Review of Waves

Properties of electromagnetic waves in vacuum:

Waves propagate through vacuum (no medium is required like sound waves)

All frequencies have the same propagation speed, c in vacuum.

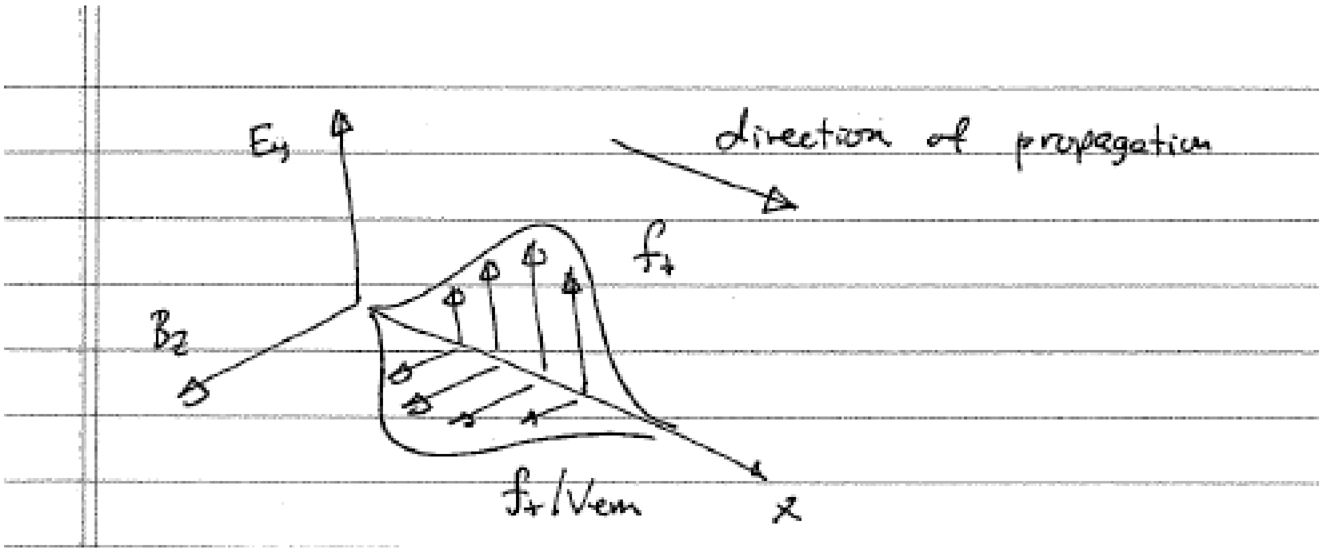
Electric and magnetic fields are oriented transverse to the direction of propagation. (transverse waves)

Waves carry both energy and momentum.

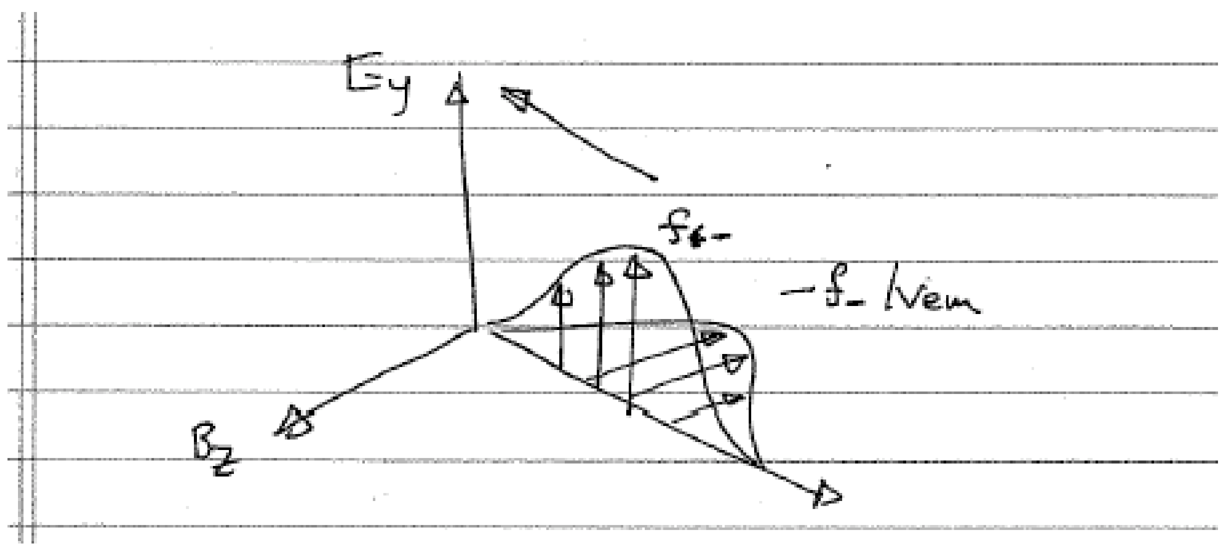
E and B fields in waves and Right Hand Rule:

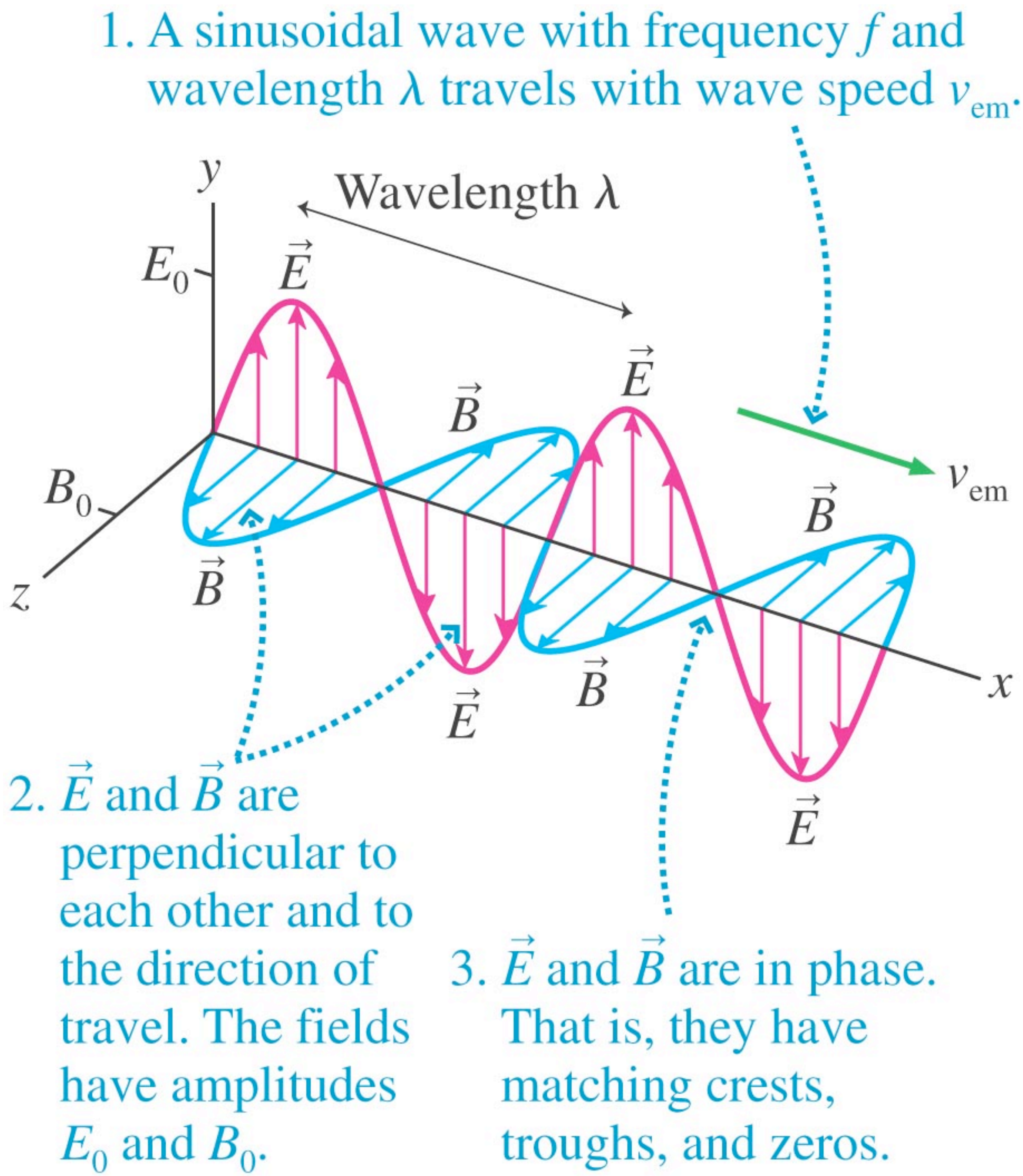
f+ solution

Wave propagates in **E** x **B** direction



f- solution

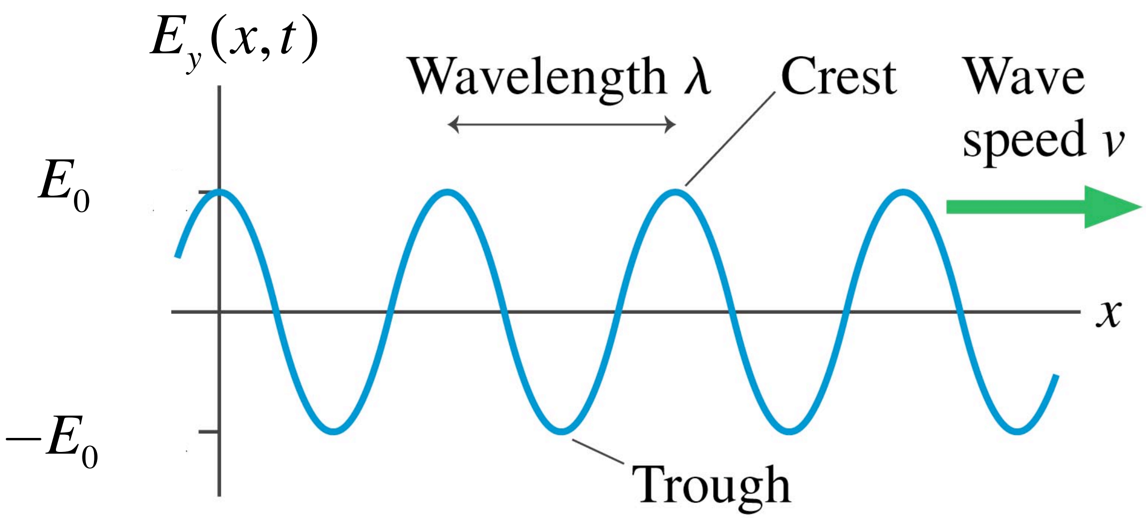



1. A sinusoidal wave with frequency f and wavelength λ travels with wave speed v_{em}.
y
Wavelength λ
E_0
$\vec{E}$
$\vec{B}$
$\vec{E}$
v_{em}
B_0
$\vec{B}$
z
$\vec{B}$
$\vec{B}$
$\vec{E}$
x
$\vec{E}$
2. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of travel. The fields have amplitudes E_0 and B_0.
3. $\vec{E}$ and $\vec{B}$ are in phase. That is, they have matching crests, troughs, and zeros.

Special Case Sinusoidal Waves

$$E_y(x,t) = f_+(x - v_{em}t) = E_0 \cos\left[k(x - v_{em}t)\right]$$

(b) A snapshot graph at one instant of time



Wavenumber and wavelength

$$k = 2\pi / \lambda$$

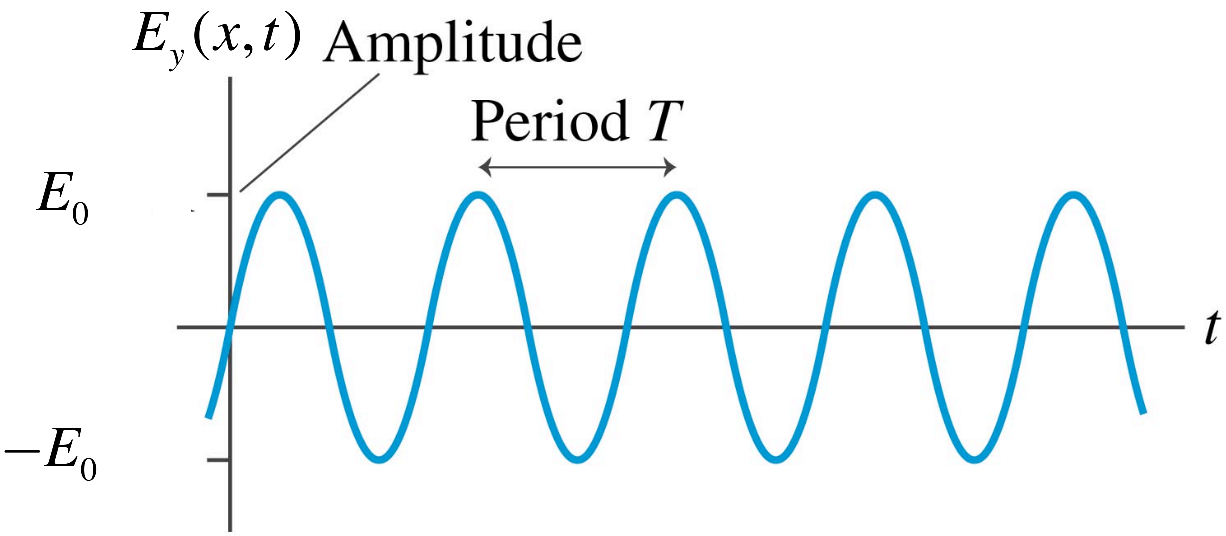
$$\lambda = 2\pi / k$$

These two contain the same information

Special Case Sinusoidal Waves

$$E_y(x,t) = f_+(x - v_{em}t) = E_0 \cos\left[k(x - v_{em}t)\right]$$

(a) A history graph at one point in space



$$2\pi = kv_{em}T$$

Introduce

$$\omega = 2\pi / T$$

$$f = 1 / T$$

Different ways of saying the same thing:

