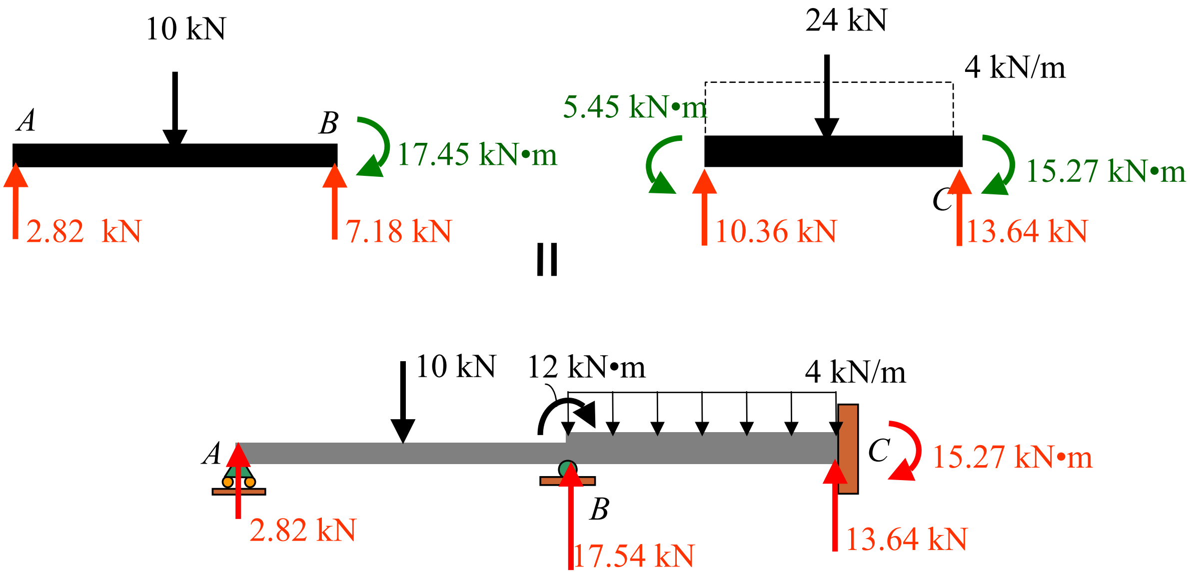
$$M_{BC} = 2EI(-\frac{3.273}{EI}) + 12 = 5.45\ kN \bullet m$$

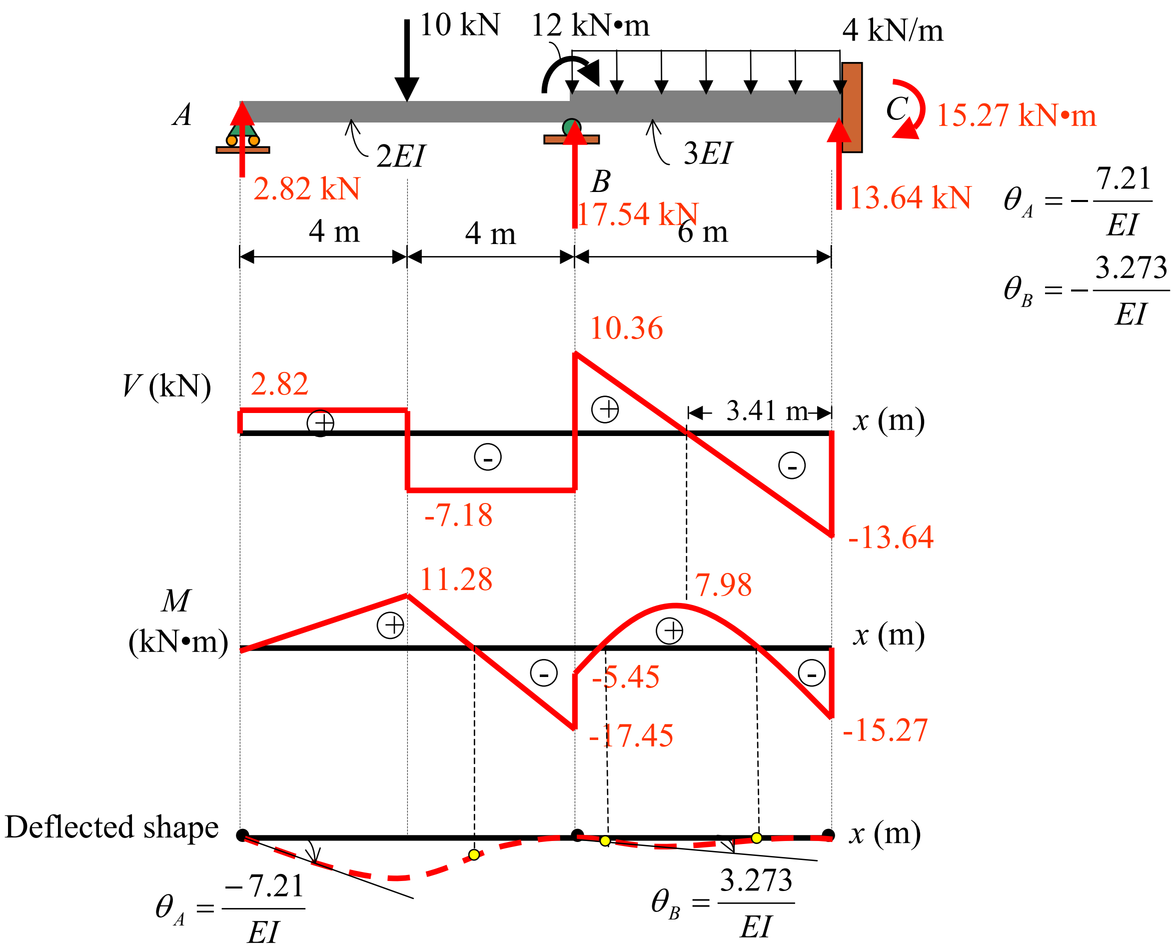
$$M_{CB} = EI(-\frac{3.273}{EI}) - 12 = -15.27\ kN \bullet m$$

$$M_{BA} = 0.75EI(-\frac{3.273}{EI}) - 15 = -17.45\ kN \bullet m$$

$$M_{BC} = 2EI(-\frac{3.273}{EI}) + 12 = 5.45\ kN \bullet m$$

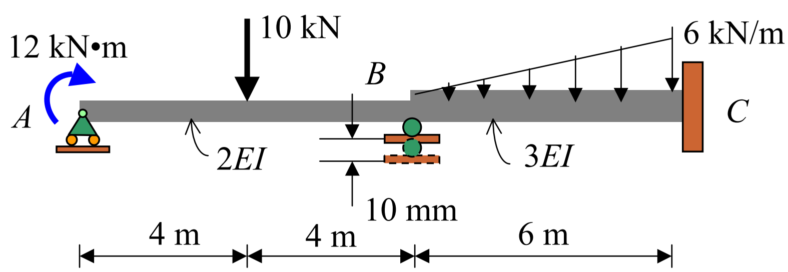
$$M_{CB} = EI(-\frac{3.273}{EI}) - 12 = -15.27\ kN \bullet m$$

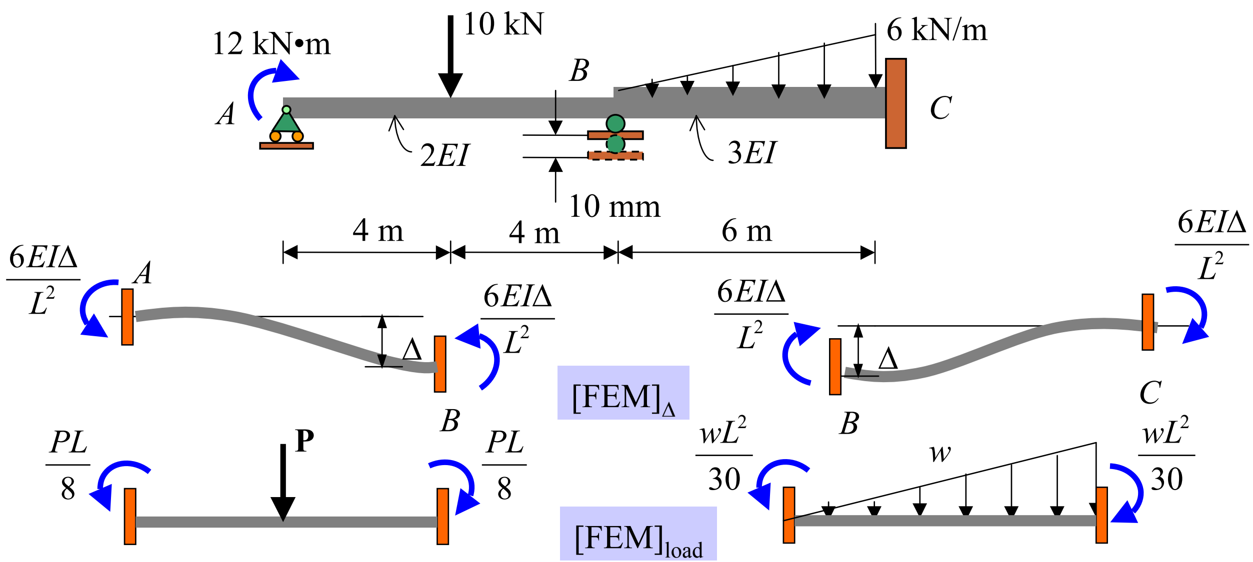



10 kN
12 kN•m
4 kN/m
A
2EI
3EI
C
15.27 kN•m
2.82 kN
B
17.54 kN
13.64 kN
4 m
4 m
6 m
$\theta_A = -\frac{7.21}{EI}$
$\theta_B = -\frac{3.273}{EI}$
V (kN)
2.82
10.36
3.41 m
x (m)
-7.18
-13.64
M
(kN•m)
11.28
7.98
-5.45
-17.45
-15.27
Deflected shape
$\theta_A = \frac{-7.21}{EI}$
$\theta_B = \frac{3.273}{EI}$

Example 5

Draw the **quantitative shear, bending moment** diagrams, and **qualitative deflected curve** for the beam shown. Support B settles 10 mm, and *EI* is constant. Take $E = 200$ GPa, $I = 200\text{x}10^6$ mm^4.



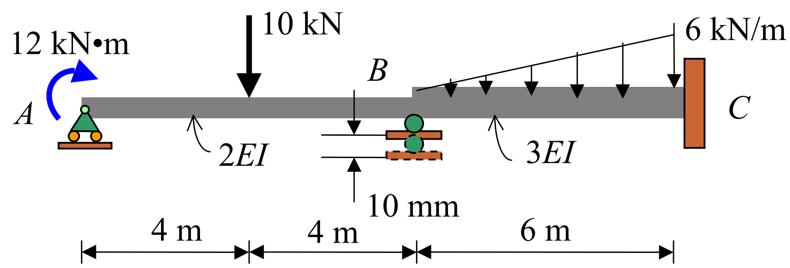


$$\cancelto{-12}{M_{AB}} = \frac{4(2EI)}{8}\theta_A + \frac{2(2EI)}{8}\theta_B + \frac{6(2EI)(0.01)}{8^2} + \frac{(10)(8)}{8} \quad ---(1)$$

$$M_{BA} = \frac{2(2EI)}{8}\theta_A + \frac{4(2EI)}{8}\theta_B + \frac{6(2EI)(0.01)}{8^2} - \frac{(10)(8)}{8} \quad ---(2)$$

$$M_{BC} = \frac{4(3EI)}{6}\theta_B + \frac{2(3EI)}{6}\cancelto{0}{\theta_C} - \frac{6(3EI)(0.01)}{6^2} + \frac{(6)(6^2)}{30} \quad ---(3)$$

$$M_{CB} = \frac{2(3EI)}{6}\theta_B + \frac{4(3EI)}{6}\cancelto{0}{\theta_C} - \frac{6(3EI)(0.01)}{6^2} - \frac{(6)(6)^2}{30} \quad ---(4)$$



$$M_{AB} = \frac{4(2EI)}{8}\theta_A + \frac{2(2EI)}{8}\theta_B + \frac{6(2EI)(0.01)}{8^2} + \frac{(10)(8)}{8} \quad ---(1)$$

