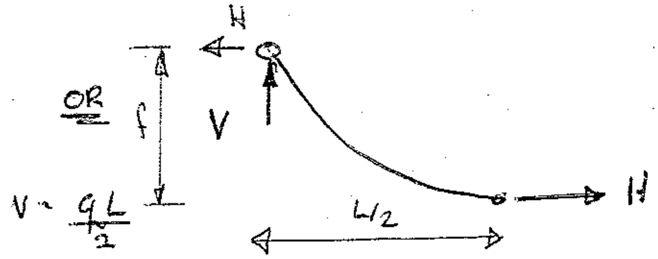
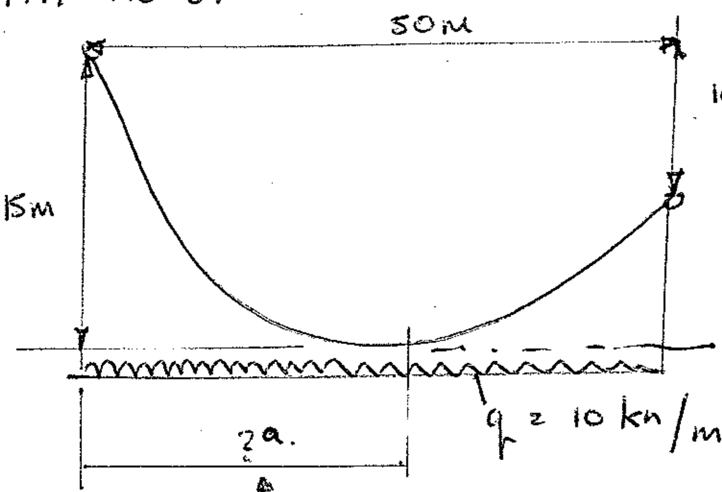


$\Sigma M = 0 \quad Hf = \left(\frac{QL}{2}\right)\frac{L}{4}$

$H = \frac{QL^2}{8F}$

b) 1977 No 6:



$15 = k(a^2)$

$5 = k(50-a)^2$

$\Rightarrow \frac{15}{5} = \frac{a^2}{(50-a)^2} \Rightarrow a.$

if known we have two useful free bodies.

<u>Parabola</u> $\quad \frac{15}{5} = \frac{a^2}{(50-a)^2} \Rightarrow$ hence a

then statics to solve for vertical reactions, and horizontal reactions.

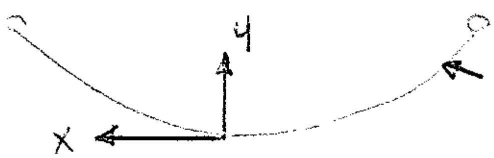
<u>Alternative method</u>:
With origin at upper support $\frac{d^2y}{dx^2} = -q/H$

Integrate twice and insert boundary conditions
$y = 0$ at $x = 0$
$y = 10$ at $x = 50$

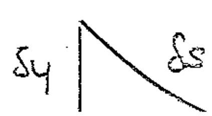
Use extra information on maximum dip to determine H

<u>*Example*</u>: Cable hanging under its own weight

$$\frac{d^2y}{dx^2} = \frac{1}{H} q(x)$$



← cable weight / unit length = q



weight of the small element
$= q \times ds$

weight of element $= q\,ds = q(x)\,dx$ $q(x) = q\left(\frac{ds}{dx}\right)$

$$\Rightarrow q_x = q\frac{ds}{dx} \rightarrow$$

Substituting

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{q}{H}\cdot\frac{ds}{dx} = \frac{q}{H}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{q}{H}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}} \qquad ds^2 = dx^2 + dy^2.$$

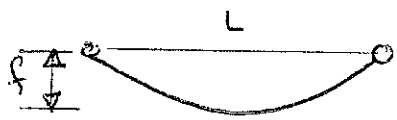
$$\Rightarrow y = \frac{1}{2c}\left[e^{cx} + e^{-cx} - 2\right] \quad \text{where } c = q/H$$

= catenary equation

Note: for large spans the catenary and the parabola are very similar.

Applications:

a) Measurement

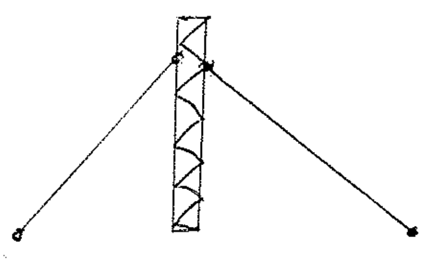


$$L_{cable} = L\left[1 + \frac{8}{3}n^2\right] \qquad n = \text{sag/span ratio} = f/L$$

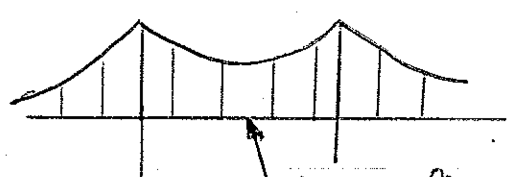
NB: $s = \int \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/2} dx;$

b) Electric cables loads, wind, ice:

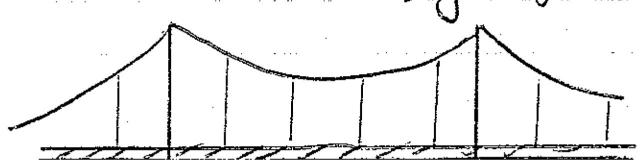
c) Guyed Structure:



d) Suspension Bridges
a) Unstiffened:



b) Stiffened



Large spans

$$y(x) = \frac{1}{2c}\left[e^{cx} + e^{-cx} - 2\right] \quad \Rightarrow \quad c = \left(q/H\right).$$

$$e^{cx} = 1 + cx + \frac{(cx)^2}{2!} + \frac{(cx)^3}{3!} + \cdot \frac{(cx)^4}{4!}$$

$$e^{-cx} = 1 - c(x) + \frac{(cx)^2}{2!} - \frac{(cx)^3}{3!} + \frac{(cx)^4}{4!}$$

$$e^{cx} + e^{-cx} = 2 + (cx)^2 + \frac{2(cx)^4}{4!} \dots$$

$$y(x) = \frac{1}{2c}\left[(cx)^2 + \frac{2(cx)^4}{4!}\right]$$

$$y(x) = \left(\frac{c^2x^2}{2c} + \frac{2c^4x^4}{2.4!\,c} + \dots\right]$$

$$= \left(\frac{cx^2}{2} + \frac{c^3x^4}{4!} + \dots\right]$$

$$= \frac{q}{H}\left[\frac{x^2}{2} + \underbrace{\left(\frac{q}{H}\right)^2\frac{1}{4!}}\, x^4\right]$$

$$\Big[\text{but. } H = \frac{qL^2}{8f} \Rightarrow \left(\frac{q}{H}\right) = \left(\frac{8f}{L^2}\right)$$

2nd order term will be very small for large spans.