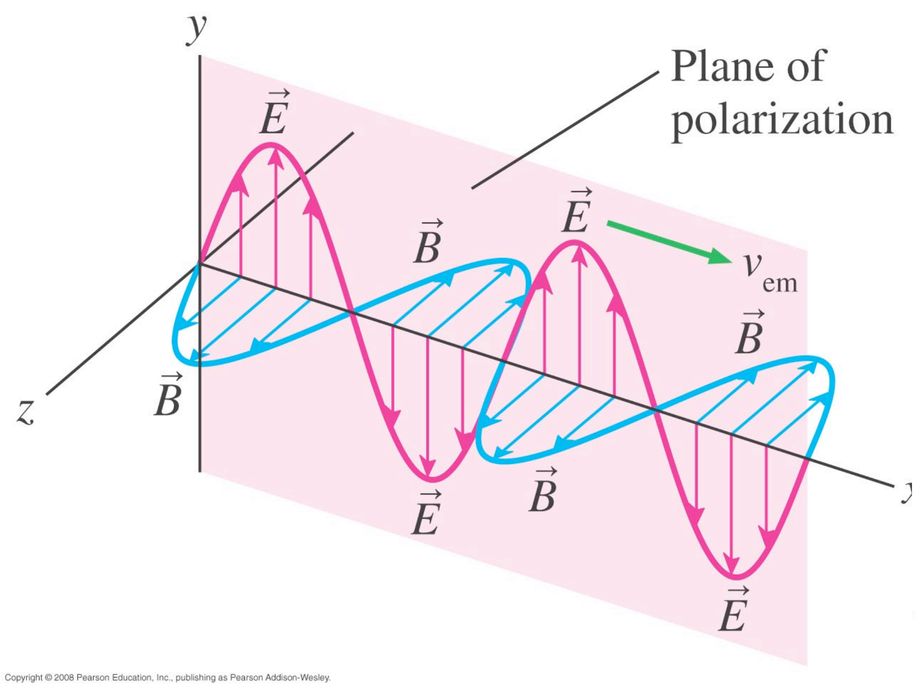
$$\omega / k = v_{em} \qquad f\lambda = v_{em}$$

Polarizations

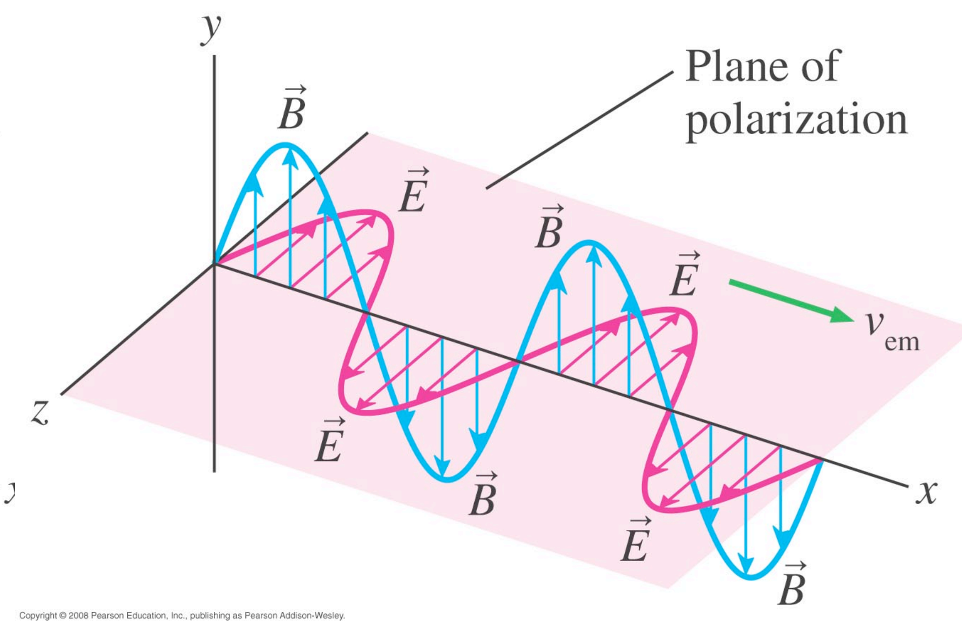
We picked this combination of fields: E_y - B_z

Could have picked this combination of fields: E_z - B_y

(a) Vertical polarization



(b) Horizontal polarization



These are called plane polarized. Fields lie in plane

Which of the following are valid EM waves?

1.

A. Yes
B. No

$$E_x(z,t) = E_0 \cos\left[k(z - v_{em}t)\right]$$

$$B_y(z,t) = \frac{E_0}{v_{em}} \cos\left[k(z - v_{em}t)\right]$$

2.

A. Yes
B. No

$$E_y(y,t) = E_0 \cos\left[k(y - v_{em}t)\right]$$

$$B_z(y,t) = \frac{E_0}{v_{em}} \cos\left[k(y - v_{em}t)\right]$$

3.

A. Yes
B. No

$$E_x(y,t) = E_0 \cos\left[k(y + v_{em} t)\right]$$

$$B_z(y,t) = \frac{E_0}{v_{em}} \cos\left[k(y + v_{em} t)\right]$$

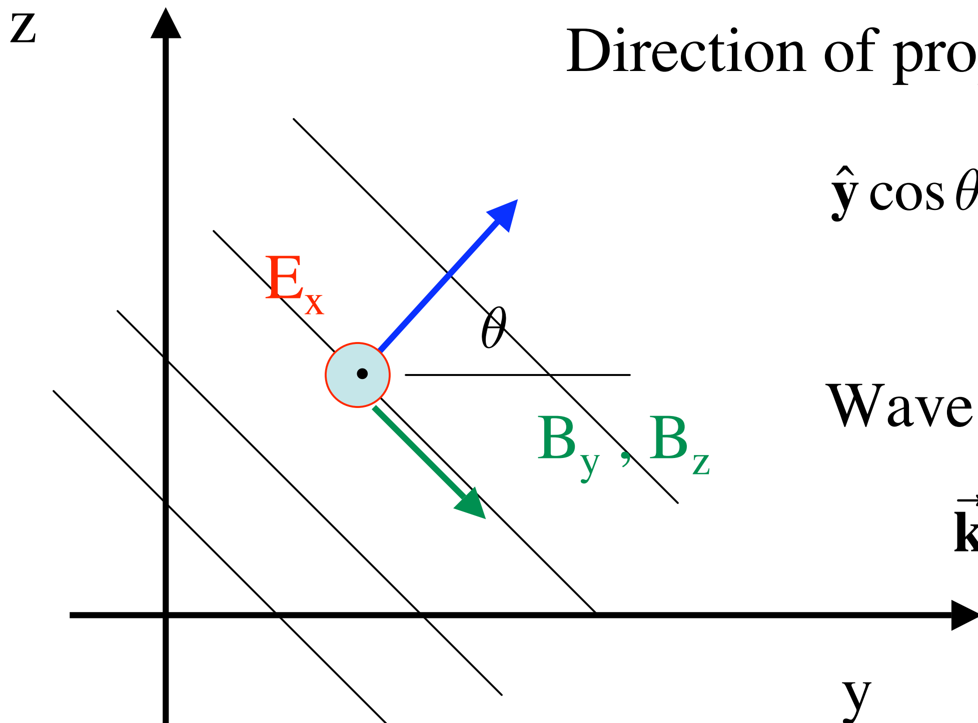
4. What direction is this wave propagating in?

A. Y
B. Z
C. X
D. None of above

$$E_x(y,z,t) = E_0 \cos\left[ky\cos\theta + kz\sin\theta - \omega t\right]$$

$$\omega = ?$$

$$\vec{\mathbf{B}} = ?$$



Direction of propagation

$$\hat{\mathbf{y}}\cos\theta + \hat{\mathbf{z}}\sin\theta$$

Wave Vector

$$\vec{\mathbf{k}} = k\left(\hat{\mathbf{y}}\cos\theta + \hat{\mathbf{z}}\sin\theta\right)$$

$$E_x(y,z,t) = E_0\cos\left[ky\cos\theta + kz\sin\theta - \omega t\right]$$

$$\omega = kv_{em}$$

$$B_y(y,z,t) = \sin\theta\frac{E_x}{v_{em}}$$

$$B_z(y,z,t) = -\cos\theta\frac{E_x}{v_{em}}$$

Energy Density and Intensity of EM Waves

Energy density associated with electric and magnetic fields

$$u_E = \frac{\varepsilon_0 \left|\vec{\mathbf{E}}\right|^2}{2} \qquad u_B = \frac{\left|\vec{\mathbf{B}}\right|^2}{2\mu_0}$$

For a wave: $$\left|\vec{\mathbf{B}}\right| = \frac{1}{v_{em}}\left|\vec{\mathbf{E}}\right| = \sqrt{\varepsilon_0 \mu_0}\left|\vec{\mathbf{E}}\right|$$

Thus:

$$u_E = u_B$$ Units: J/m^3

Energy density in electric and magnetic fields are equal for a wave in vacuum.

We now want to expand the picture in the following way:

EM waves propagate in 3D not just 1D as we have considered.
- Diffraction - waves coming from a finite source spread out.

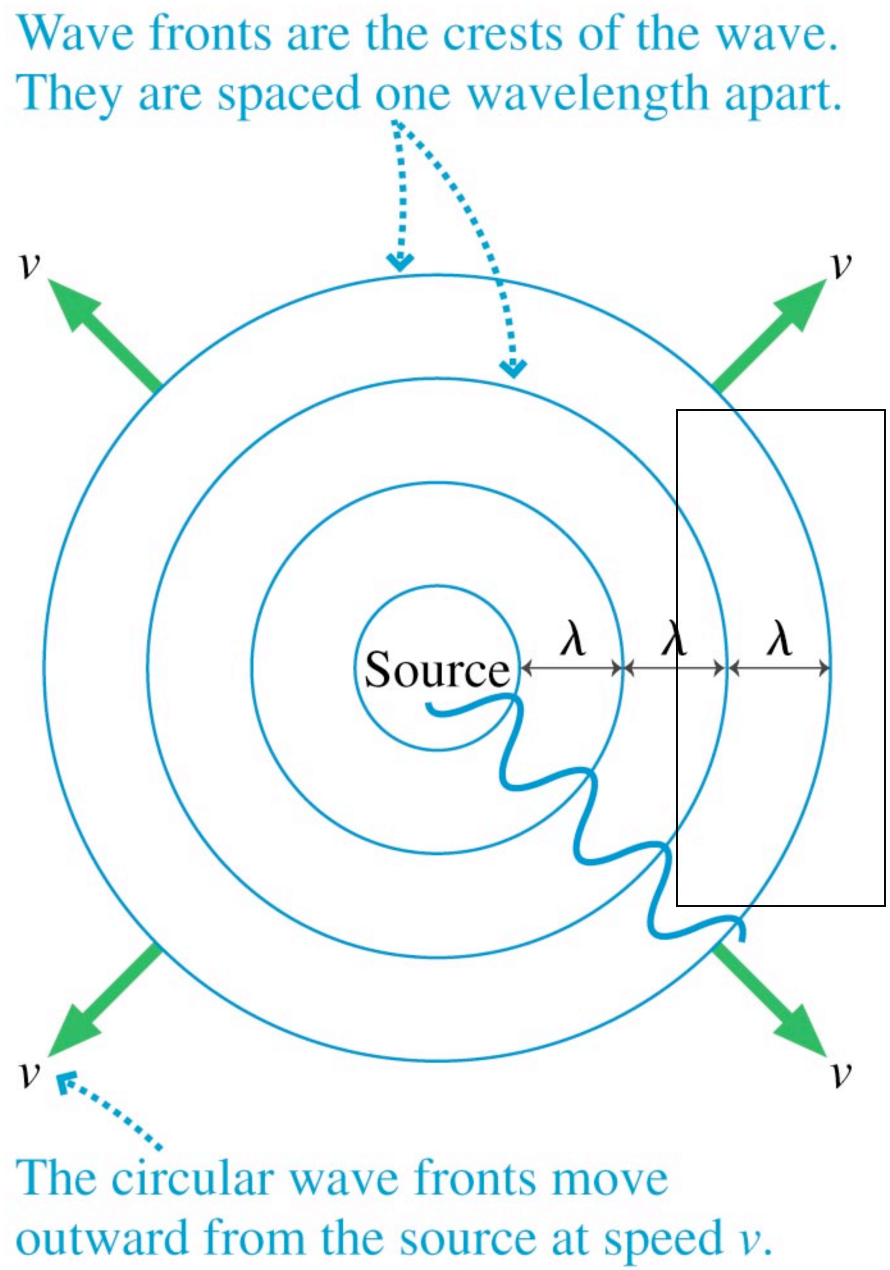
EM waves propagate through material and are modified.
- Dispersion - waves are slowed down by media, different frequency waves travel with different speeds

- Reflection - waves encounter boundaries between media. Some energy is reflected.
- Refraction - wave trajectories are bent when crossing from one medium to another.