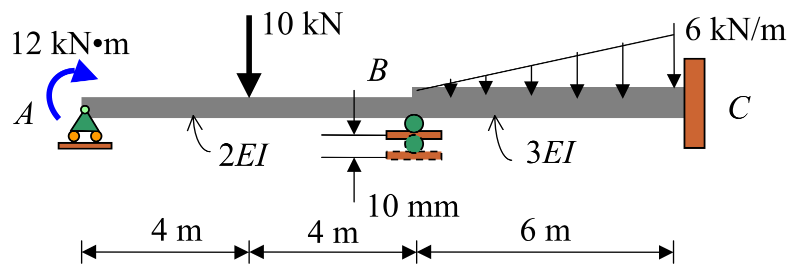
$$M_{BA} = \frac{2(2EI)}{8}\theta_A + \frac{4(2EI)}{8}\theta_B + \frac{6(2EI)(0.01)}{8^2} - \frac{(10)(8)}{8} \quad ---(2)$$

Substitute $EI = (200\text{x}10^6 \text{ kPa})(200\text{x}10^{-6}\text{ m}^4) = 200\text{x}200 \text{ kN}\cdot\text{m}^2$:

$$M_{AB} = \frac{4(2EI)}{8}\theta_A + \frac{2(2EI)}{8}\theta_B + 75 + 10 \quad ---(1)$$

$$M_{BA} = \frac{2(2EI)}{8}\theta_A + \frac{4(2EI)}{8}\theta_B + 75 - 10 \quad ---(2)$$

$$\frac{2(2)-(1)}{2}: \quad M_{BA} = \frac{3(2EI)}{8}\theta_B + \overbrace{75 - (75/2) - 10 - (10/2) - 12/2}^{16.5} ---(2a)$$

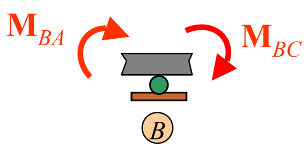


$M_{BA} = (3/4)(2EI)\theta_B + 16.5$

$M_{BC} = (4/6)(3EI)\theta_B - 192.8$

$\boxed{+\circlearrowright\ \Sigma \mathbf{M}_B = 0:\ - M_{BA} - M_{BC} = 0}$

$(3/4 + 2)EI\theta_B + 16.5 - 192.8 = 0$



$\theta_B = 64.109/EI$

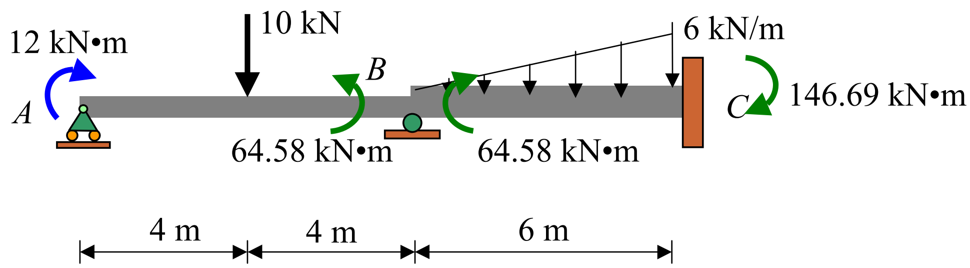
Substitute θ_B in (1): $\theta_A = -129.06/EI$

Substitute θ_A and θ_B in (5), (3), (4):

$M_{BA} = 64.58$ kN•m,

$M_{BC} = -64.58$ kN•m

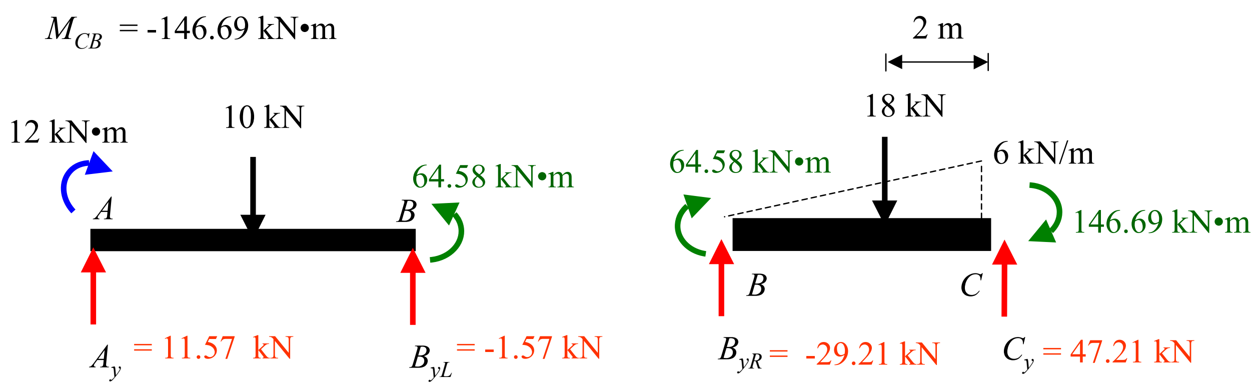
$M_{CB} = -146.69$ kN•m

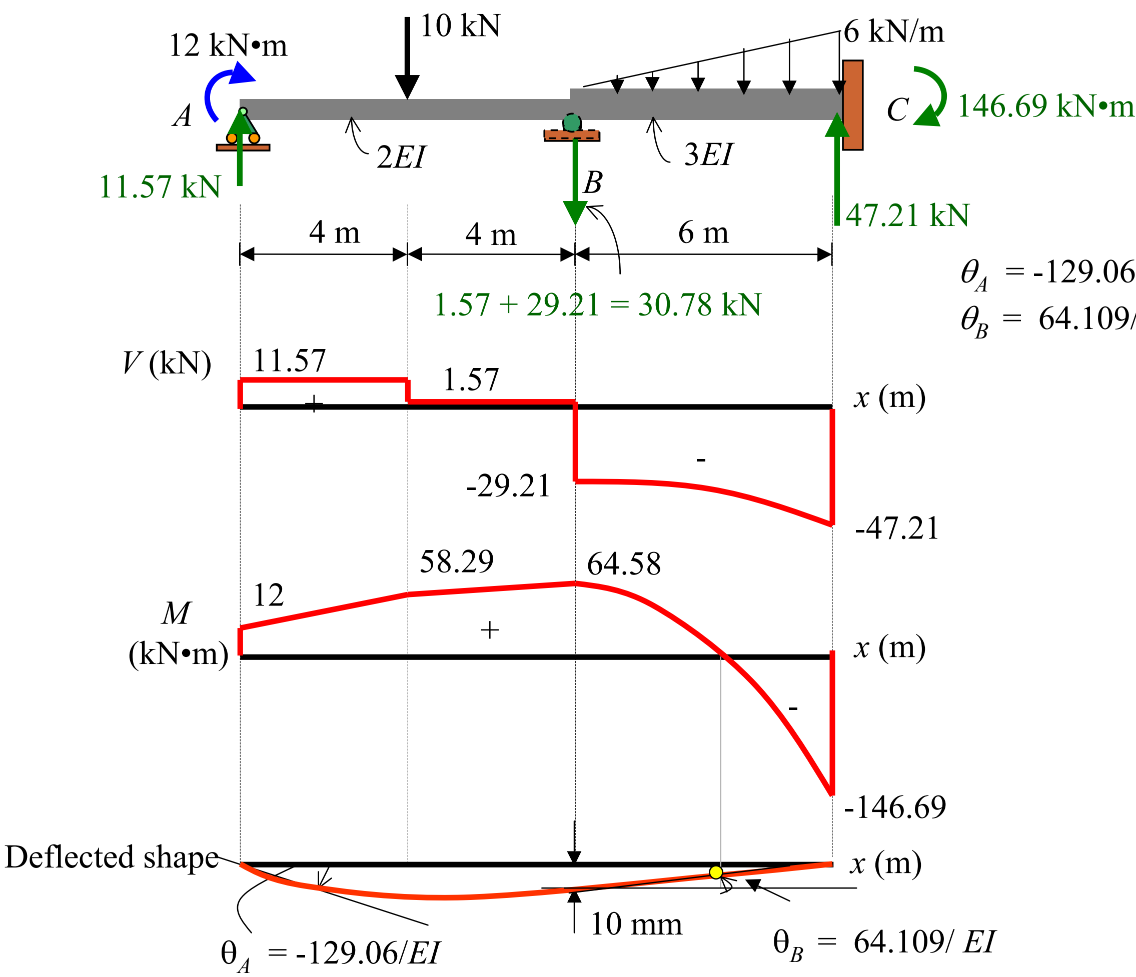


M_{BA} = 64.58 kN•m,

M_{BC} = -64.58 kN•m

M_{CB} = -146.69 kN•m

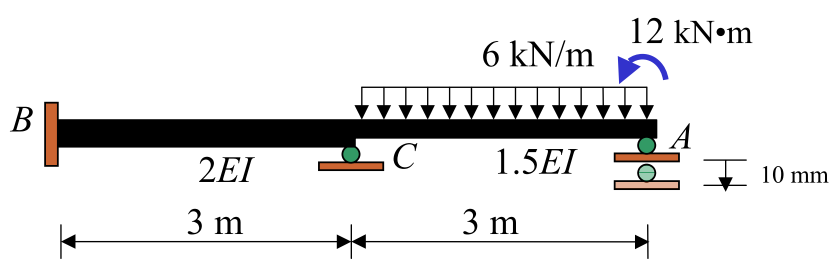


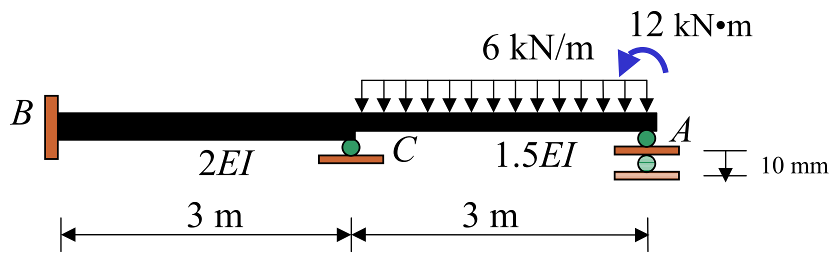


12 kN•m
10 kN
6 kN/m
A
C
146.69 kN•m
2EI
3EI
B
11.57 kN
47.21 kN
4 m
4 m
6 m
1.57 + 29.21 = 30.78 kN
θA = -129.06/EI
θB = 64.109/ EI
V (kN)
11.57
1.57
+
x (m)
-29.21
-
-47.21
58.29
64.58
12
M
(kN•m)
+
x (m)
-
-146.69
Deflected shape
x (m)
10 mm
θA = -129.06/EI
θB = 64.109/ EI

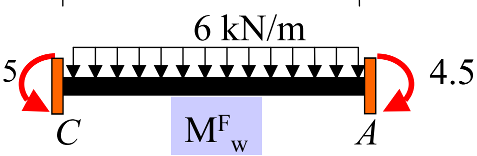
<u>Example 6</u>

For the beam shown, support A settles 10 mm downward, use the slope-deflection method to
(a)Determine all the **slopes** at supports
(b)Determine all the **reactions** at supports
(c)Draw its **quantitative shear, bending moment diagrams**, and **qualitative deflected shape**. (3 points)
Take E= 200 GPa, I = 50(10^6) mm^4.

