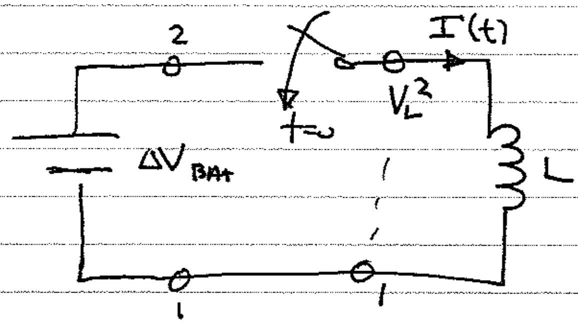
Inductors in Circuits

What happens when I connect an inductor to a Voltage source



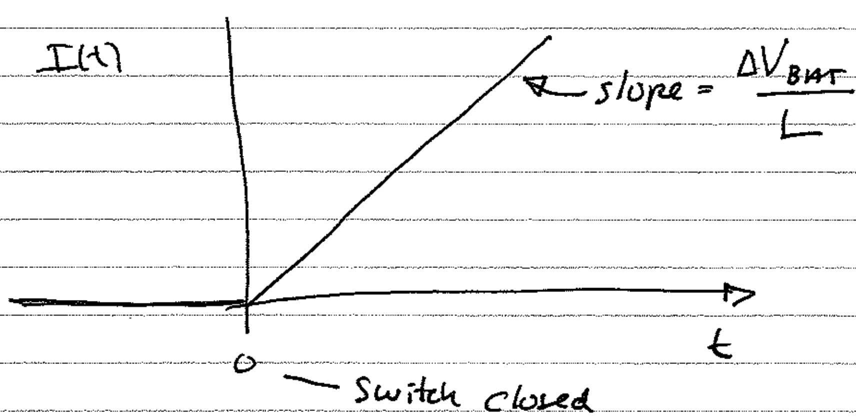
$$V_L = L\frac{dI}{dt}$$

Before switch is closed $I(t) = 0$. Once swith is closed $V_L = \Delta V_{BAT}$ Potential difference between is same for inductor and battery

$$\Delta V_{BAT} = L\frac{dI}{dt}$$

$$I(t) = I(0) + (\Delta V_{BAT}/L)\,t$$

initial current
in this case $I(0)=0$

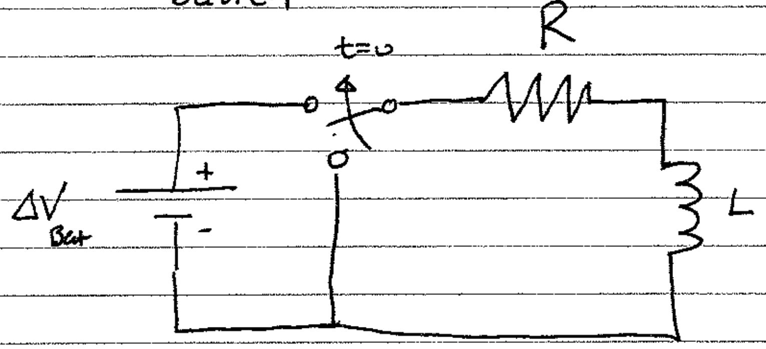


Can I open Switch?

Ans NO!

An L-R circuit

Consider the series connection of a resistor and an inductor with a Battery

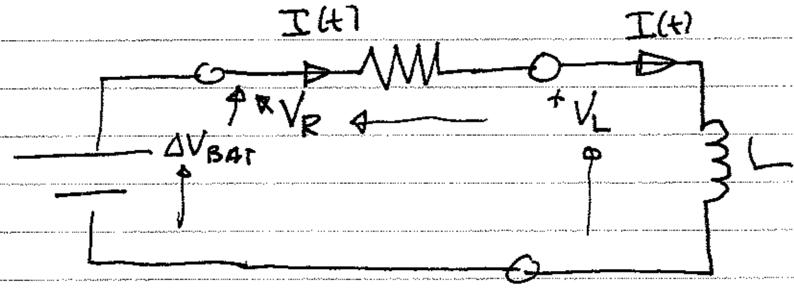


At $t=0$ the switch is closed.

Assume that just before the switch is closed

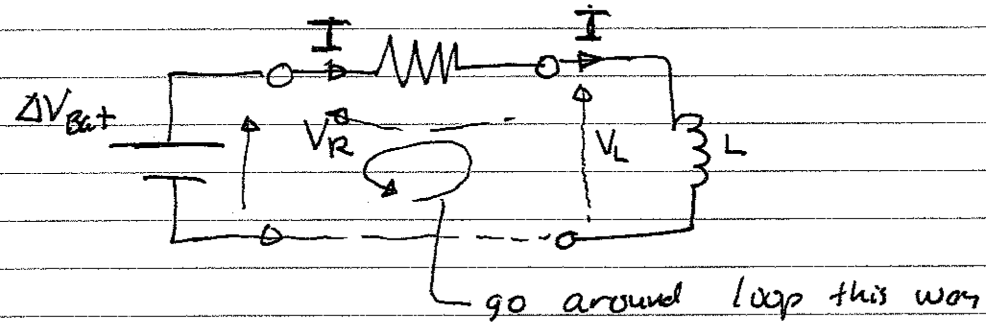
Why? All inductors have some resistance, battery too.