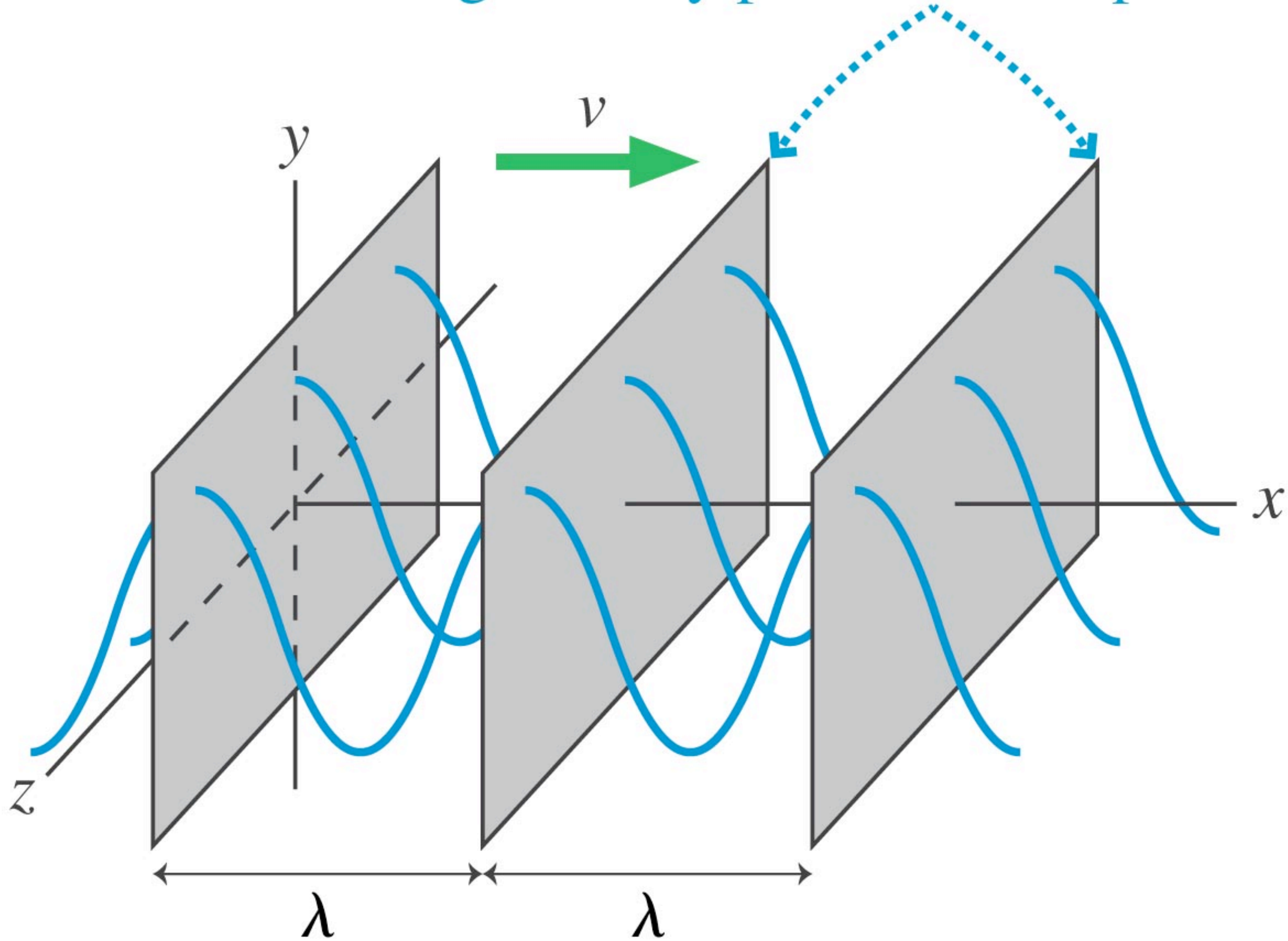
EM waves can take multiple paths and arrive at the same point.
- Interference - contributions from different paths add or cancel.

Waves emanating from a point source

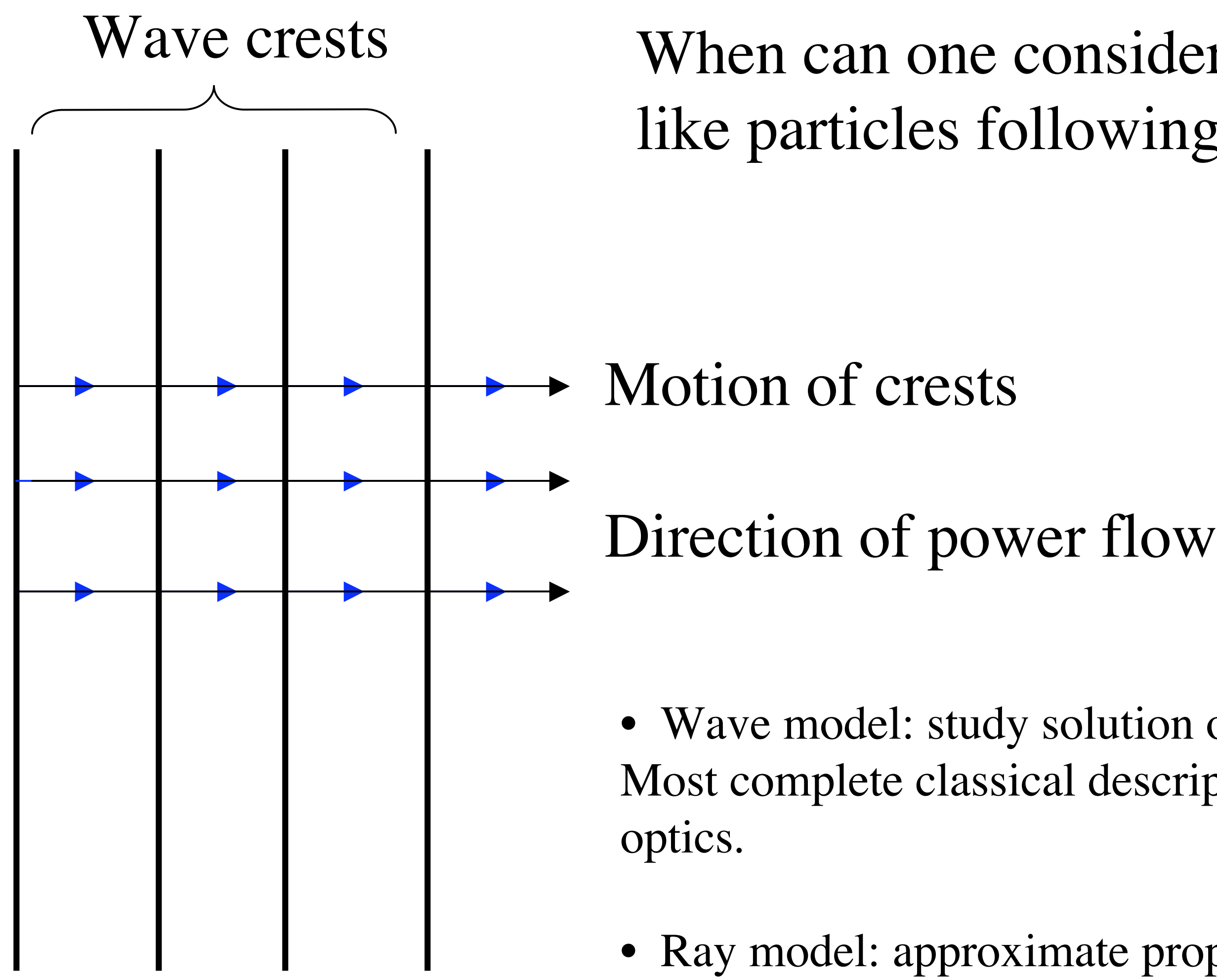
(a)



Very far from the source, small segments of spherical wave fronts appear to be planes. The wave is cresting at every point in these planes.



When can one consider waves to be like particles following a trajectory?

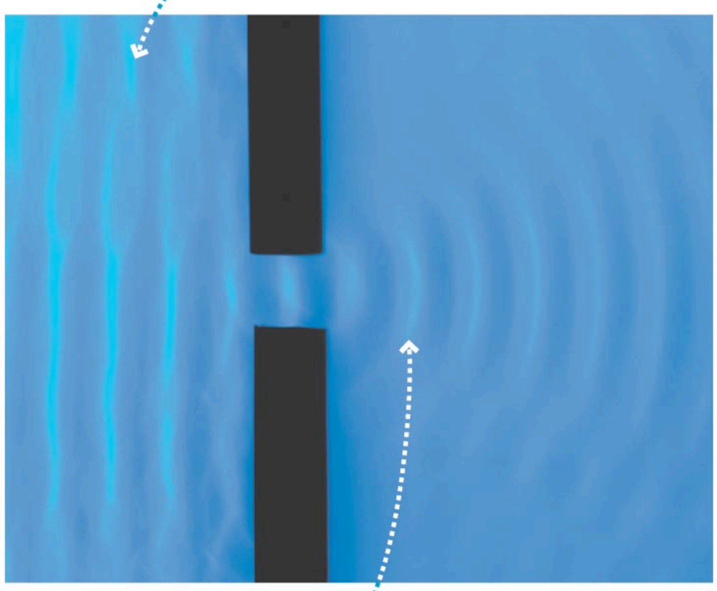


- Wave model: study solution of Maxwell equations. Most complete classical description. Called physical optics.

- Ray model: approximate propagation of light as that of particles following specific paths or “rays”. Called geometric optics.

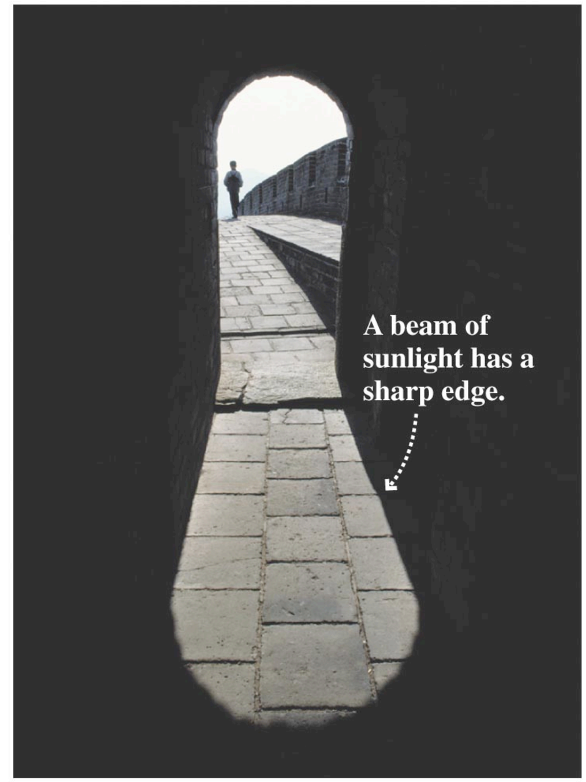
- Quantum optics: Light actually comes in chunks called photons

(a) Plane waves approach from the left.



Circular waves spread out on the right.

(b)

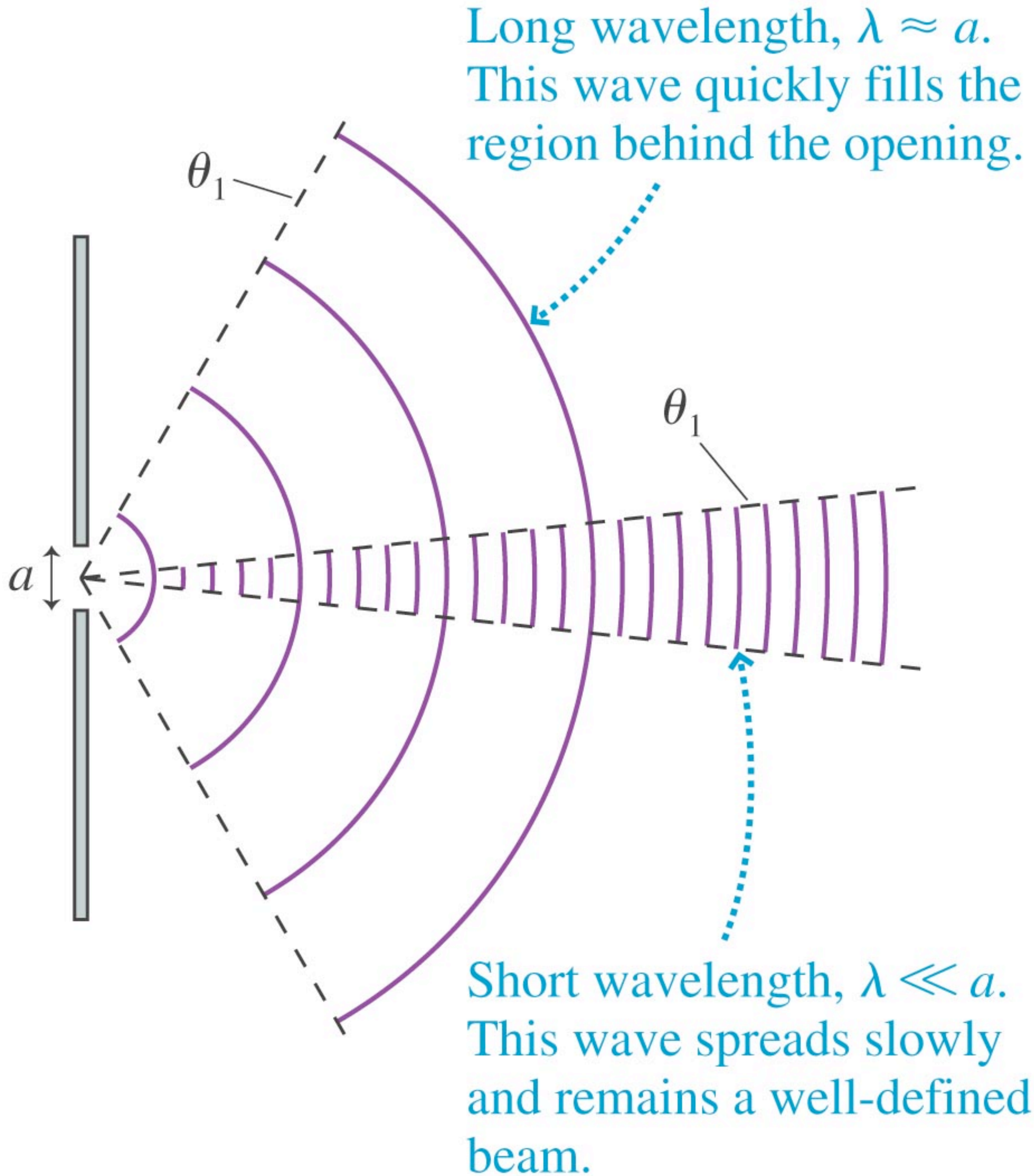


λ Comparable to opening size

What is the difference? Diffraction.