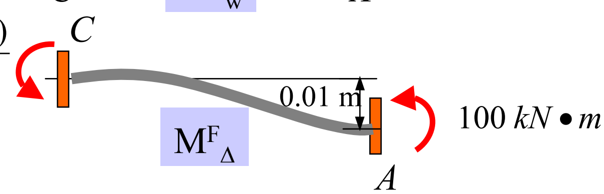




$$\frac{6(3^2)}{12} = 4.5$$



$$\frac{6(1.5\times 200\times 50)(0.01)}{3^2}$$
$$= 100\ kN \bullet m$$

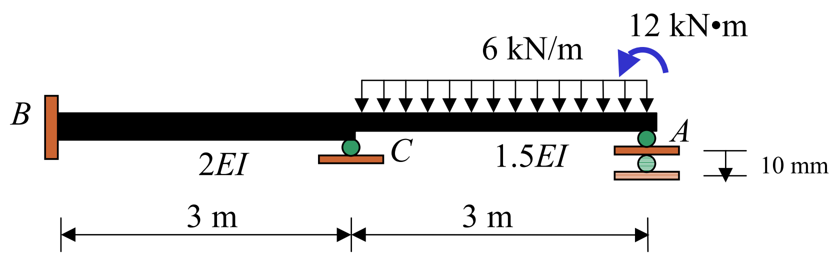


$$M_{CB} = \frac{4(2EI)}{3}\theta_C \quad ---(1)$$

$$M_{CA} = \frac{4(1.5EI)}{3}\theta_C + \frac{2(1.5EI)}{3}\theta_A + 4.5 + 100 \quad ---(2)$$

$$\overset{12}{\cancel{M}_{AC}} = \frac{2(1.5EI)}{3}\theta_C + \frac{4(1.5EI)}{3}\theta_A - 4.5 + 100 \quad ---(3)$$

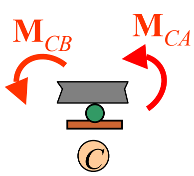
$$\frac{2(2)-(2)}{2}: \quad M_{CA} = \frac{3(1.5EI)}{3}\theta_C + \frac{3(4.5)}{2} + \frac{100}{2} + \frac{12}{2} \quad ---(2a)$$



$$M_{CB} = \frac{4(2EI)}{3}\theta_C \quad ---(1)$$

$$M_{CA} = \frac{3(1.5EI)}{3}\theta_C + \frac{3(4.5)}{2} + \frac{100}{2} + \frac{12}{2} \quad ---(2a)$$

• **Equilibrium equation:**



$$M_{CB} + M_{CA} = 0$$

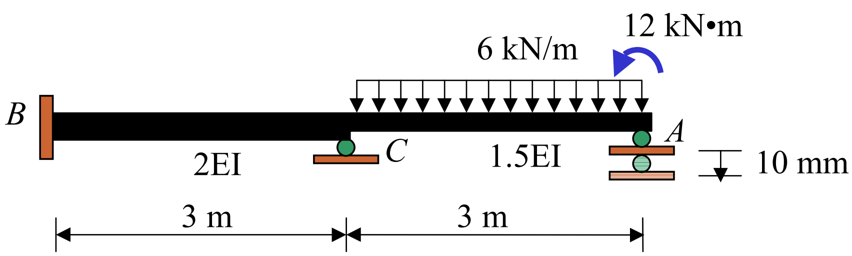
$$\frac{(8+4.5)EI}{3}\theta_C + \frac{3(4.5)}{2} + \frac{100}{2} + \frac{12}{2} = 0$$

$$\theta_C = \frac{-15.06}{EI} = -0.0015\,rad$$

Substitute θ_C in eq.(3)

$$12 = \frac{2(1.5EI)}{3}(\frac{-15.06}{EI}) + \frac{4(1.5EI)}{3}\theta_A - 4.5 + 100 \quad ---(3)$$

$$\theta_A = \frac{-34.22}{EI} = -0.0034\,rad$$

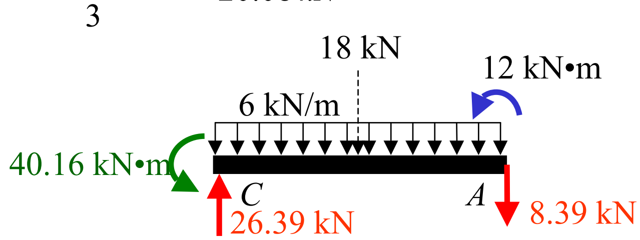


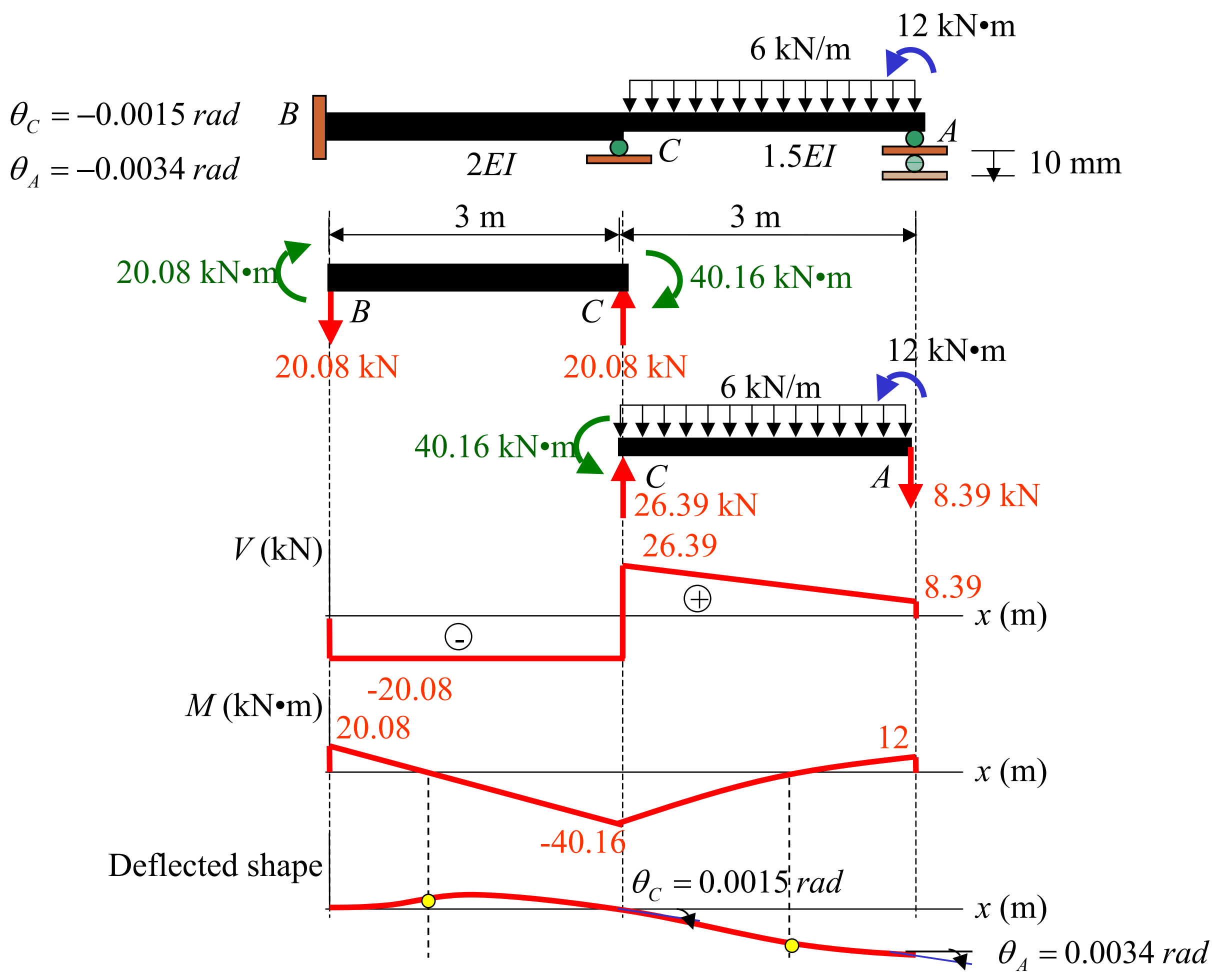
$$\theta_C = \frac{-15.06}{EI} = -0.0015\ rad \qquad \theta_A = \frac{-34.22}{EI} = -0.0034\ rad$$

$$M_{BC} = \frac{2(2EI)}{3}\theta_C = \frac{2(2EI)}{3}\left(\frac{-15.06}{EI}\right) = -20.08\ kN \bullet m$$

$$M_{CB} = \frac{4(2EI)}{3}\theta_C = \frac{4(2EI)}{3}\left(\frac{-15.06}{EI}\right) = -40.16\ kN \bullet m$$

$20.08\ kN$ $\qquad \dfrac{40.16 + 20.08}{3} = 20.08\ kN$





$\theta_C = -0.0015\ rad$
$\theta_A = -0.0034\ rad$
B
12 kN•m
6 kN/m
C
A
2EI
1.5EI
10 mm
3 m
3 m
20.08 kN•m
40.16 kN•m
B
C
20.08 kN
20.08 kN
12 kN•m
6 kN/m
40.16 kN•m
C
A
26.39 kN
8.39 kN
V (kN)
26.39
8.39
x (m)
-20.08
M (kN•m)
20.08
12
x (m)
-40.16
Deflected shape
$\theta_C = 0.0015\ rad$
x (m)
$\theta_A = 0.0034\ rad$

Example 7

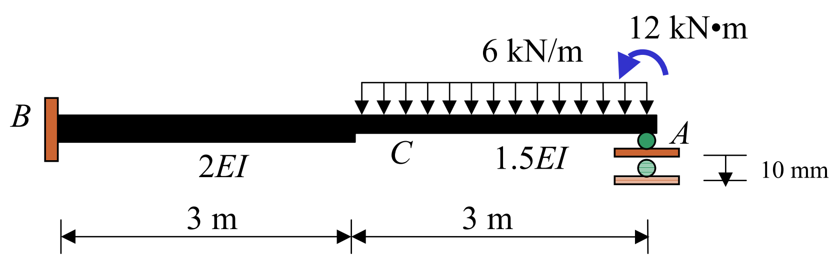
For the beam shown, support A settles 10 mm downward, use the slope-deflection method to

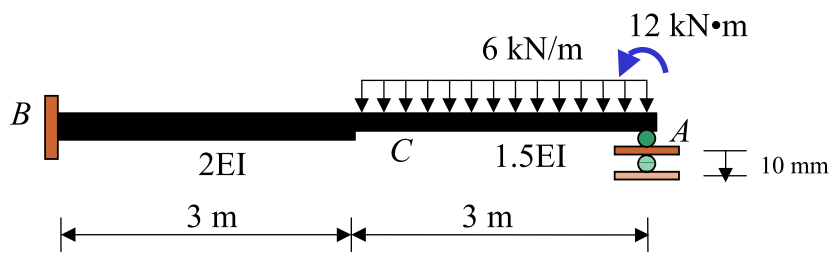
(a)Determine all the **slopes** at supports

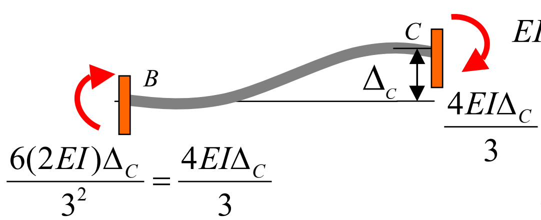
(b)Determine all the **reactions** at supports

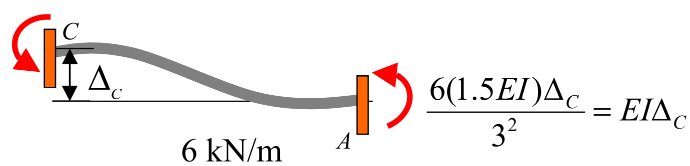
(c)Draw its **quantitative shear, bending moment diagrams**, and **qualitative deflected shape**.

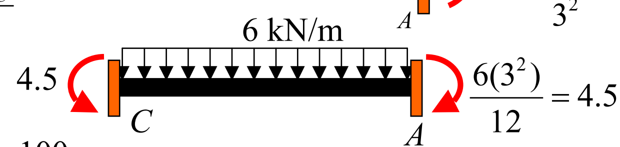
Take E= 200 GPa, I = 50(10^6) mm^4.