λ Much smaller than opening size

Diffraction

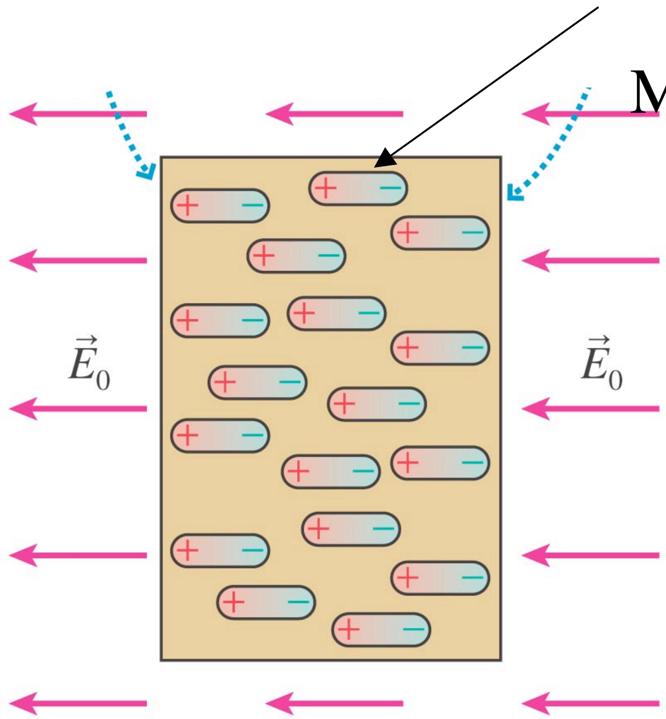


Propagation of light through dielectric media

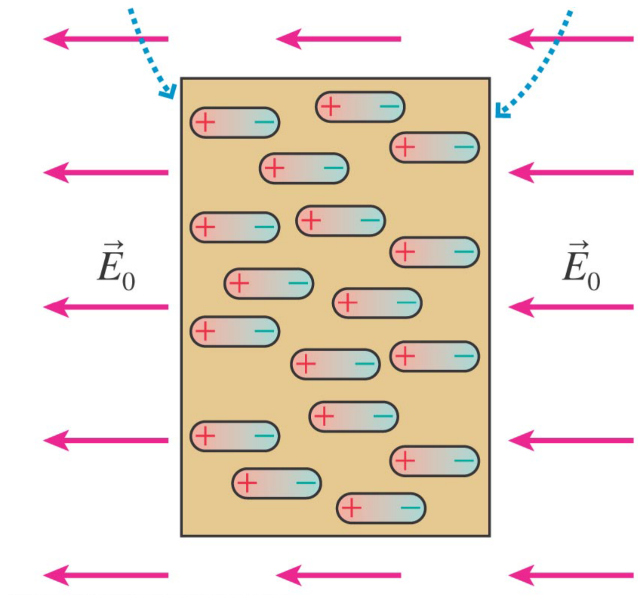
In a dielectric the Electric field causes molecules/atoms to become polarized.

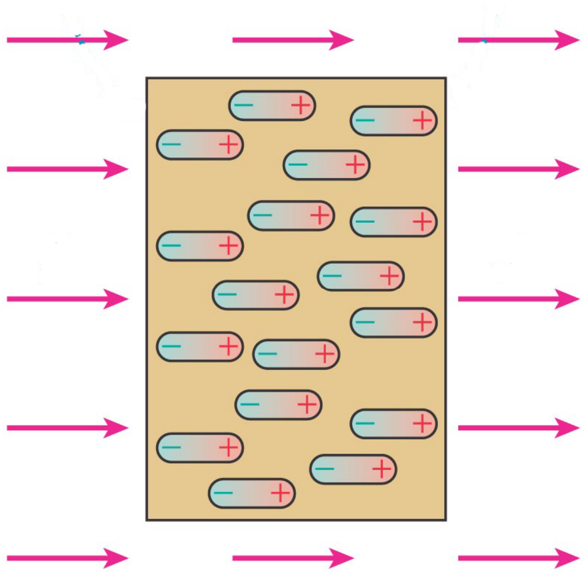
Molecules align with field

Medium becomes "polarized"



What happens when the electric field oscillates in time?

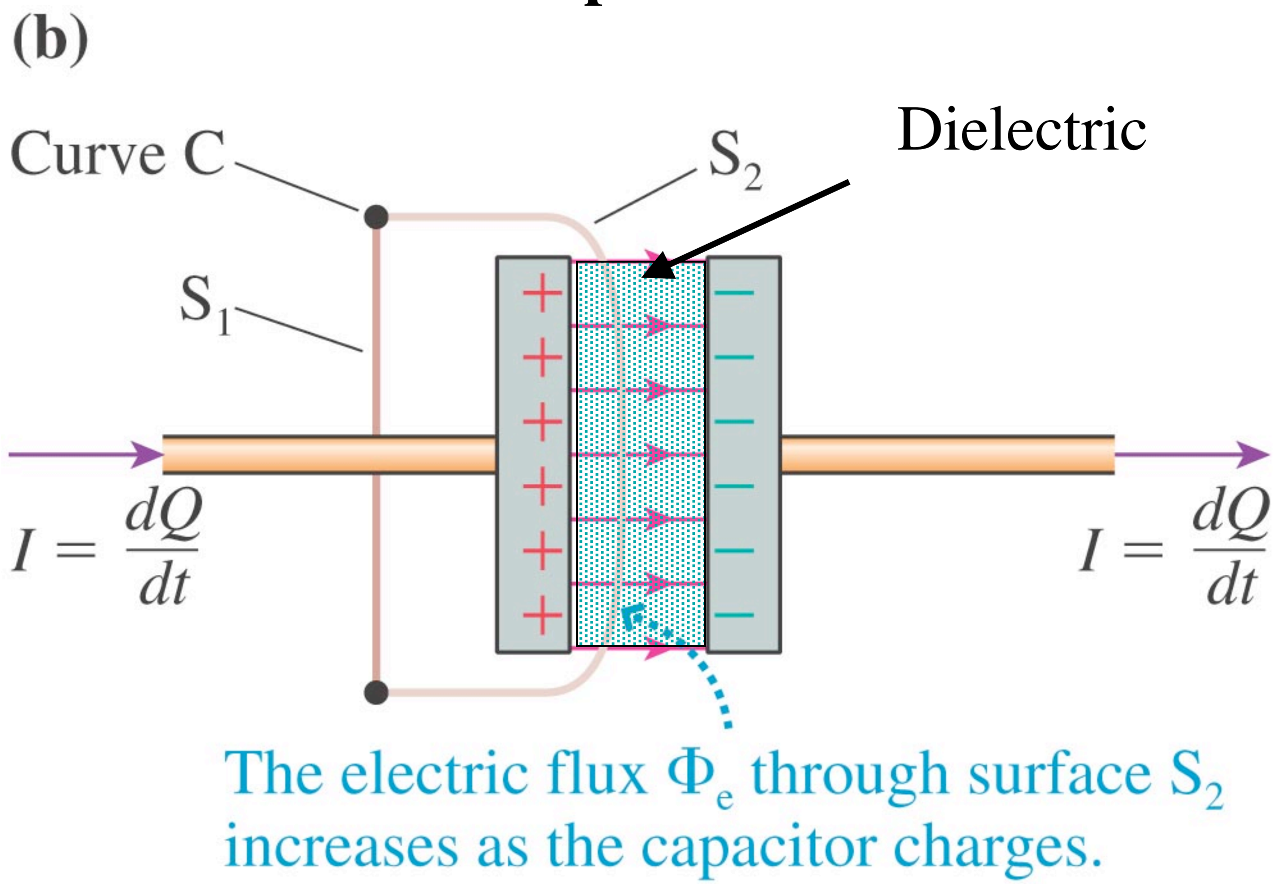




A current is induced.

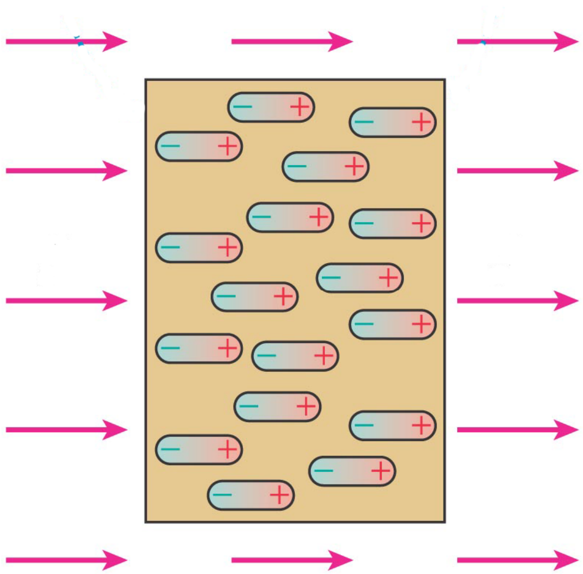
$$I \propto \frac{\partial E}{\partial t}$$

How is the Ampere-Maxwell Law modified?



For S_2 there is both displacement current and real current

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I_{\text{through}} + I_{\text{disp}}) = \mu_0 \left(I_{\text{through}} + \epsilon_0 \frac{d\Phi_e}{dt} \right) \qquad (35.22)$$



$$I \propto \frac{\partial E}{\partial t}$$

Real current given by dielectric constant

Electric flux

$$I_{through} = \varepsilon_0 (\kappa - 1) \frac{d\Phi_e}{dt}$$

κ Dielectric constant

In a dielectric material

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \mu_0 (\varepsilon_0 \kappa \frac{d\Phi_e}{dt})$$

κ Dielectric constant

<u>Bottom Line:</u>

Replace ε_0 by $\varepsilon_0 \kappa$

Static dielectric constants

To complicate matters, dielectric constant depends on frequency.

Dielectric function depends on frequency

TABLE 30.1 Properties of dielectrics

Material	Dielectric constant κ	Dielectric strength E_{max} (10^6 V/m)
Vacuum	1	—
Air (1 atm)	1.0006	3
Teflon	2.1	60
Polystyrene plastic	2.6	24
Mylar	3.1	7
Paper	3.7	16
Pyrex glass	4.7	14
Pure water (20°C)	80	—
Titanium dioxide	110	6
Strontium titanate	300	8

Consequences for EM Plane waves

$$E_y(x,t) = f_+(x - v_{em}t) + f_-(x + v_{em}t)$$

$$B_z(x,t) = \frac{1}{v_{em}}\left(f_+(x - v_{em}t) - f_-(x + v_{em}t)\right)$$

$$v_{em} = 1/\sqrt{\mu_0\varepsilon_0\kappa} = c/\sqrt{\kappa}$$

Propagation speed changes in a material

Refraction

Ratio of E to B changes

Reflection

For sinusoidal waves the following is still true

$$f\lambda = v_{em}$$

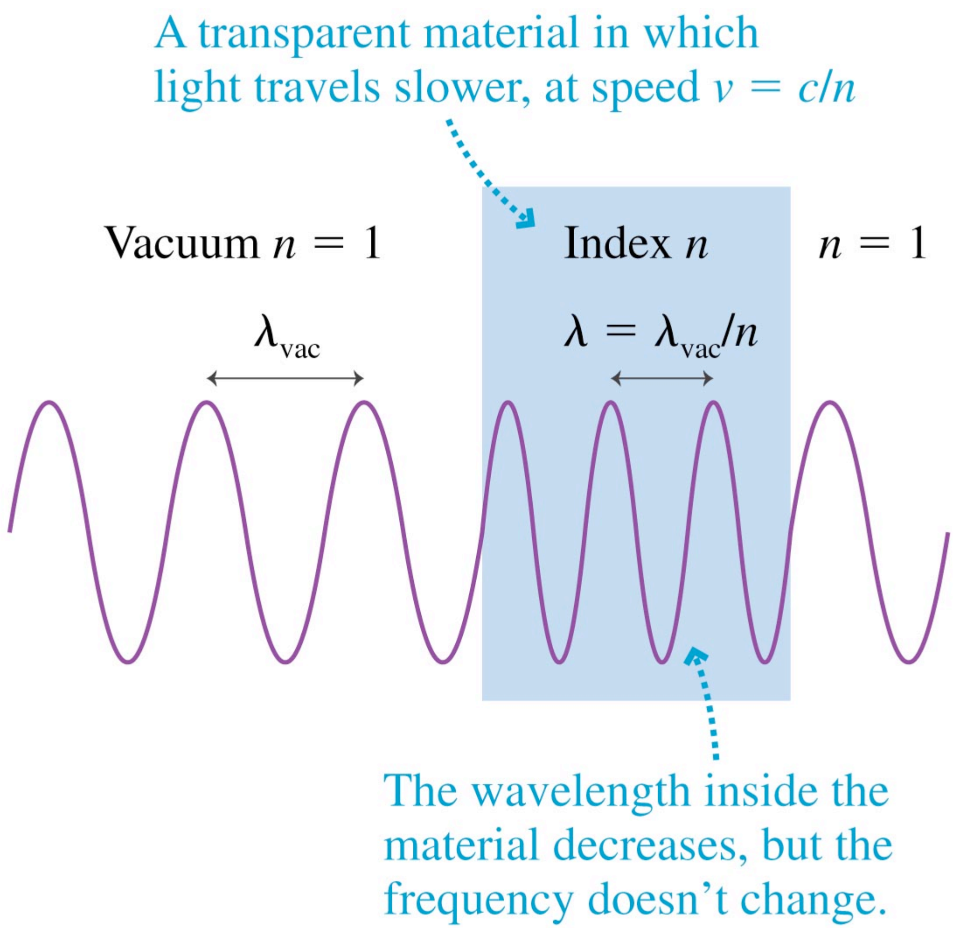
$$\omega / k = v_{em}$$

Index of refraction

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}} = \frac{c}{v_{em}} = \sqrt{\kappa}$$

TABLE 20.2 Typical indices of refraction

Material	Index of refraction
Vacuum	1 exactly
Air	1.0003
Water	1.33
Glass	1.50
Diamond	2.42



For sinusoidal waves the following is still true

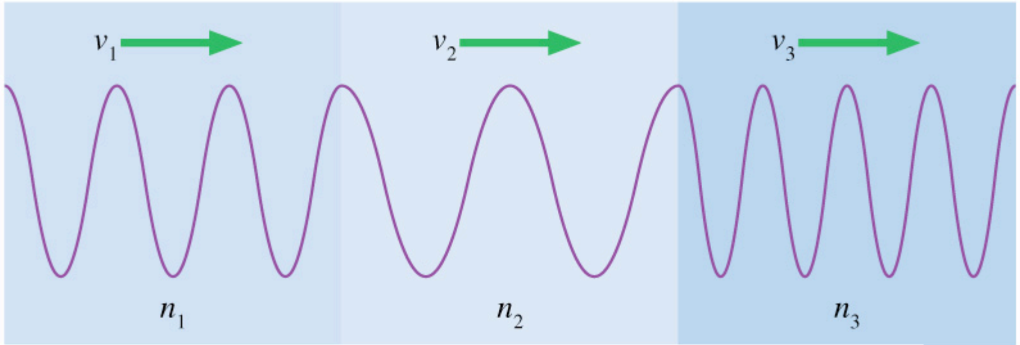
$$f\lambda = v_{em}$$

$$\omega / k = v_{em}$$

Frequency is the same in both media

Wavelength changes

A light wave travels through three transparent materials of equal thickness. Rank in order, from the largest to smallest, the indices of refraction n_1, n_2, and n_3.



A. $n_1 > n_2 > n_3$

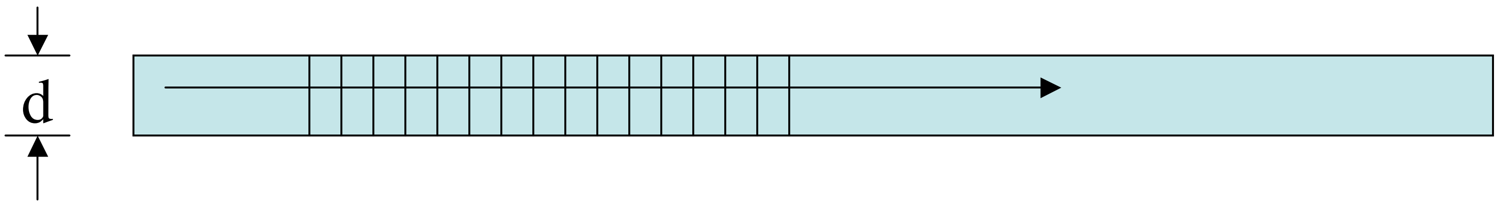
B. $n_2 > n_1 > n_3$

C. $n_3 > n_1 > n_2$

D. $n_3 > n_2 > n_1$

E. $n_1 = n_2 = n_3$

Example: Optical fiber



Fused silica: n=1.48

In vacuum

Vacuum wavelength $\lambda = 1550$ nm

Wavelength in fiber $\lambda/n = 1045$nm

D = 10 μm

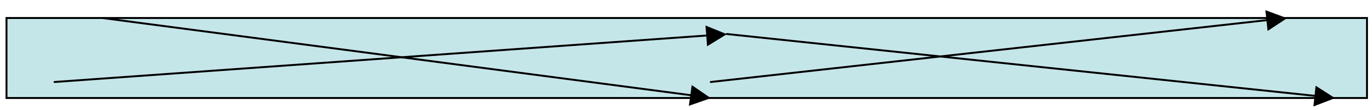
P=1mW inside fiber

I=P/A=?

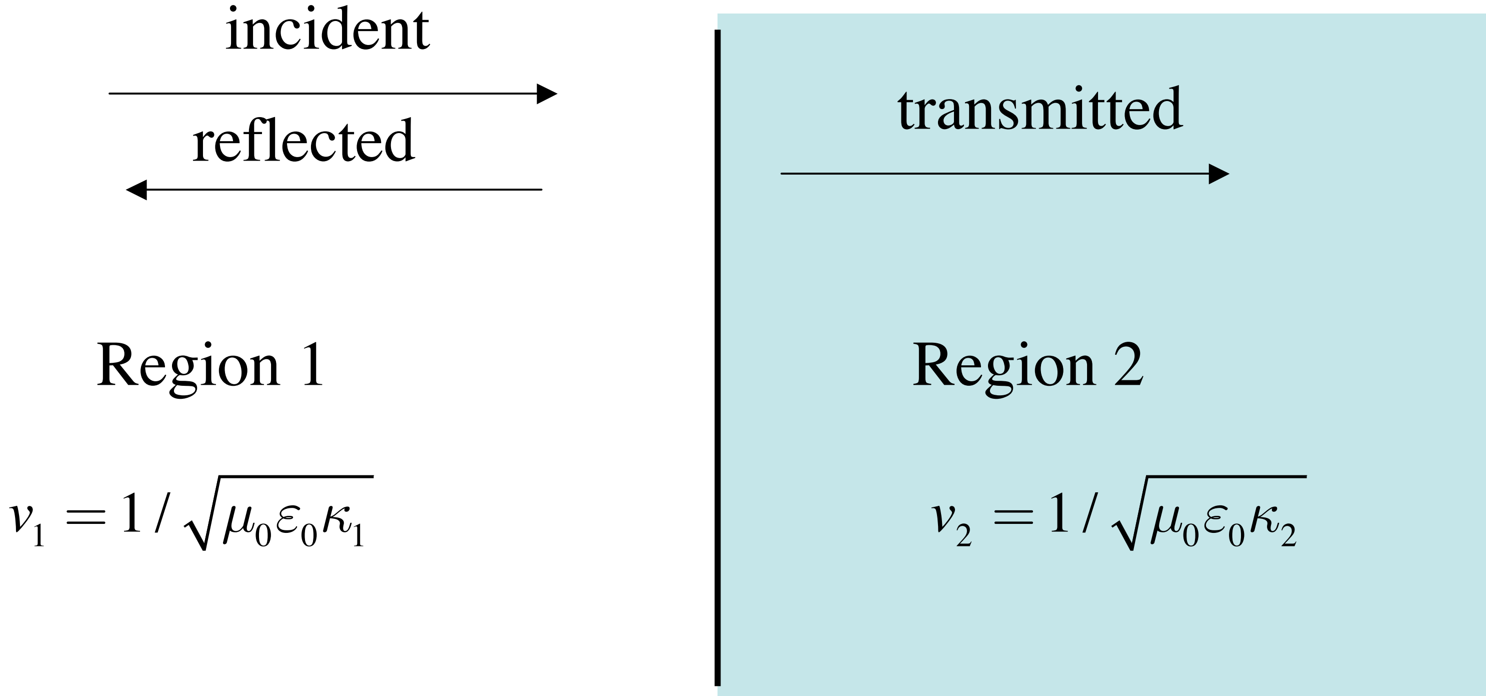
E=?

$$I = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\vec{\mathbf{E}}\right|^2$$

Rays bounce back and forth



Reflection from surface

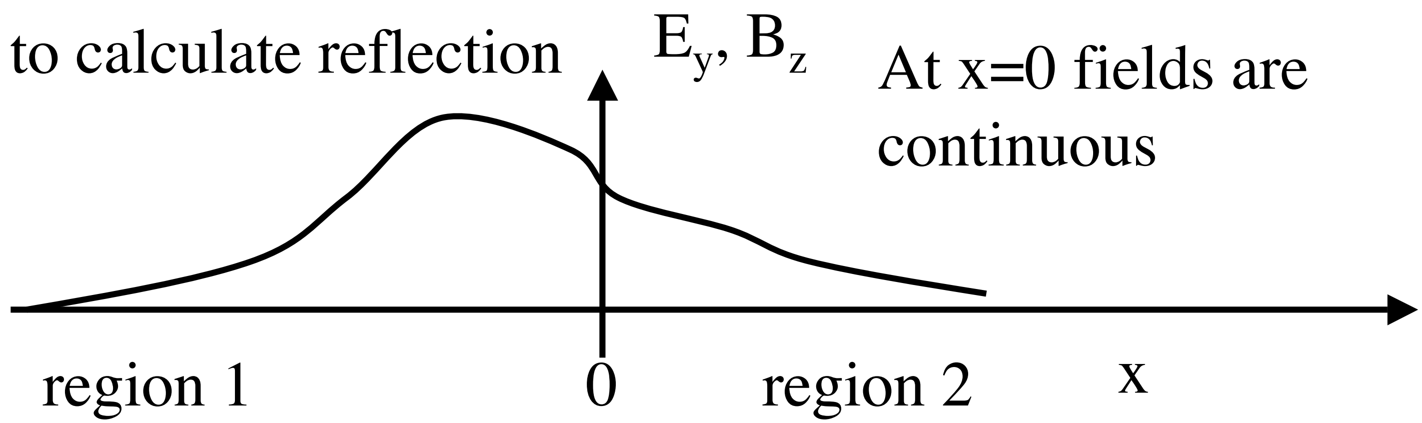


Reflection coefficient $\rho = \dfrac{v_2 - v_1}{v_2 + v_1}$

What if $\kappa_1 = \kappa_2\ , \quad v_1 = v_2$

"Index matched"

How to calculate reflection



At x=0 fields are continuous

For x<0

incident

reflected

$$E_y(x,t) = f_+(x - v_1 t) + f_-(x + v_1 t)$$

$$B_z(x,t) = \frac{1}{v_1}\left(f_+(x - v_1 t) - f_-(x + v_1 t)\right)$$

$$v_1 = 1/\sqrt{\mu_0 \varepsilon_0 \kappa_1}$$

For x>0

transmitted

$$E_y(x,t) = f_t(x - v_2 t)$$

$$B_z(x,t) = \frac{1}{v_2} f_t(x - v_2 t)$$

$$v_1 = 1/\sqrt{\mu_0 \varepsilon_0 \kappa_1}$$

Continuity of E_y at x=0

x<0 x>0

$$E_y(0,t) = f_+(-v_1 t) + f_-(v_1 t) = f_t(-v_2 t)$$

Continuity of B_z at x=0

x<0 x>0

$$B_z(0,t) = \frac{1}{v_1}\left(f_+(-v_1 t) - f_-(v_1 t)\right) = \frac{1}{v_2} f_t(-v_2 t)$$

Solve for reflected & transmitted pulses

$$f_-(v_1 t) = \rho f_+(-v_1 t)$$
$$f_t(-v_2 t) = (1+\rho) f_+(-v_1 t)$$

$$\rho = \frac{v_2 - v_1}{v_2 + v_1}$$

Example: what fraction of power is reflected at boundary between glass and air?