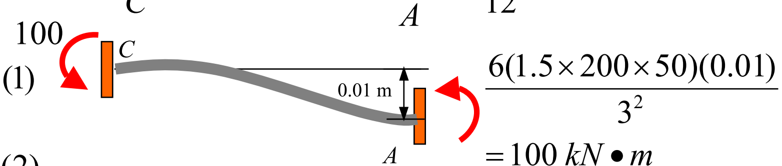












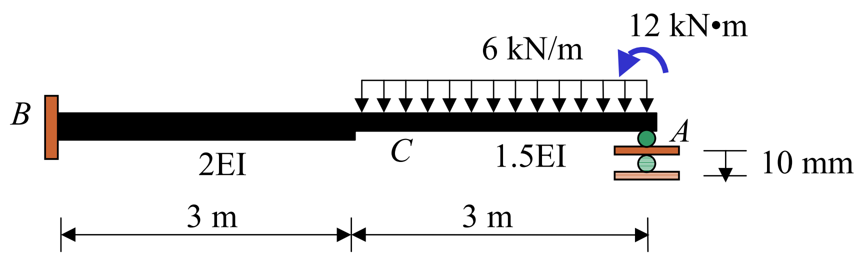
$$M_{BC} = \frac{2(2EI)}{3}\theta_C - \frac{4EI}{3}\Delta_C \quad ---(1)$$

$$M_{CB} = \frac{4(2EI)}{3}\theta_C - \frac{4EI}{3}\Delta_C \quad ---(2)$$

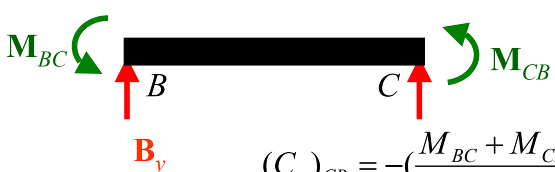
$$M_{CA} = \frac{4(1.5EI)}{3}\theta_C + \frac{2(1.5EI)}{3}\theta_A + EI\Delta_C + 4.5 + 100 --(3)$$

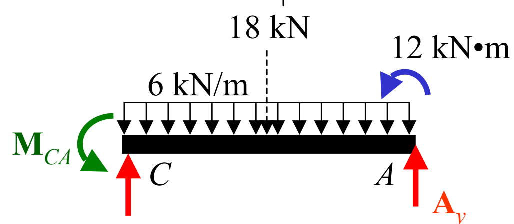
$$\overset{12}{\cancel{M}}_{AC} = \frac{2(1.5EI)}{3}\theta_C + \frac{4(1.5EI)}{3}\theta_A + EI\Delta_C - 4.5 + 100 --(4)$$

$$\frac{2(3)-(4)}{2} : M_{CA} = \frac{3(1.5EI)}{3}\theta_C + \frac{EI}{2}\Delta_C + \frac{3(4.5)}{2} + \frac{100}{2} + \frac{12}{2} \quad ---(3a)$$



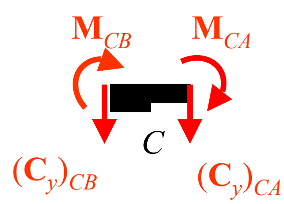
• **Equilibrium equation:**





$$(C_y)_{CB} = -(\frac{M_{BC}+M_{CB}}{3})$$

$$(C_y)_{CA} = \frac{M_{CA}+12+18(1.5)}{3} = \frac{M_{CA}+39}{3}$$



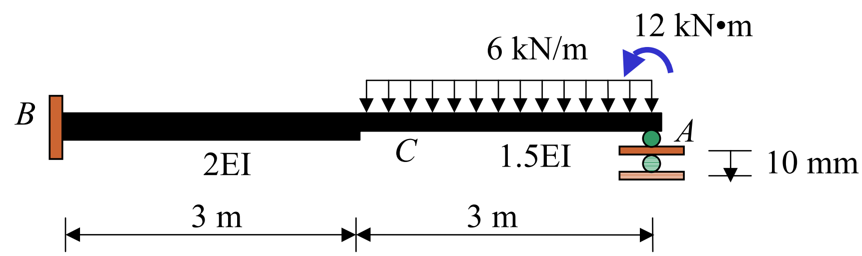
$$\Sigma M_C = 0: \quad M_{CB} + M_{CA} = 0 \quad ---(1^*)$$

$$\Sigma C_y = 0: \quad (C_y)_{CB} + (C_y)_{CA} = 0 \quad ---(2^*)$$

$$\textit{Substitute in } (1^*) \quad 4.167EI\theta_C - 0.8333EI\Delta_C = -62.15 \quad ---(5)$$

$$\textit{Substitute in } (2^*) \quad -2.5EI\theta_C + 3.167EI\Delta_C = -101.75 \quad ---(6)$$

$$\textit{From } (5) \textit{ and } (6) \quad \theta_C = -25.51/EI = -0.00255\ rad \qquad \Delta_C = -52.27/EI = -5.227\ mm$$



• **Solve equation**

$$\theta_C = \frac{-25.51}{EI} = -0.00255\ rad$$

$$\Delta_C = \frac{-52.27}{EI} = -5.227\ mm$$

Substitute θ_C and Δ_C in (4)

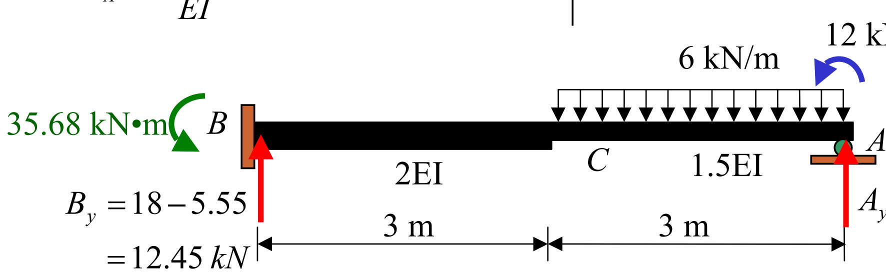
$$\theta_A = \frac{-2.86}{EI} = -0.000286\ rad$$

Substitute θ_C and Δ_C in (1), (2) and (3a)

$$M_{BC} = 35.68\ kN \bullet m$$

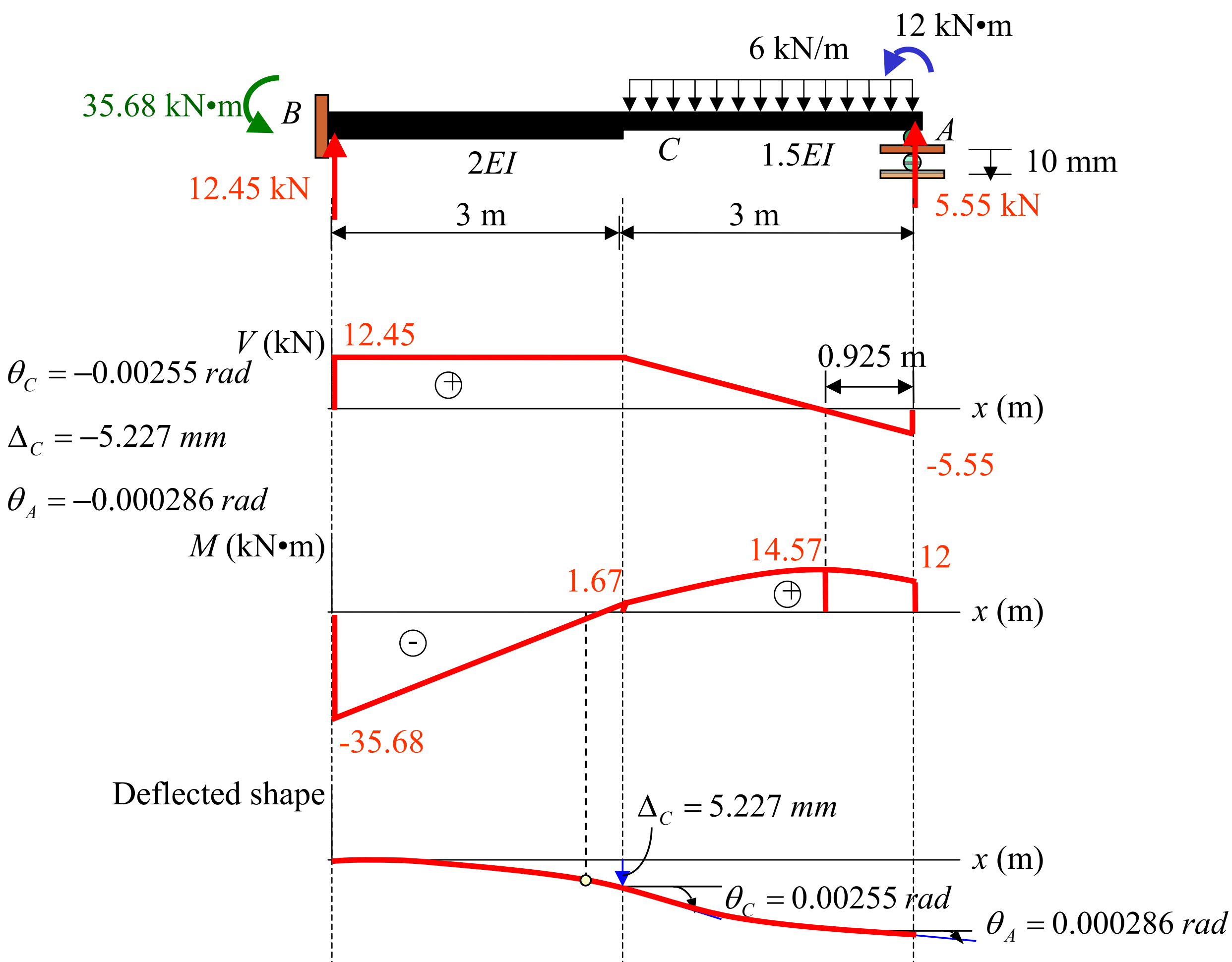
$$M_{CB} = 1.67\ kN \bullet m$$

$$M_{CA} = -1.67\ kN \bullet m$$

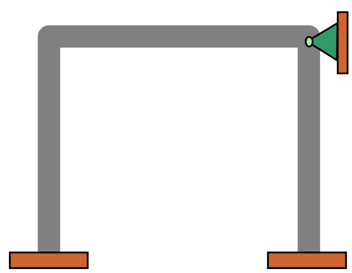


$$B_y = 18 - 5.55 = 12.45\ kN$$

$$A_y = \frac{18(4.5) - 12 - 35.68}{6} = 5.55\ kN$$


12 kN•m
6 kN/m
35.68 kN•m
B
A
C
2EI
1.5EI
10 mm
12.45 kN
5.55 kN
3 m
3 m
V (kN)
12.45
0.925 m
x (m)
-5.55
$\theta_C = -0.00255\ rad$
$\Delta_C = -5.227\ mm$
$\theta_A = -0.000286\ rad$
M (kN•m)
14.57
12
1.67
x (m)
-35.68
Deflected shape
$\Delta_C = 5.227\ mm$
x (m)
$\theta_C = 0.00255\ rad$
$\theta_A = 0.000286\ rad$

Example of Frame: No Sidesway

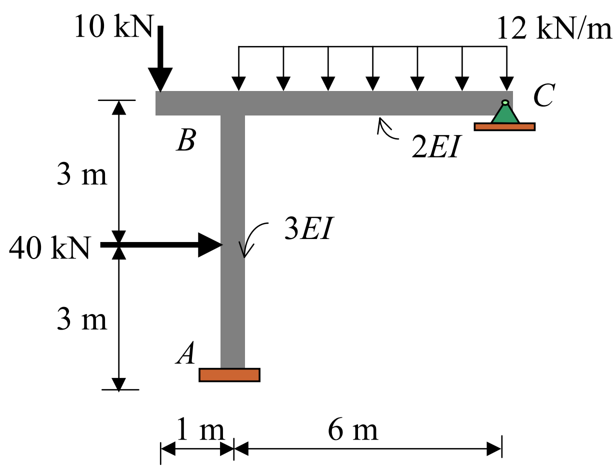


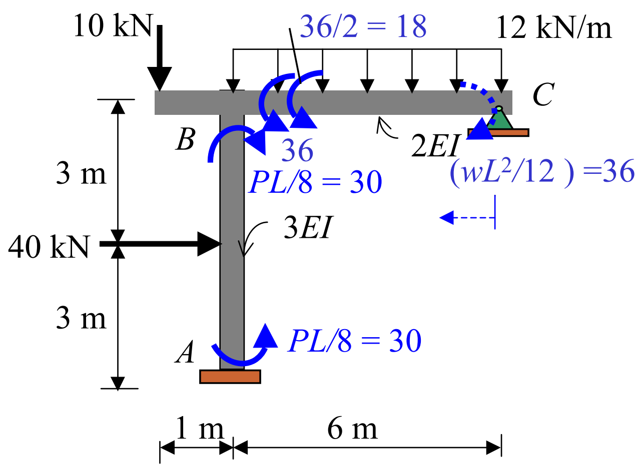
Example 6

For the frame shown, use the slope-deflection method to

(a) Determine the **end moments** of each member and **reactions** at supports

(b) Draw the **quantitative bending moment diagram**, and also draw the **qualitative deflected shape** of the entire frame.





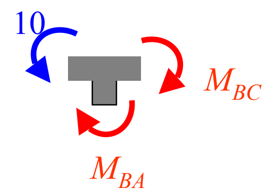
• **Slope-Deflection Equations**

$$M_{AB} = \frac{2(3EI)}{6}\theta_B + 30 \quad ---(1)$$

$$M_{BA} = \frac{4(3EI)}{6}\theta_B - 30 \quad ---(2)$$

$$M_{BC} = \frac{3(2EI)}{6}\theta_B + 36 + 18 \quad ---(3)$$

• **Equilibrium equations**



$$10 - M_{BA} - M_{BC} = 0 \quad ---(1^*)$$

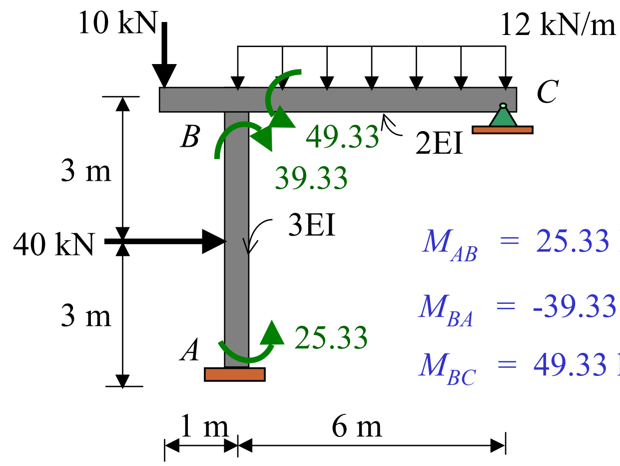
Substitute (2) and (3) in (1*)

$$10 - 3EI\theta_B + 30 - 54 = 0$$

$$\theta_B = \frac{-14}{(3EI)} = \frac{-4.667}{EI}$$

$$Substitute\,\theta_B = \frac{-4.667}{EI}\,in\ (1)\ to\ (3)$$

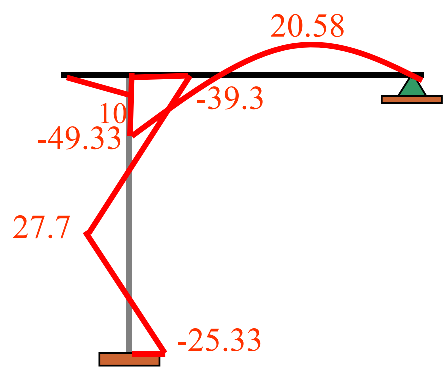
$$M_{AB} = 25.33\ kN \bullet m$$
$$M_{BA} = -39.33\ kN \bullet m$$
$$M_{BC} = 49.33\ kN \bullet m$$



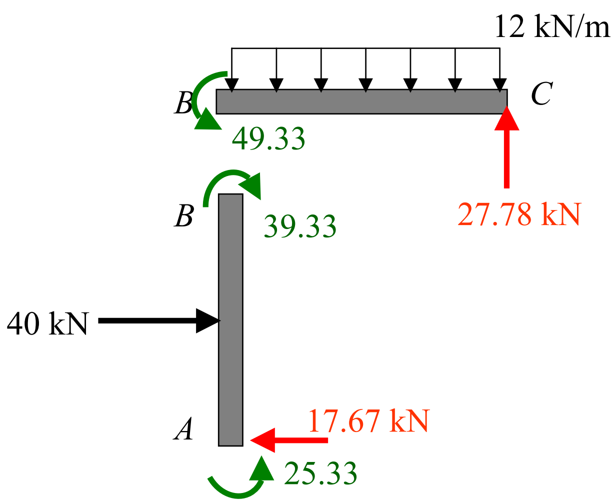
$$M_{AB} = 25.33 \text{ kN}\cdot\text{m}$$

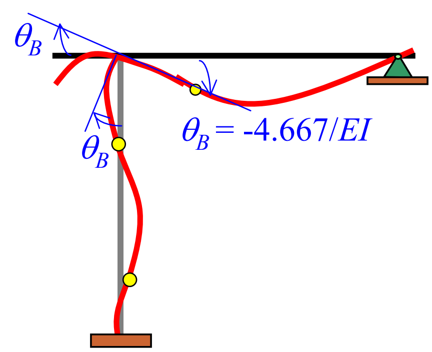
$$M_{BA} = -39.33 \text{ kN}\cdot\text{m}$$

$$M_{BC} = 49.33 \text{ kN}\cdot\text{m}$$



Bending moment diagram

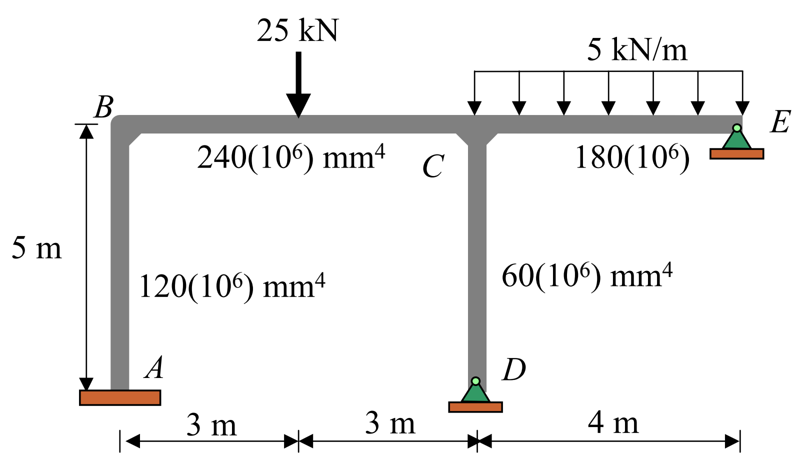


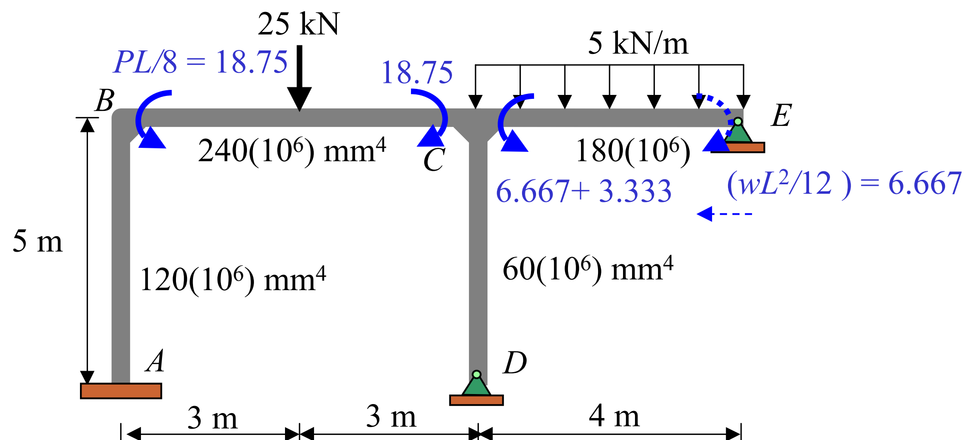


Deflected curve

Example 7

Draw the **quantitative shear, bending moment** diagrams and **qualitative deflected curve** for the frame shown. $E = 200$ GPa.





$$M_{AB} = \frac{4(2EI)}{5}\theta_A + \frac{2(2EI)}{5}\theta_B$$

$$M_{BA} = \frac{2(2EI)}{5}\theta_A + \frac{4(2EI)}{5}\theta_B$$

$$M_{BC} = \frac{4(4EI)}{6}\theta_B + \frac{2(4EI)}{6}\theta_C + 18.75$$

$$M_{CB} = \frac{2(4EI)}{6}\theta_B + \frac{4(4EI)}{6}\theta_C - 18.75$$

$$M_{CD} = \frac{3(EI)}{5}\theta_C$$

$$M_{CE} = \frac{3(3EI)}{4}\theta_C + 10$$

$$\boxed{M_{BA} + M_{BC} = 0}$$

$$(\frac{8}{5} + \frac{16}{6})EI\theta_B + (\frac{8}{6})EI\theta_C = -18.75 \quad ---(1)$$

$$\boxed{M_{CB} + M_{CD} + M_{CE} = 0}$$

$$(\frac{8}{6})EI\theta_B + (\frac{16}{6} + \frac{3}{5} + \frac{9}{4})EI\theta_C = 8.75 \quad ---(2)$$

$$\text{From (1) and (2)}: \quad \theta_B = \frac{-5.29}{EI} \qquad \theta_C = \frac{2.86}{EI}$$

Substitute θ_B = -1.11/*EI,* θ_c = -20.59/*EI* below

$$M_{AB} = \frac{4(2EI)}{5}\cancelto{0}{\theta_A} + \frac{2(2EI)}{5}\theta_B \quad \dashrightarrow \quad M_{AB} = -4.23 \text{ kN•m}$$

$$M_{BA} = \frac{2(2EI)}{5}\cancelto{0}{\theta_A} + \frac{4(2EI)}{5}\theta_B \quad \dashrightarrow \quad M_{BA} = -8.46 \text{ kN•m}$$

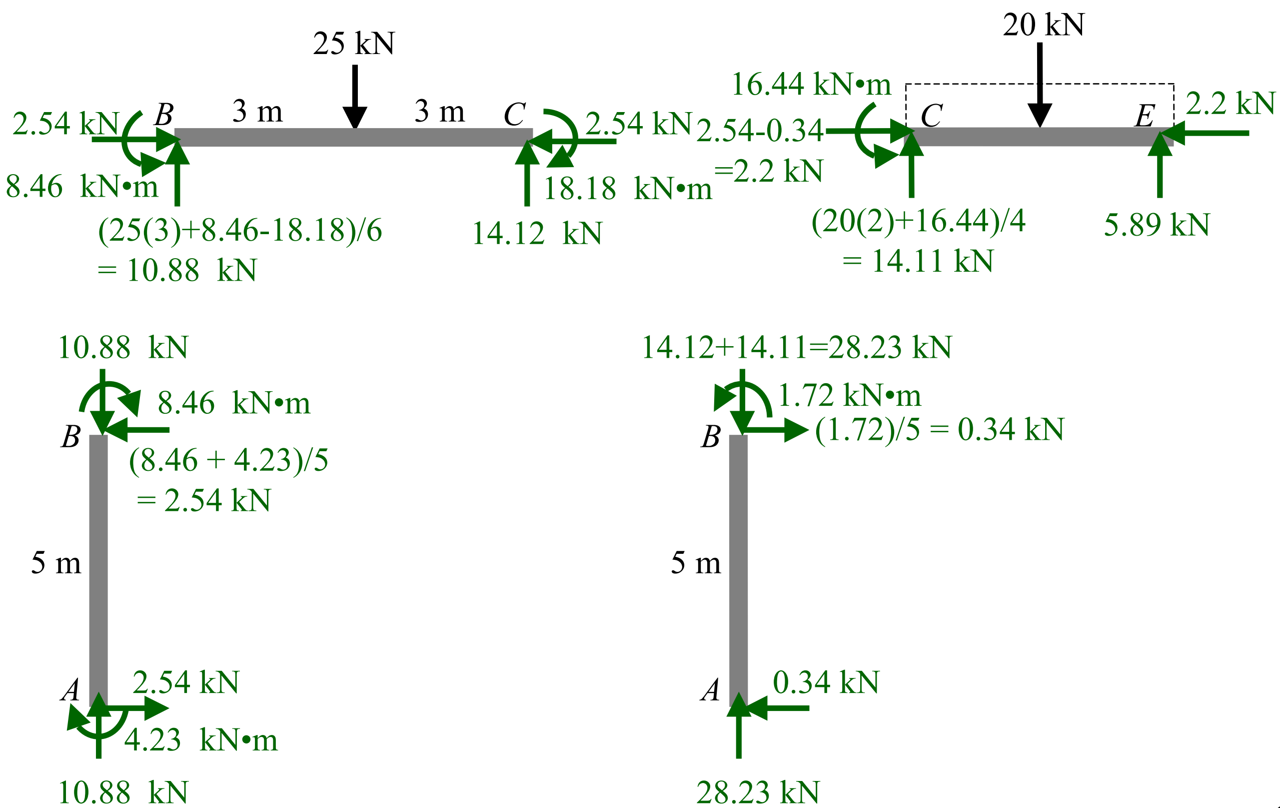
$$M_{BC} = \frac{4(4EI)}{6}\theta_B + \frac{2(4EI)}{6}\theta_C + 18.75 \quad \dashrightarrow \quad M_{BC} = 8.46 \text{ kN•m}$$