Reflection coefficient $$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2}$$

Air: n_1=1.0003 Glass: n_2 =1.5

$$\rho = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1-1.5}{1+1.5} = -0.2$$

ratio of reflected to incident electric field amplitude

I=Power/Area

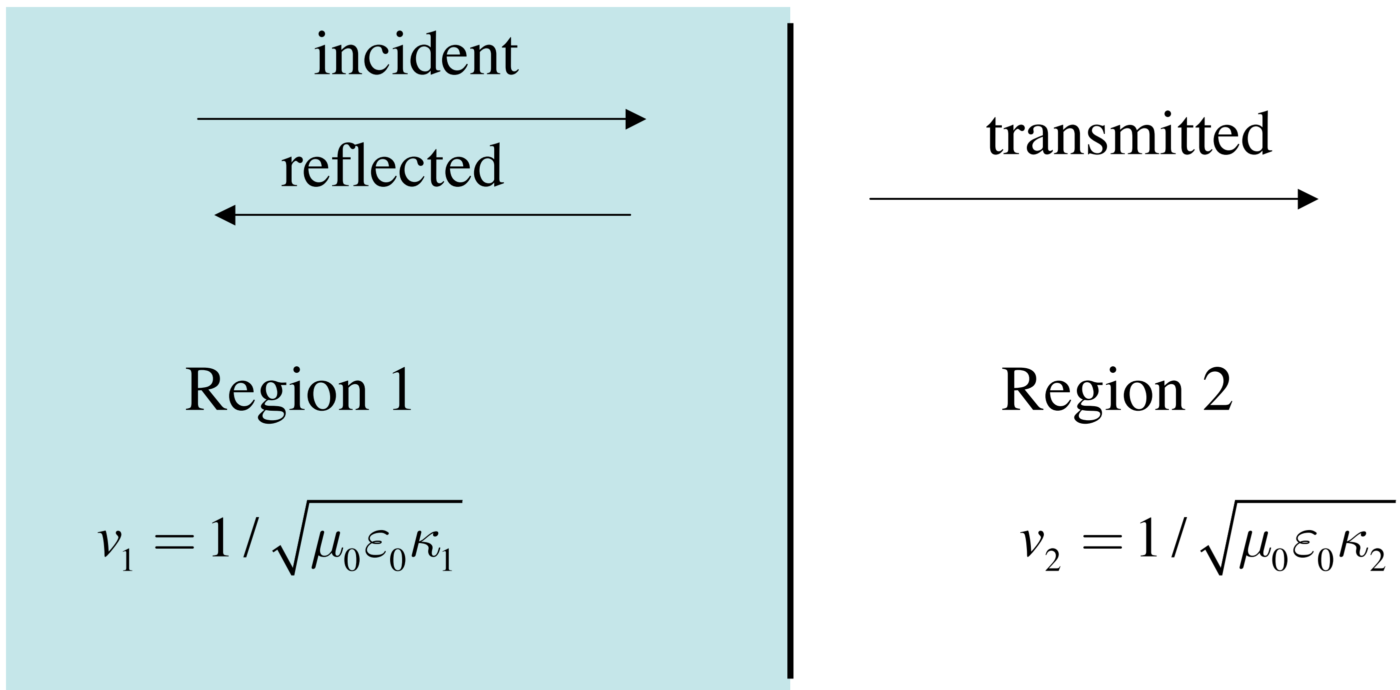
$$I = \sqrt{\frac{\varepsilon_0}{\mu_0}} \left|\vec{\mathbf{E}}\right|^2$$

$$I_{\text{Refl}} = \rho^2 I_{\text{Inc}}$$

$$I_{\text{Refl}} = 0.04 I_{\text{Inc}}$$

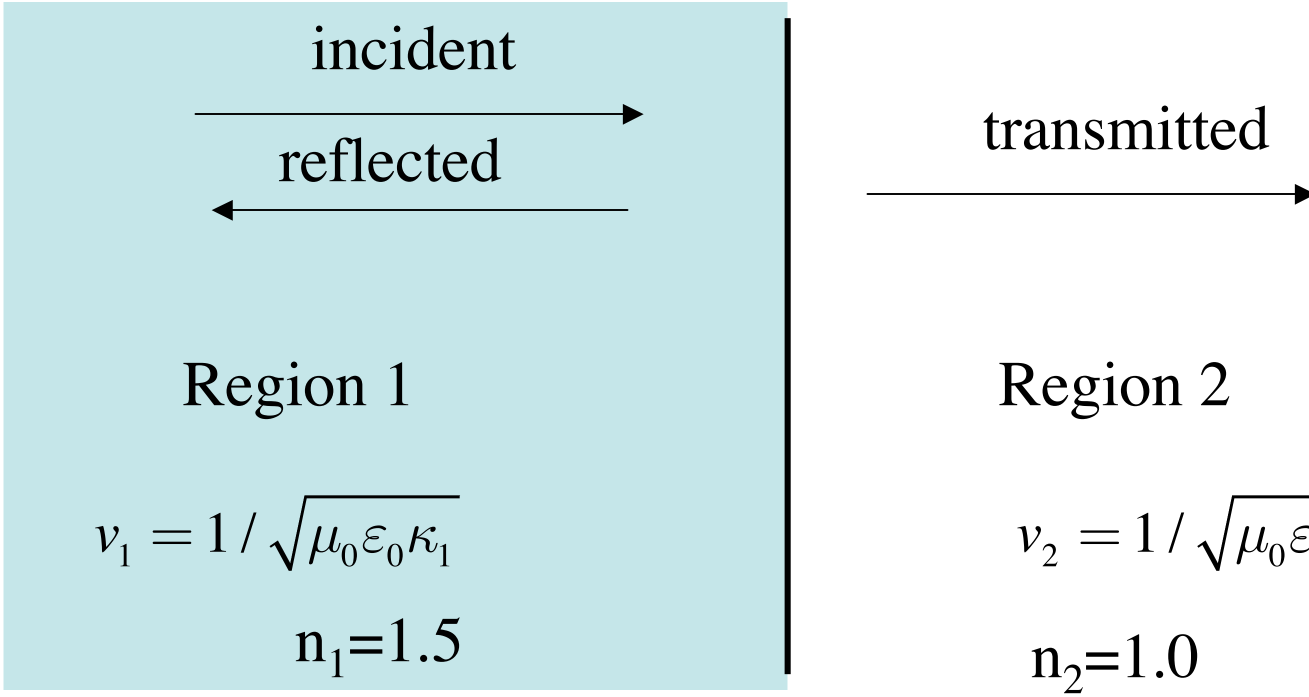
ratio of reflected to incident intensity

Suppose the wave was incident on the boundary from the glass side



n_1=1.5 n_2=1.0

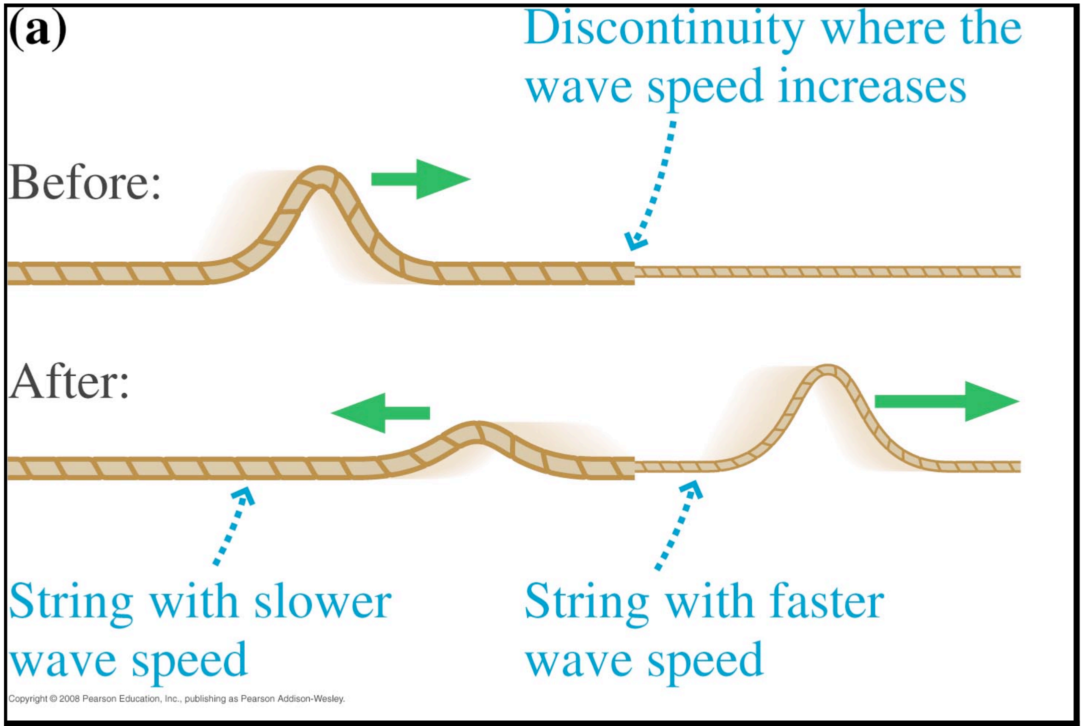
What is reflection coefficient? $\rho = \dfrac{v_2 - v_1}{v_2 + v_1} = \dfrac{n_1 - n_2}{n_1 + n_2}$



What is reflection coefficient?

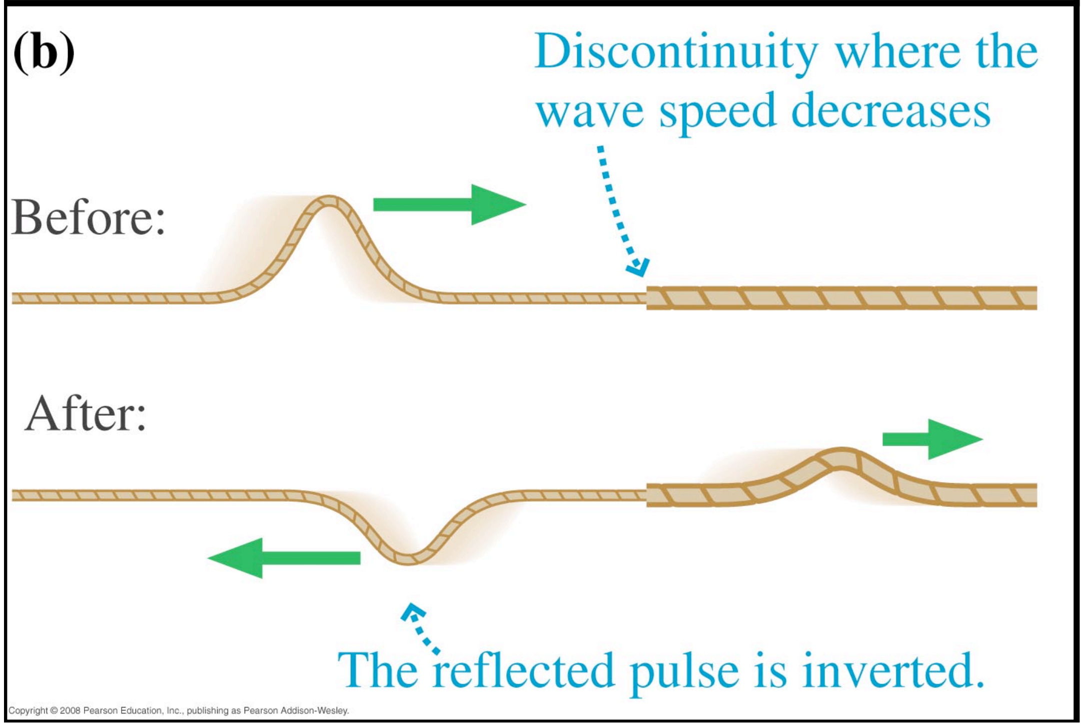
A. ρ=0
B. ρ =.2
C. ρ =-.2
D. ρ =5

$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2}$$



Region 2 has higher velocity

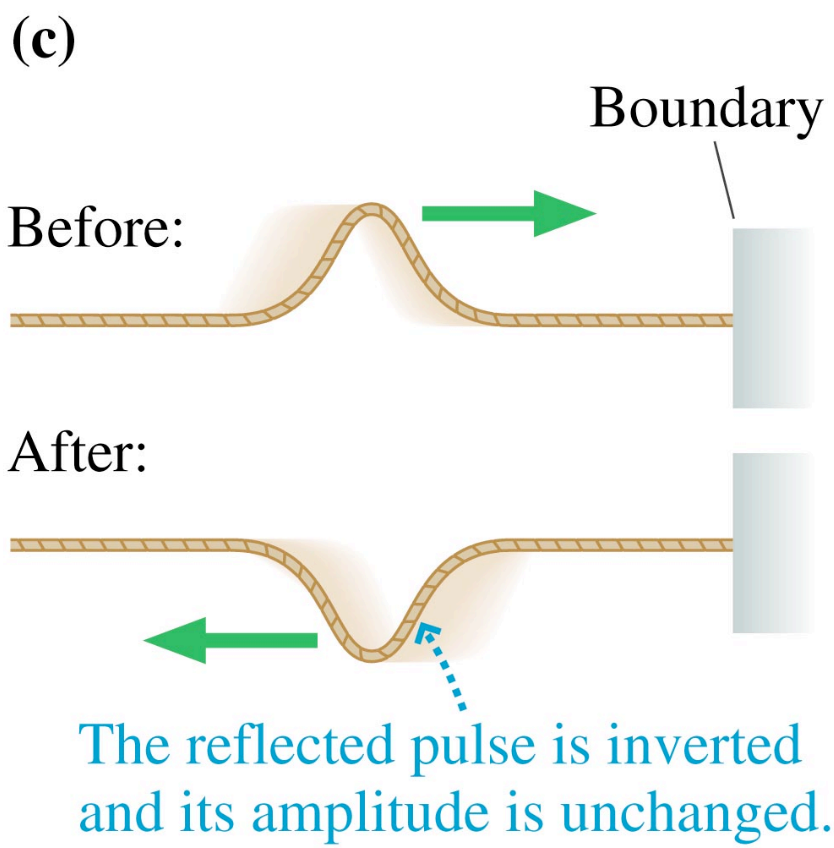
$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2} > 0$$