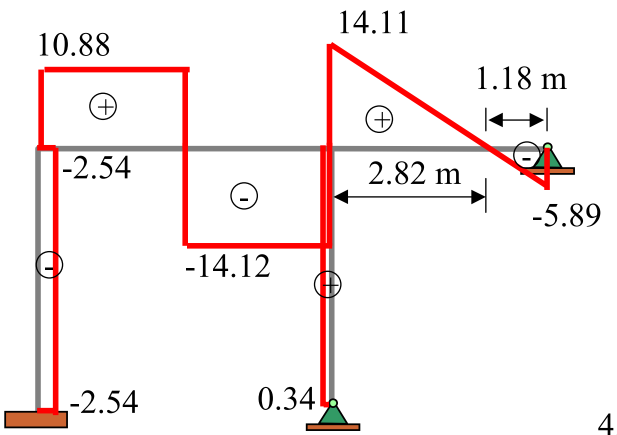
$$M_{CB} = \frac{2(4EI)}{6}\theta_B + \frac{4(4EI)}{6}\theta_C - 18.75 \quad \dashrightarrow \quad M_{CB} = -18.18 \text{ kN•m}$$

$$M_{CD} = \frac{3(EI)}{5}\theta_C \quad \dashrightarrow \quad M_{CD} = 1.72 \text{ kN•m}$$

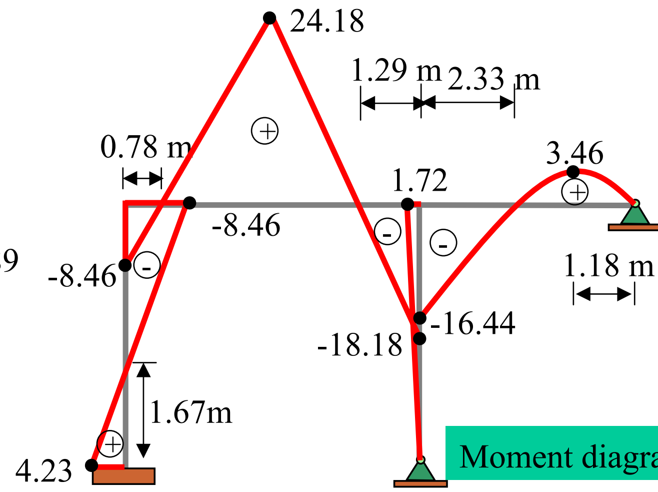
$$M_{CE} = \frac{3(3EI)}{4}\theta_C + 10 \quad \dashrightarrow \quad M_{CE} = 16.44 \text{ kN•m}$$

M_{AB} = -4.23 kN•m, M_{BA} = -8.46 kN•m, M_{BC} = 8.46 kN•m, M_{CB} = -18.18 kN•m,
M_{CD} = 1.72 kN•m, M_{CE} = 16.44 kN•m

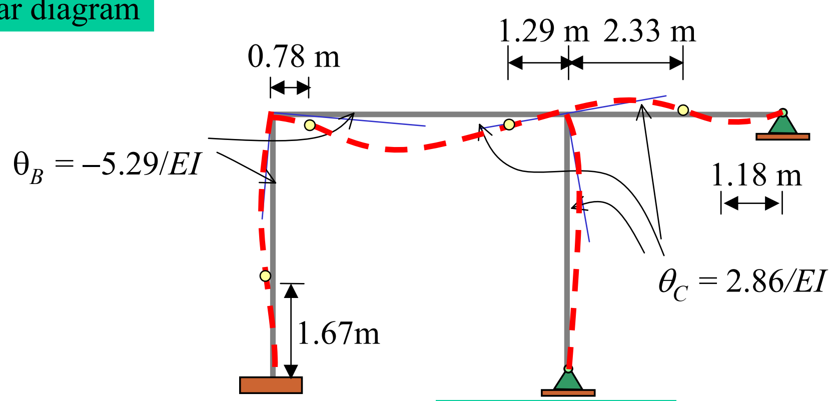




Shear diagram

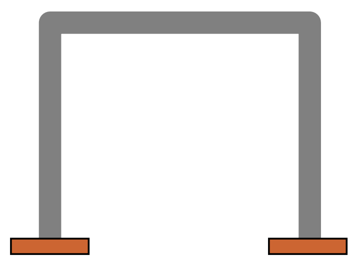


Moment diagram



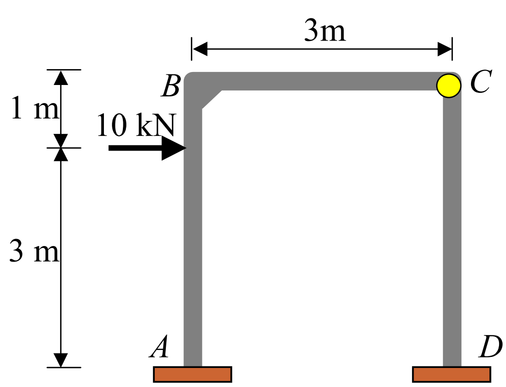
Deflected curve

Example of Frames: Sidesway

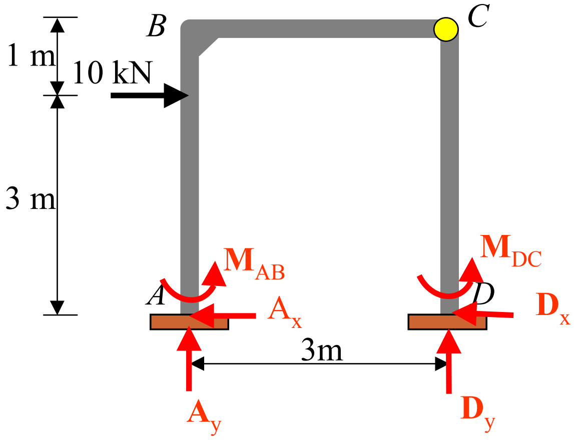


<u>Example 8</u>

Determine the moments at each joint of the frame and draw the **quantitative bending moment** diagrams and **qualitative deflected curve** . The joints at *A* and *D* are fixed and joint *C* is assumed pin-connected. *EI* is constant for each member



• **Overview**



• **Unknowns**

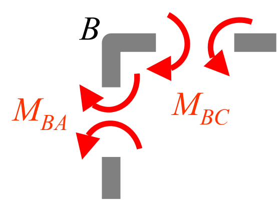
$$\theta_B \text{ and } \Delta$$

• **Boundary Conditions**

$$\theta_A = \theta_D = 0$$

• **Equilibrium Conditions**

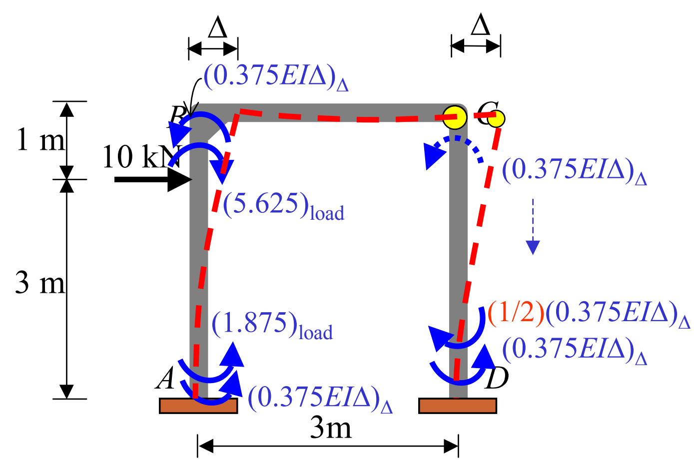
- Joint *B*



$$\Sigma M_B = 0: \quad M_{BA} + M_{BC} = 0 \quad ---(1^*)$$

- Entire Frame

$$\overset{+}{\rightarrow} \quad \Sigma F_x = 0: \quad 10 - A_x - D_x = 0 \quad ---(2^*)$$



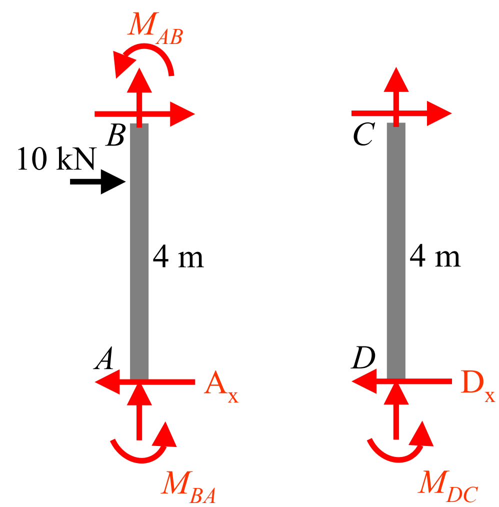
- **Slope-Deflection Equations**

$$M_{AB} = \frac{2(EI)}{4}\theta_B + \cancelto{5.625}{\frac{10(3)(1^2)}{4^2}} + \cancelto{0.375EI\Delta}{\frac{6EI\Delta}{4^2}} \quad ---(1)$$

$$M_{BA} = \frac{4(EI)}{4}\theta_B - \cancelto{5.625}{\frac{10(3^2)(1)}{4^2}} + \cancelto{0.375EI\Delta}{\frac{6EI\Delta}{4^2}} \quad ---(2)$$

$$M_{BC} = \frac{3(EI)}{3}\theta_B \quad ---(3)$$

$$M_{DC} = 0.375EI\Delta - \frac{1}{2}0.375EI\Delta = 0.1875EI\Delta \quad ---(4)$$



$$\circlearrowleft + \quad \Sigma M_B = 0:$$

$$A_x = \frac{(M_{AB} + M_{BA})}{4}$$

$$A_x = 0.375EI\theta_B + 0.1875EI\Delta + 1.563 ---(5)$$

$$\circlearrowleft + \quad \Sigma M_C = 0:$$

$$D_x = \frac{M_{DC}}{4} = 0.0468EI\Delta \quad ---(6)$$

Equilibrium Conditions:

$$M_{BA} + M_{BC} = 0 \quad ---(1^*)$$

$$10 - A_x - D_x = 0 \quad ---(2^*)$$

Slope-Deflection Equations:

$$M_{AB} = \frac{2(EI)}{4}\theta_B + 5.625 + 0.375EI\Delta \quad ---(1)$$

$$M_{BA} = \frac{4(EI)}{4}\theta_B - 5.625 + 0.375EI\Delta \quad ---(2)$$

$$M_{BC} = \frac{3(EI)}{3}\theta_B \quad ---(3)$$

$$M_{DC} = 0.1875EI\Delta \quad ---(4)$$

Horizontal reaction at supports:

$$A_x = 0.375EI\theta_B + 0.1875EI\Delta + 1.563 ---(5)$$

$$D_x = 0.0468EI\Delta \quad ---(6)$$

- **Solve equation**

Substitute (2) and (3) in (1*)

$$2EI\,\theta_B + 0.375EI\,\Delta = 5.625 \quad ----(7)$$

Substitute (5) and (6) in (2*)

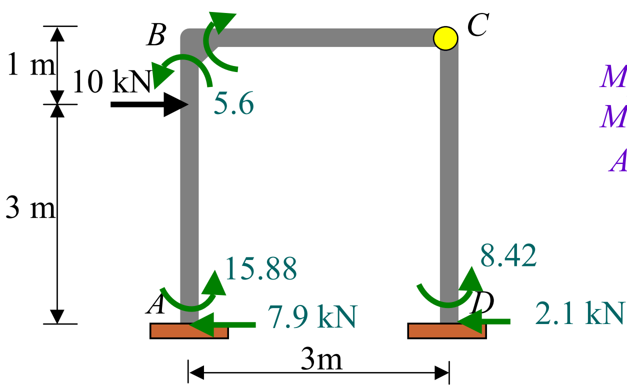
$$-0.375EI\theta_B - 0.235EI\Delta = -8.437 \quad ---(8)$$

From (7) and (8) can solve;