Region 2 has lower velocity

$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2} < 0$$

What if $\kappa_2 , n_2 \to \infty$

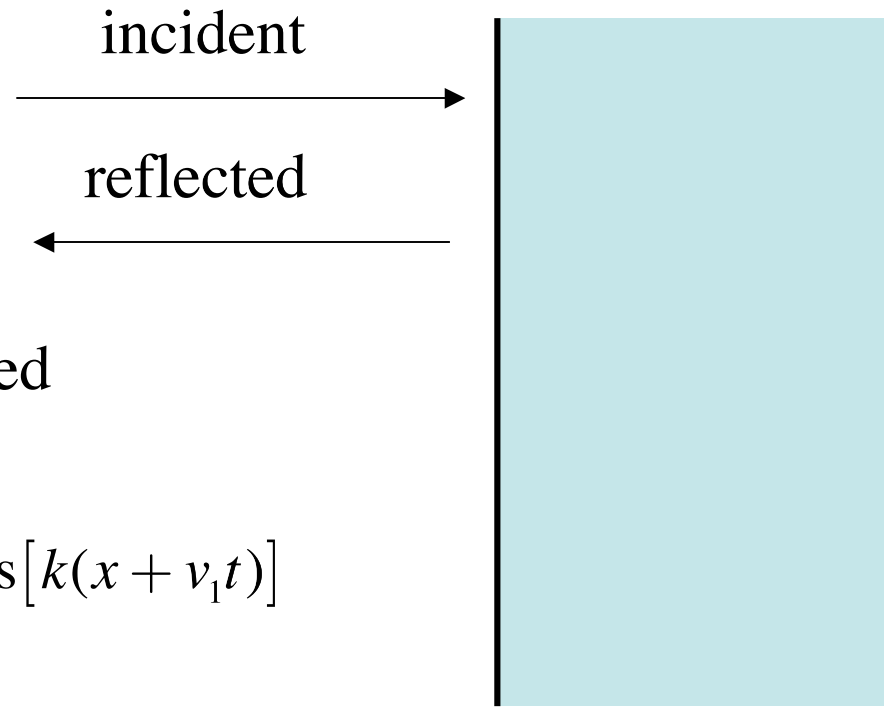


$$\rho = \frac{n_1 - n_2}{n_1 + n_2} \to -1$$

Same as for reflection from a conductor

Interference in 1 Dimension

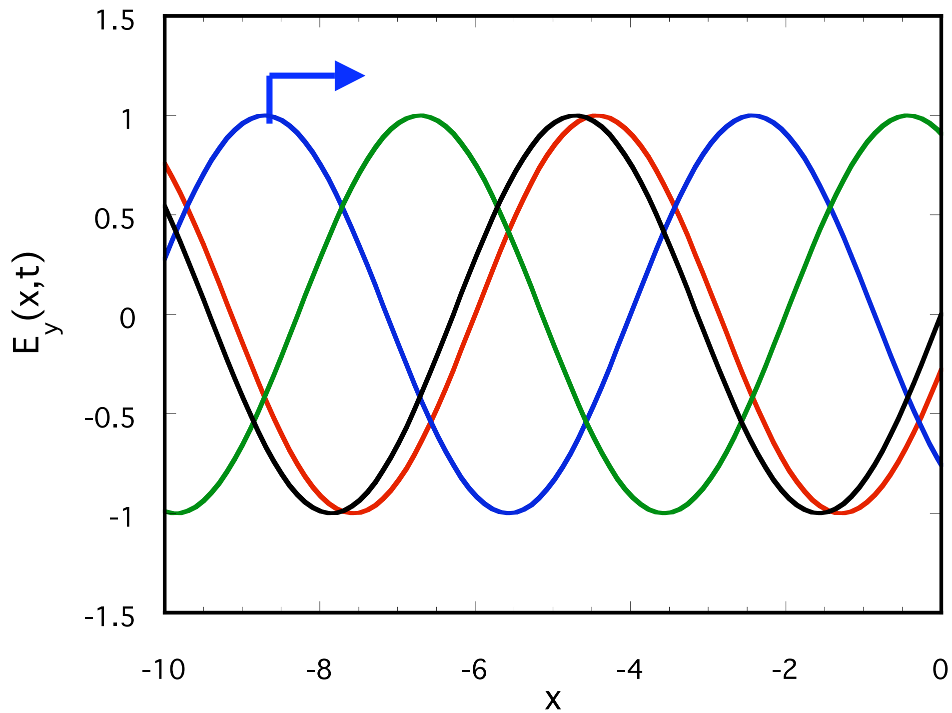
Incident and reflected fields add (superposition)



incident reflected

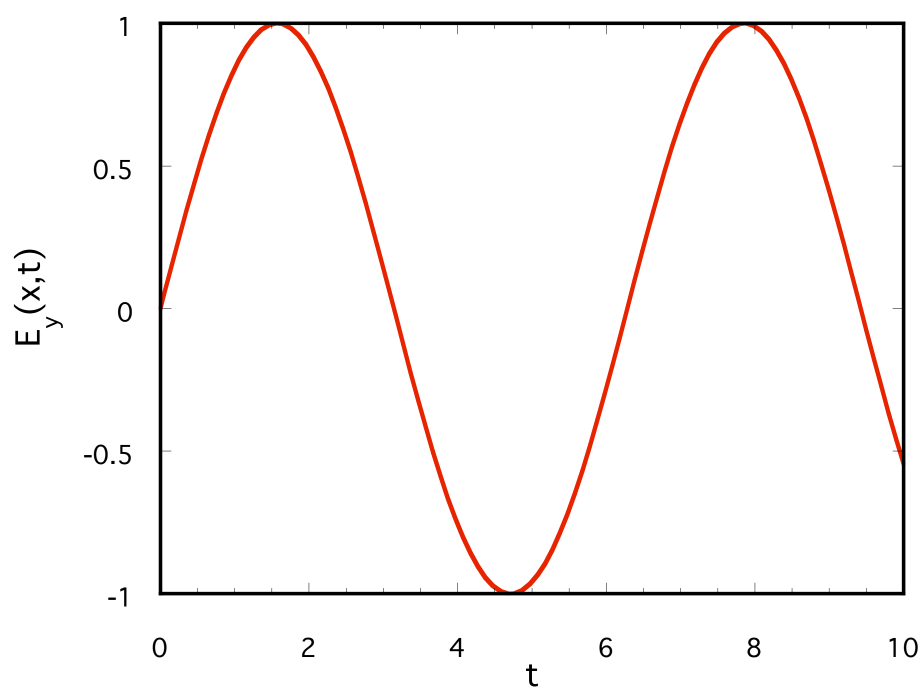
$$E_y(x,t) = E_0 \cos\left[k(x - v_1 t)\right] + \rho E_0 \cos\left[k(x + v_1 t)\right]$$

Incident and reflected waves will interfere, changing the peak electric field at different points



Case #1: no reflection ρ=0.
No interference,

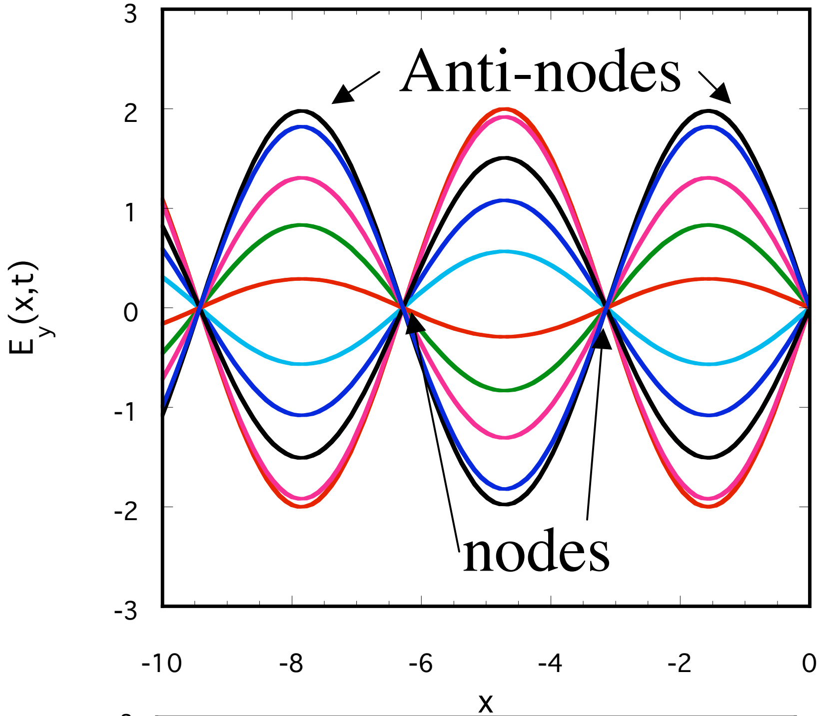
E plotted versus x for several values of t



E plotted versus t for a single value of x.

Same peak value independent of x

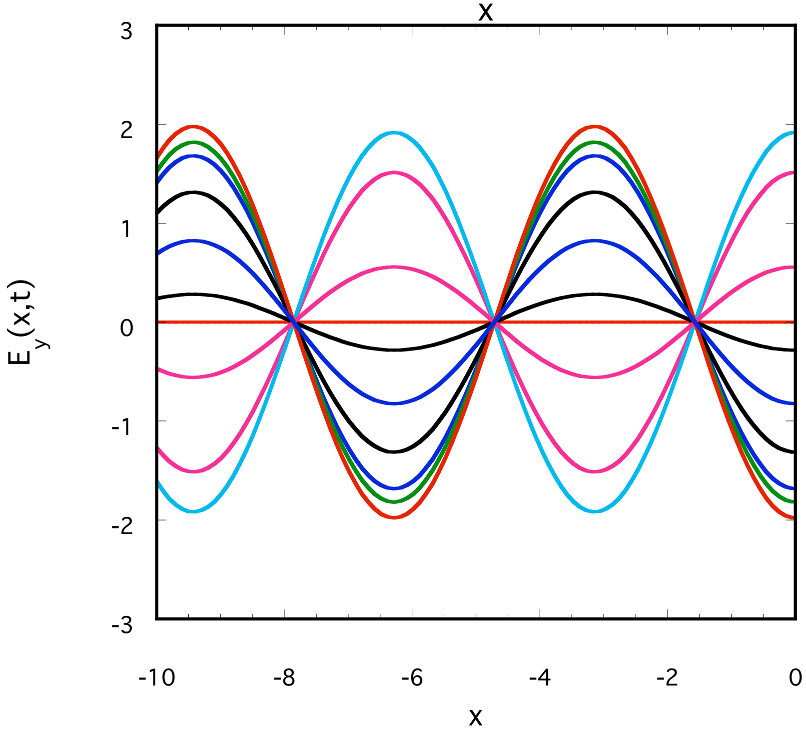
Traveling wave



Case #2: total reflection ρ= -1.

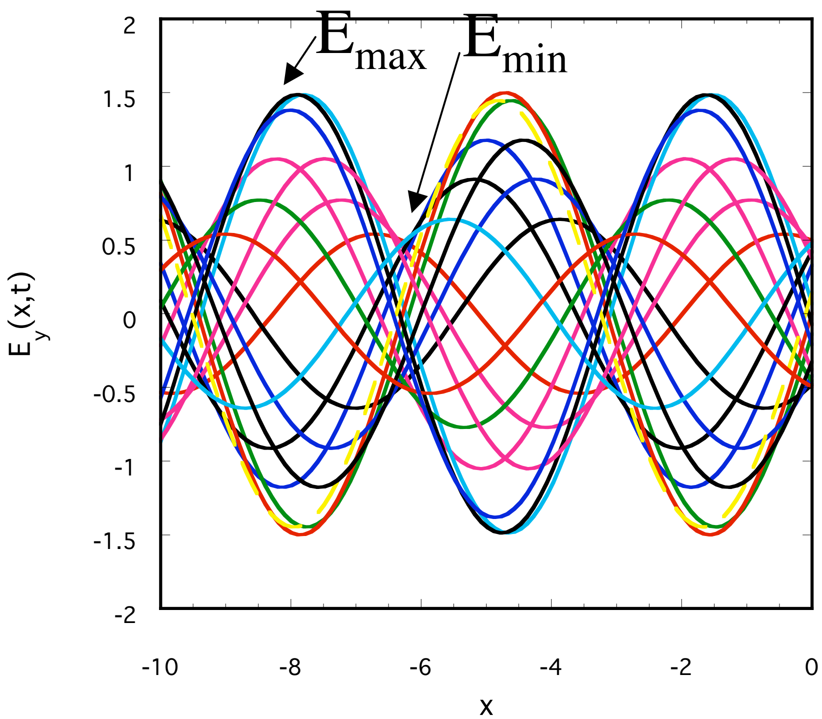
E plotted versus x for
several values of t

How far apart are the nodes?
Anti-nodes?
What is peak E?