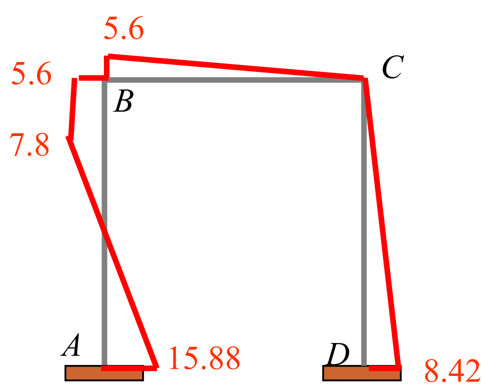
$$\theta_B = \frac{-5.6}{EI} \qquad \Delta = \frac{44.8}{EI}$$

$$Substitute\,\theta_B = \frac{-5.6}{EI}\; and\; \Delta = \frac{44.8}{EI} \quad in\ (1) to\ (6)$$

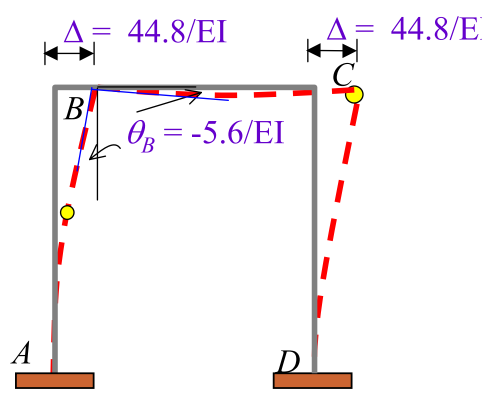
M_{AB} = 15.88 kN•m
M_{BA} = 5.6 kN•m
M_{BC} = -5.6 kN•m
M_{DC} = 8.42 kN•m
A_x = 7.9 kN
D_x = 2.1 kN



M_{AB} = 15.88 kN•m, M_{BA} = 5.6 kN•m,
M_{BC} = -5.6 kN•m, M_{DC} = 8.42 kN•m,
A_x = 7.9 kN, D_x = 2.1 kN,



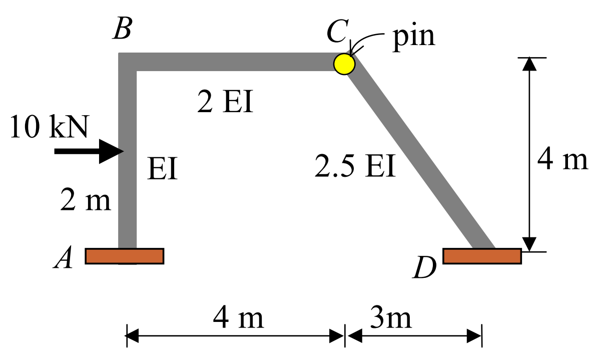
Bending moment diagram



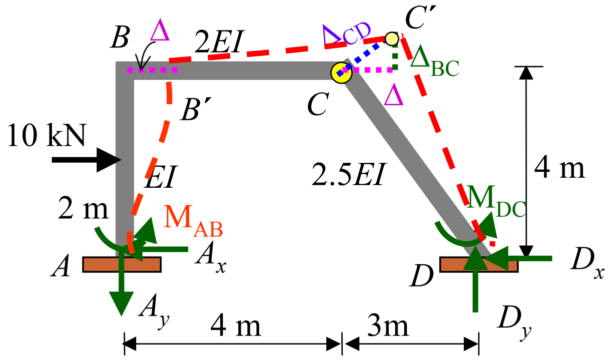
Deflected curve

Example 9

From the frame shown use the slope-deflection method to:
(a) Determine the **end moments** of each member and **reactions** at supports
(b) Draw the **quantitative bending moment diagram**, and also draw the **qualitative deflected** shape of the entire frame.



• **Overview**



• **Unknowns**

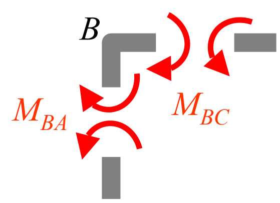
$$\theta_B \text{ and } \Delta$$

• **Boundary Conditions**

$$\theta_A = \theta_D = 0$$

• **Equilibrium Conditions**

- Joint *B*

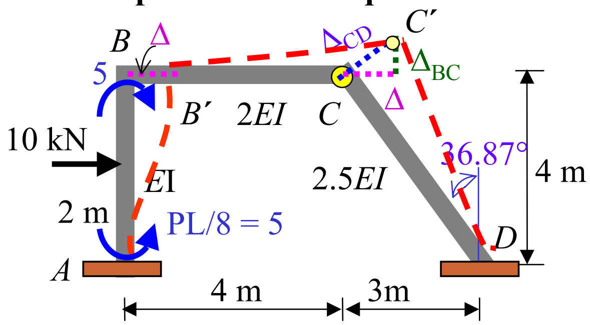


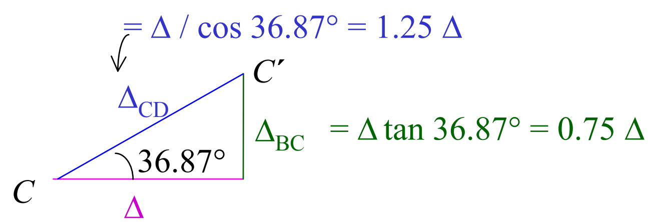
$$\Sigma M_B = 0: \quad M_{BA} + M_{BC} = 0 \quad ---(1^*)$$

- Entire Frame

$$\overset{+}{\rightarrow} \quad \Sigma F_x = 0: \quad 10 - A_x - D_x = 0 \quad ---(2^*)$$

• **Slope-Deflection Equation**



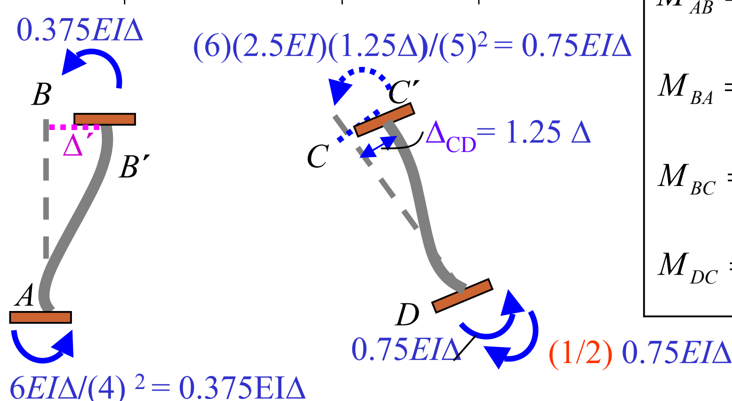


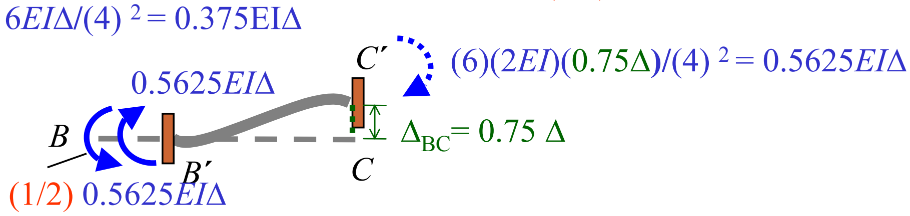
$$M_{AB} = \frac{2(EI)}{4}\theta_B + 0.375EI\Delta + 5 \quad ---(1)$$

$$M_{BA} = \frac{4(EI)}{4}\theta_B + 0.375EI\Delta - 5 \quad ---(2)$$

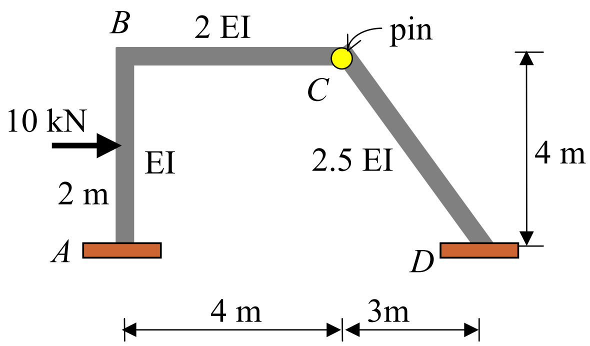
$$M_{BC} = \frac{3(2EI)}{4}\theta_B - 0.2813EI\Delta \quad ---(3)$$

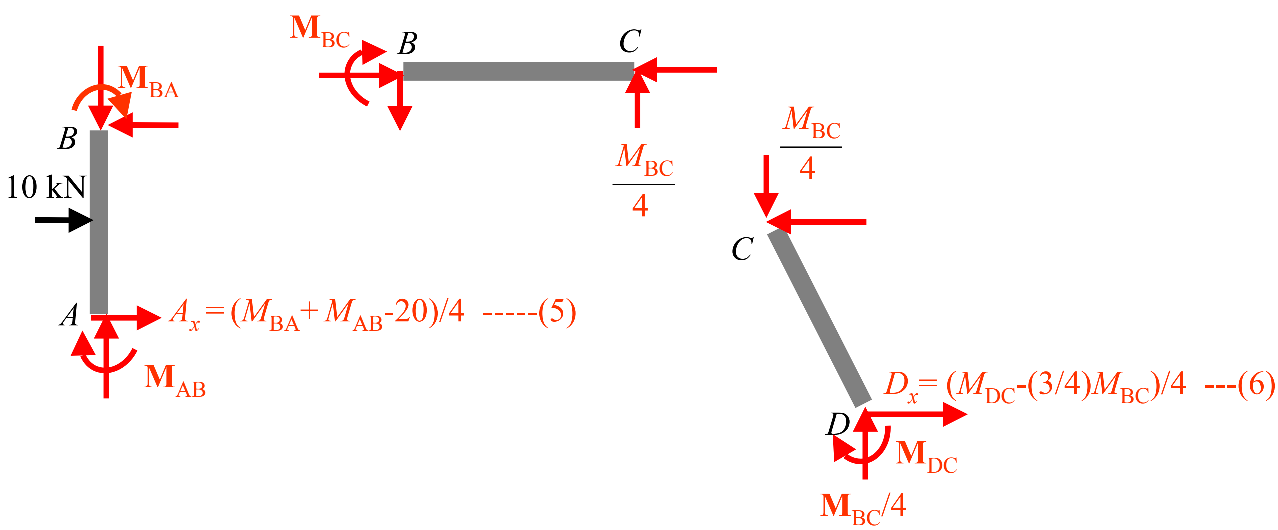
$$M_{DC} = 0.375EI\Delta \quad ---(4)$$





• **Horizontal reactions**





Equilibrium Conditions:

$$M_{BA} + M_{BC} = 0 \quad ---(1^*)$$

$$10 - A_x - D_x = 0 \quad ---(2^*)$$

Slope-Deflection Equation:

$$M_{AB} = \frac{2(EI)}{4}\theta_B + 5 + \frac{6EI\Delta}{4^2} \quad ---(1)$$

$$M_{BA} = \frac{4(EI)}{4}\theta_B - 5 + \frac{6EI\Delta}{4^2} \quad ---(2)$$

$$M_{BC} = \frac{3(2EI)}{4}\theta_B - \frac{3(2EI)(0.75\Delta)}{4^2} \quad ---(3)$$

$$M_{DC} = \frac{3(2.5EI)(1.25\Delta)}{5^2} \quad ---(4)$$

Horizontal reactions at supports:

$$A_x = \frac{(M_{BA} + M_{AB} - 20)}{4} ---(5)$$

$$D_x = \frac{M_{DC} - \frac{3}{4}M_{BC}}{4} \quad ---(6)$$

• **Solve equations**

Substitute (2) and (3) in (1*)

$$2.5EI\theta_B + 0.0938EI\Delta - 5 = 0 \quad ---(7)$$

Substitute (5) and (6) in (2*)

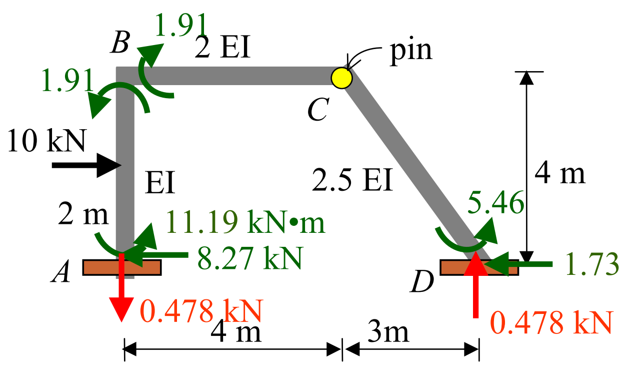
$$0.0938EI\theta_B + 0.334EI\Delta - 5 = 0 \quad ---(8)$$

From (7) and (8) can solve;

$$\theta_B = \frac{1.45}{EI} \qquad \Delta = \frac{-14.56}{EI}$$

$$Substitute\, \theta_B = \frac{1.45}{EI} \; and \; \Delta = \frac{-14.56}{EI} \quad in\ (1) to\ (6)$$

M_{AB} = 15.88 kN•m
M_{BA} = 5.6 kN•m
M_{BC} = -5.6 kN•m
M_{DC} = 8.42 kN•m
A_x = 7.9 kN
D_x = 2.1 kN



M_{AB} = 11.19 kN•m

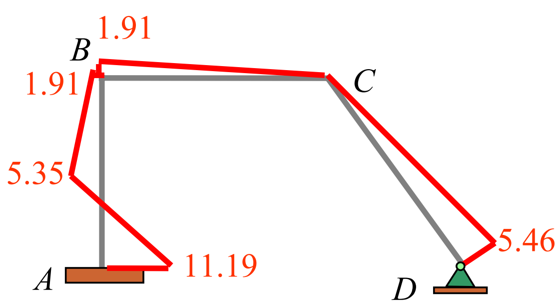
M_{BA} = 1.91 kN•m

M_{BC} = -1.91 kN•m

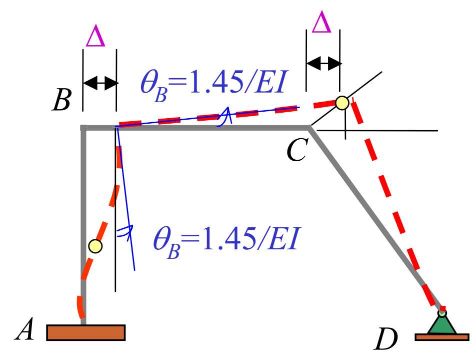
M_{DC} = 5.46 kN•m

A_x = 8.28 kN•m

D_x = 1.72 kN•m



Bending-moment diagram



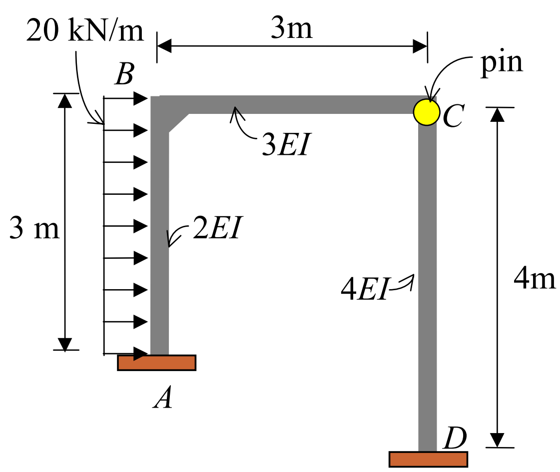
Deflected shape

Example 10

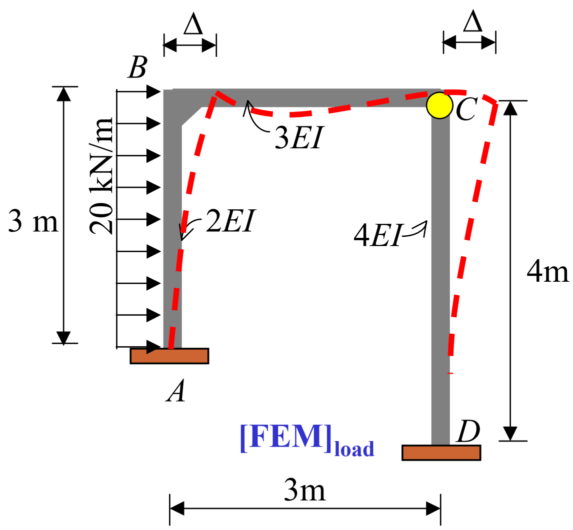
From the frame shown use the moment distribution method to:

(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected curve.**



- **Overview**



- **Unknowns**

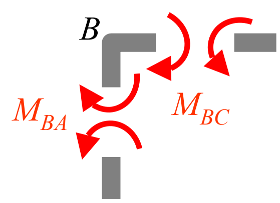
$$\theta_B \text{ and } \Delta$$

- **Boundary Conditions**

$$\theta_A = \theta_D = 0$$

- **Equilibrium Conditions**

- Joint *B*

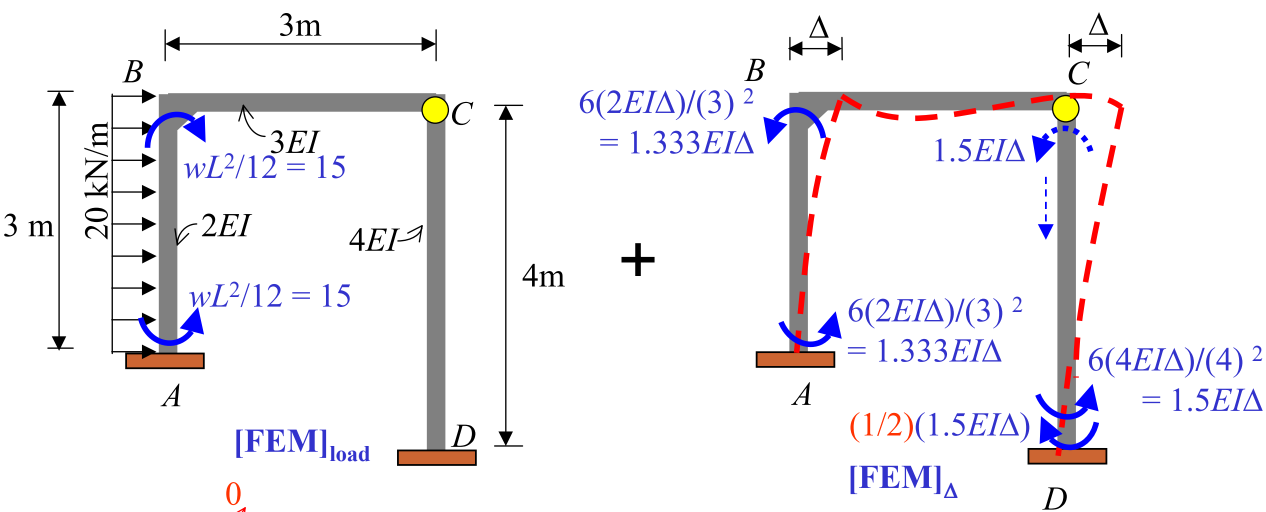


$$\Sigma M_B = 0: \quad M_{BA} + M_{BC} = 0 \quad --- (1^*)$$

- Entire Frame

$$\overset{+}{\rightarrow} \quad \Sigma F_x = 0: \quad 60 - A_x - D_x = 0 \quad --- (2^*)$$

• **Slope-Deflection Equation**



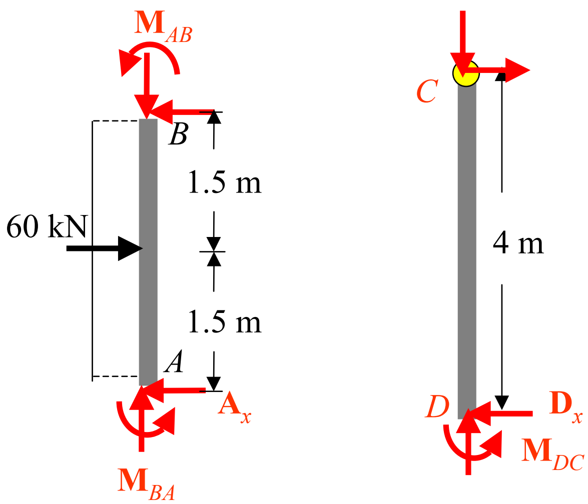
$$M_{AB} = \frac{4(2EI)}{3}\cancelto{0}{\theta_A} + \frac{2(2EI)}{3}\theta_B + 15 + 1.333EI\Delta \quad = 1.333EI\theta_B + 15 + 1.333EI\Delta \quad \text{----------(1)}$$

$$M_{BA} = \frac{2(2EI)}{3}\cancelto{0}{\theta_A} + \frac{4(2EI)}{3}\theta_B - 15 + 1.333EI\Delta \quad = 2.667EI\theta_B - 15 + 1.333EI\Delta \quad \text{----------(2)}$$

$$M_{BC} = \frac{3(3EI)}{3}\theta_B \quad = 3EI\theta_B \quad \text{----------(3)}$$

$$M_{DC} = \frac{3(4EI)}{4}\cancelto{0}{\theta_D} + 0.75EI\Delta = 0.75EI\Delta \quad \text{----------(4)}$$

• **Horizontal reactions**



$+\ \Sigma M_B = 0:$

$$A_x = \frac{M_{BA} + M_{AB} + 60(1.5)}{3}$$

$$A_x = 1.333EI\theta_B + 0.889EI\Delta + 30 \quad ---(5)$$

$+\ \Sigma \mathbf{M}_C = 0:$

$$D_x = \frac{M_{DC}}{4} = 0.188EI\Delta \quad ---(6)$$

Equilibrium Conditions

$$M_{BA} + M_{BC} = 0 \quad ---(1^*)$$
$$60 - A_x - D_x = 0 \quad ---(2^*)$$

Equation of moment

$$M_{AB} = 1.333EI\theta_B + 15 + 1.333EI\Delta \quad ---(1)$$

$$M_{BA} = 2.667EI\theta_B - 15 + 1.333EI\Delta \quad ---(2)$$

$$M_{BC} = 3EI\theta_B \quad ---(3)$$

$$M_{DC} = 0.75EI\Delta \quad ---(4)$$

Horizontal reaction at support

$$A_x = 1.333EI\theta_B + 0.889EI\Delta + 30 \quad ---(5)$$

$$D_x = 0.188EI\Delta \quad ---(6)$$

• **Solve equation**

Substitute (2) and (3) in (1*)

$$5.667EI\theta_B + 1.333EI\Delta = 15 \quad ---(7)$$

Substitute (5) and (6) in (2*)

$$-1.333EI\theta_B - 1.077EI\Delta = -30 \quad ---(8)$$

From (7) and (8), solve equations;

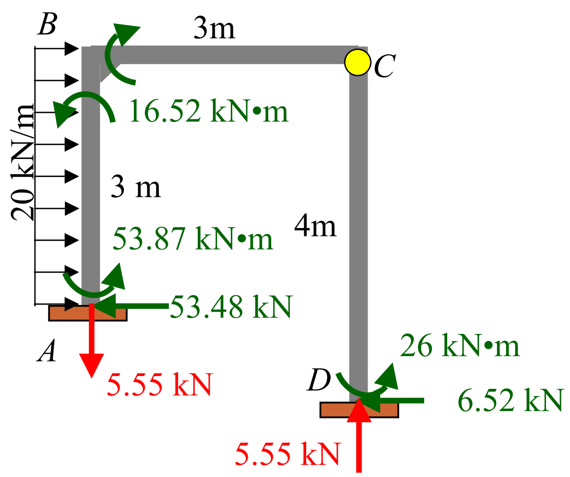
$$\theta_B = \frac{-5.51}{EI} \qquad \Delta = \frac{34.67}{EI}$$

$$Substitute\, \theta_B = \frac{-5.51}{EI} \; and \; \Delta = \frac{34.67}{EI} \quad in\ (1) to\ (6)$$

$$M_{AB} = 53.87\ kN \bullet m$$
$$M_{BA} = 16.52\ kN \bullet m$$
$$M_{BC} = -16.52\ kN \bullet m$$
$$M_{DC} = 26.0\ kN \bullet m$$

A_x = 53.48 kN

D_x = 6.52 kN



$$M_{AB} = 53.87\ kN \bullet m$$
$$M_{BA} = 16.52\ kN \bullet m$$
$$M_{BC} = -16.52\ kN \bullet m$$
$$M_{DC} = 26.0\ kN \bullet m$$
$$A_x = 53.48\ \text{kN}$$
$$D_x = 6.52\ \text{kN}$$