Case #3: total reflection ρ= +1.

E plotted versus x for
several values of t



Case #4: partial reflection ρ= -0.5

E plotted versus x for several values of t

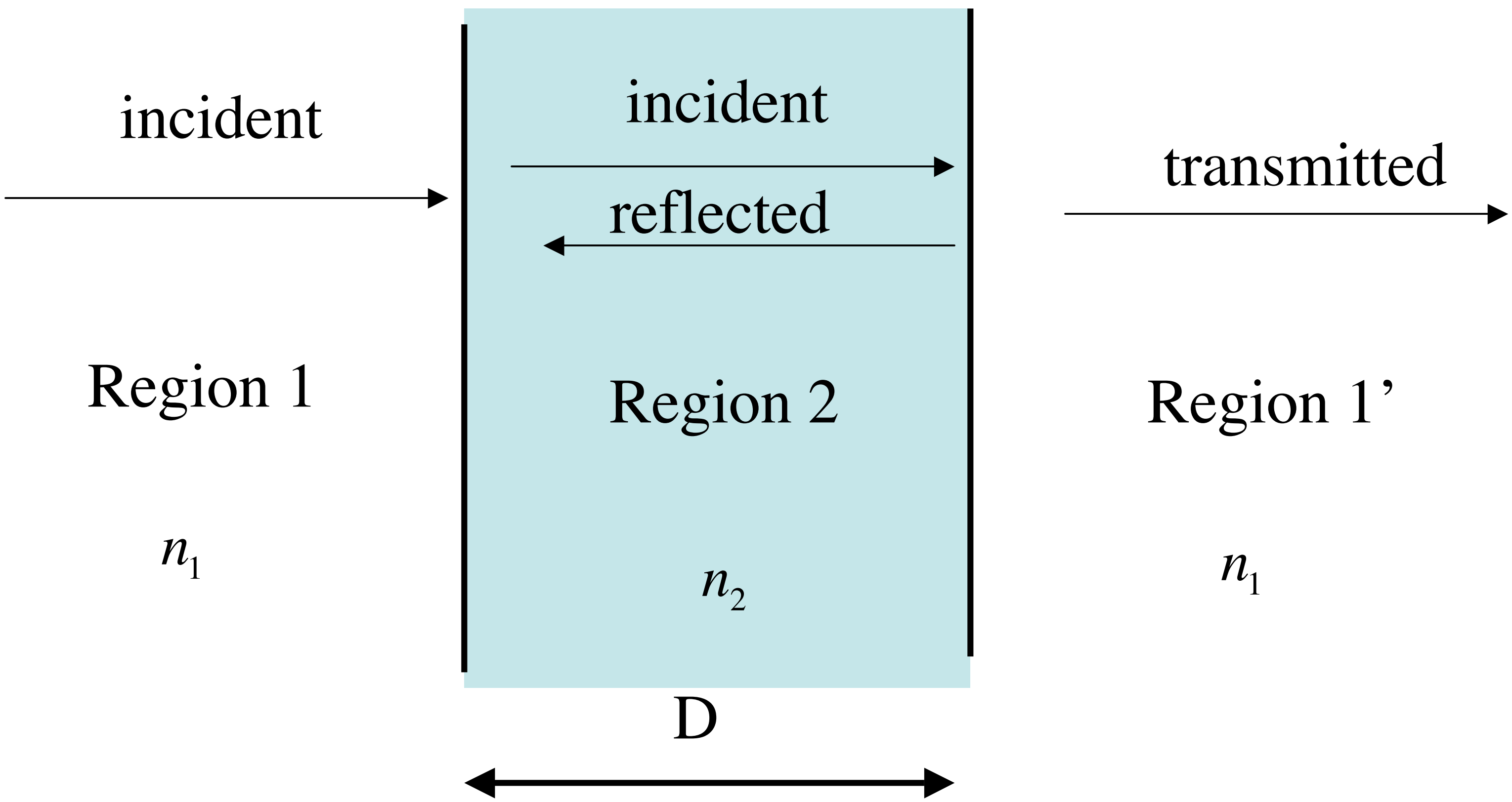
How far apart are the minima?
The Maxima?
What is peak E?

When reflection is not total there are still local maxima and minima.

$$\frac{E_{max}}{E_{min}} = \frac{1+|\rho|}{1-|\rho|} = \text{VSWR}$$

Voltage Standing Wave Ratio
Pronounced “vizwarr”

Half Wavelength Window Eliminates Reflection



$$D = m\lambda_2 / 2 = m\lambda_{Vac} / (2n_2)$$

When dielectric #2 is an integer number of half-wavelengths thick, no reflection

CVD diamond window

Manufactured by elementsix

http://www.e6.com/wps/wcm/connect/E6_Content_EN/Home

Summary

1. Waves are modified by dielectric constant of medium, κ.
2. All our Maxwell equations are valid provided we replace

$$\varepsilon_0 \rightarrow \varepsilon_0 \kappa$$

3. Speed of waves is lowered. (Index of refraction - *n*)

$$n = \frac{speed\ of\ light\ in\ vacuum}{speed\ of\ light\ in\ material} = \frac{c}{v_{em}} = \sqrt{\kappa}$$

4. Frequency of wave does not change in going from one medium to another. Wavelength does. $\lambda = \lambda_{vac} / n$

5. Waves are reflected at the boundary between two media. (Reflection coefficient)

$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2}$$

6. Reflected waves interfere with incident waves.

Distance between interference maxima/minima $\lambda / 2$

Ratio of maximum peak field to minimum peak field
(Voltage standing wave ratio)

$$\frac{E_{\max}}{E_{\min}} = \frac{1+|\rho|}{1-|\rho|} = \text{VSWR}$$

Solution of the Wave equation

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

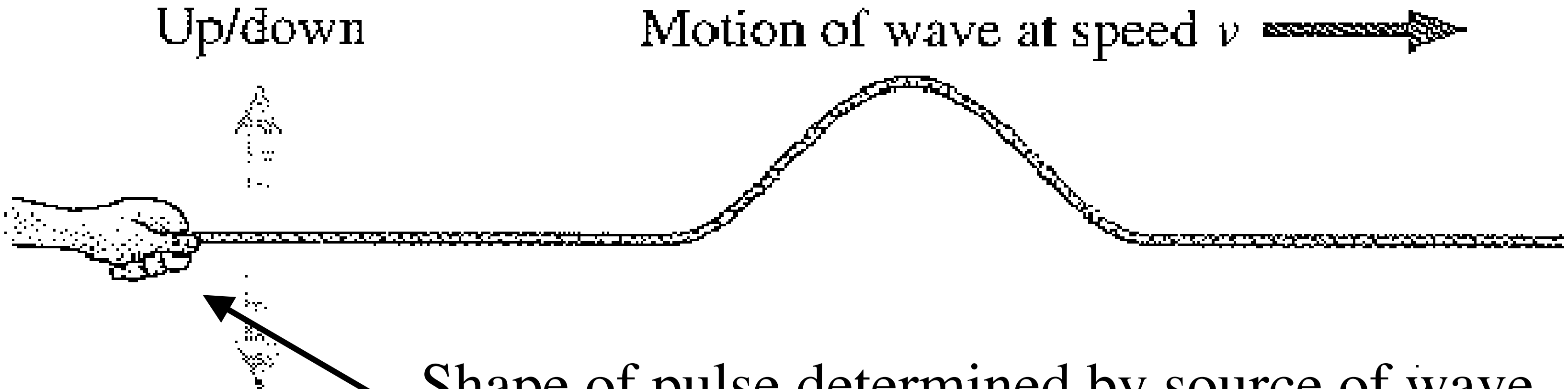
$$E_y(x,t) = f_+(x - v_{em}t) + f_-(x + v_{em}t)$$

Where $f_{+,-}$ are any two functions you like, and

$$v_{em} = 1/\sqrt{\mu_0 \varepsilon_0}$$

v_{em} is a property of space. $v_{em} = 2.9979 \times 10^8 \ m/s$

$f_{+,-}$ Represent forward and backward propagating wave (pulses). They depend on how the waves were launched



Shape of pulse determined by source of wave.

$$E_y(x,t) = f_+(x - v_{em}t)$$

Speed of pulse determined by medium

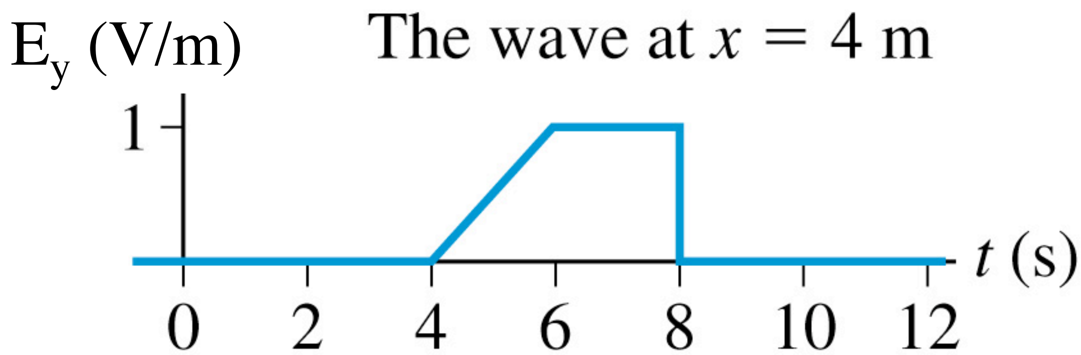
$$v_{em} = 1/\sqrt{\mu_0 \varepsilon_0}$$

What is the magnetic field of the wave?

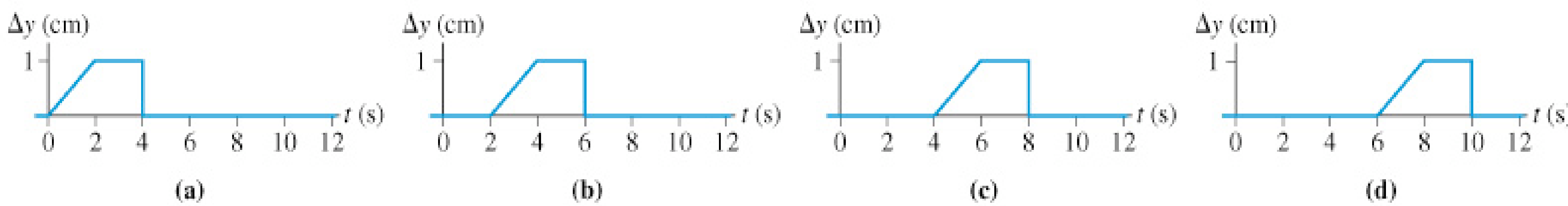
$$E_y(x,t) = f_+(x - v_{em}t) + f_-(x + v_{em}t)$$

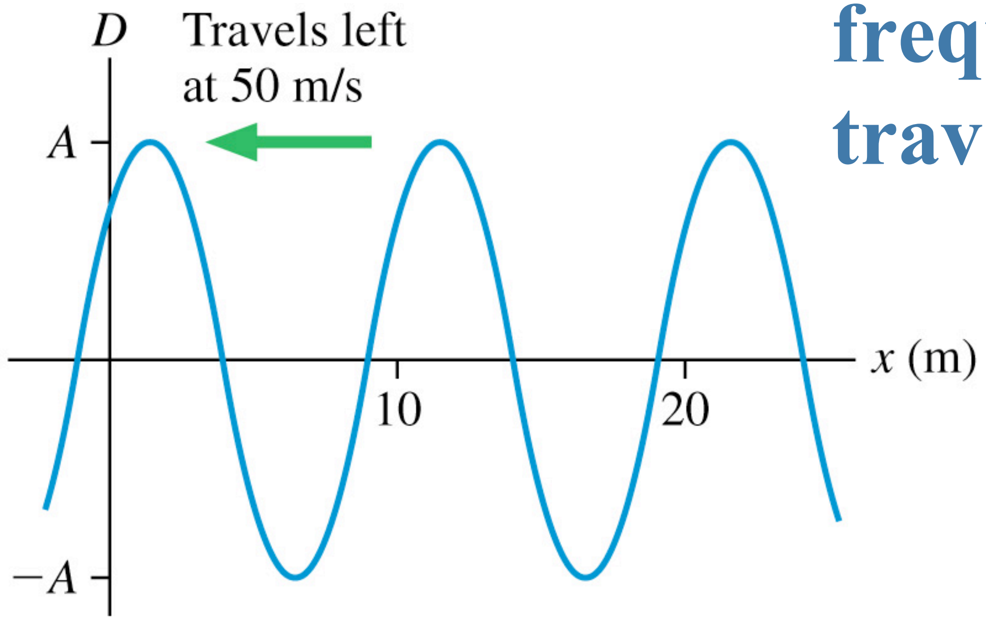
$$B_z(x,t) = \frac{1}{v_{em}}\left(f_+(x - v_{em}t) - f_-(x + v_{em}t)\right)$$

Notice minus sign



The graph at the top is the history graph at $x = 4$ m of a wave traveling to the right at a speed of 2 m/s. Which is the history graph of this wave at $x = 0$ m?





What is the frequency of this traveling wave?

A. 0.1 Hz

B. 0.2 Hz

C. 2 Hz

D. 5 Hz

E. 10 Hz

What is the phase difference between the crest of a wave and the adjacent trough?

A. 0

B. π

C. $\pi/4$

D. $\pi/2$

E. $3\pi/2$