When Switch is closed



Same current entering R must also enter L. Continuity of charge.

What about Voltage



Arrows indicate direction in which voltage is defined for a 2-terminal device.

Kirchoff's Voltage Law: Sum of voltages around a closed loop is ZERO. (Sound familar?)

opposite arrows

$$V_L + V_R - \Delta V_{BAT} = 0$$

$$\boxed{L\frac{dI}{dt} + IR - \Delta V_{BAT} = 0}$$

differential equation determines $I(t)$

initial condition $I(0)=0$

This is a <u>linear differential</u> equation with <u>constant coefficients</u>

Linear: Only first power of unknown and it's derivatives appear (no I^2, I^3, etc.)

Constant Coefficients: L and R are not functions of time.

Solution:

Generally one can find a general solution by adding together ~~what~~ a "particular" solution and a homogeneous solution.

$$I(t) = I_p(t) + I_h(t)$$

The particular solution is any solution that satisfies

BC

$$L\frac{dI_p}{dt} + I_p R = \Delta V_{BAT}$$

The homogeneous solution satisfies

$$L\frac{dI_h}{dt} + I_h R = 0$$

You can verify that $I(t) = I_p(t) + I_h(t)$ satisfies the original equation.

Since $I_p(t)$ can be any solution let's pick a simple one

$$I_p = \frac{\Delta V_{Bat}}{R} = \text{const}$$

note: in this case $dI_p/dt = 0$ and particular equation is satisfied.

Now consider homogeneous equation

$$L\frac{dI_h}{dt} + RI_h = 0$$

This is satisfied by the decaying exponential

$$I_h(t) = A\, e^{-Rt/L}$$

where A is a constant. ~~So add~~

$\cancel{I(t) = \Delta V_{Bat}}$

So, adding particular & homogeneous

$$I(t) = I_p(t) + I_h(t) = \frac{\Delta V_{Bat}}{R} + A e^{-Rt/L}$$

Now, we pick A to satisfy the initial condition

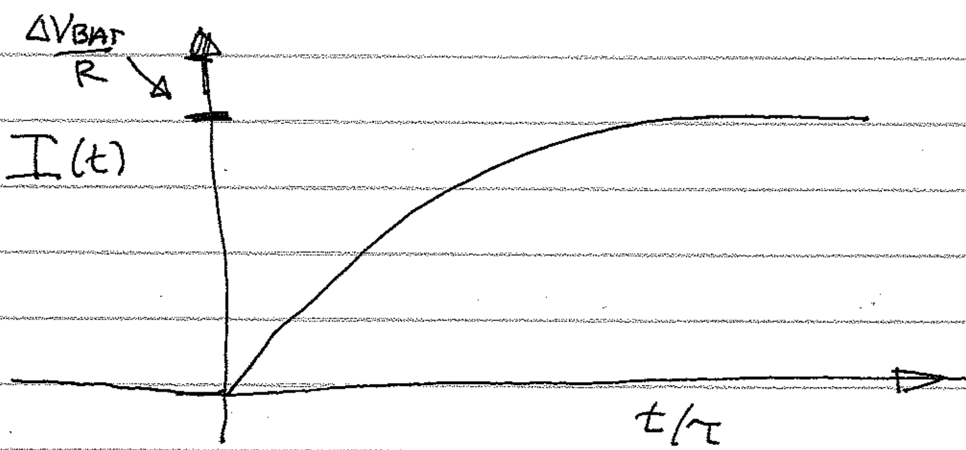
$$I(0) = 0 \qquad I(0) = \frac{\Delta V_{Bat}}{R} + A$$

Therefore $A = -\frac{\Delta V_{BAT}}{R}$

Solution:

$$I(t) = \frac{\Delta V_{Bat}}{R}\left(1 - e^{-t/\tau}\right)$$

where $\tau = (L/R)$



EXAMPLE: $\Delta V_{Bat} = 1.5\ V$

$R = 100\Omega$

$L = 0.1$ henries

$\tau = L/R = 0.1/100 = .001$ sec $= 1$ msec

$\Delta V_{Bat}/R = 1.5/100 = .015\ A = 15\,mA$

~~what is goo~~

$$I(t) = \frac{\Delta V_{Bat}}{R}\left(1 - e^{-t/\tau}\right)$$

Voltage across resistor

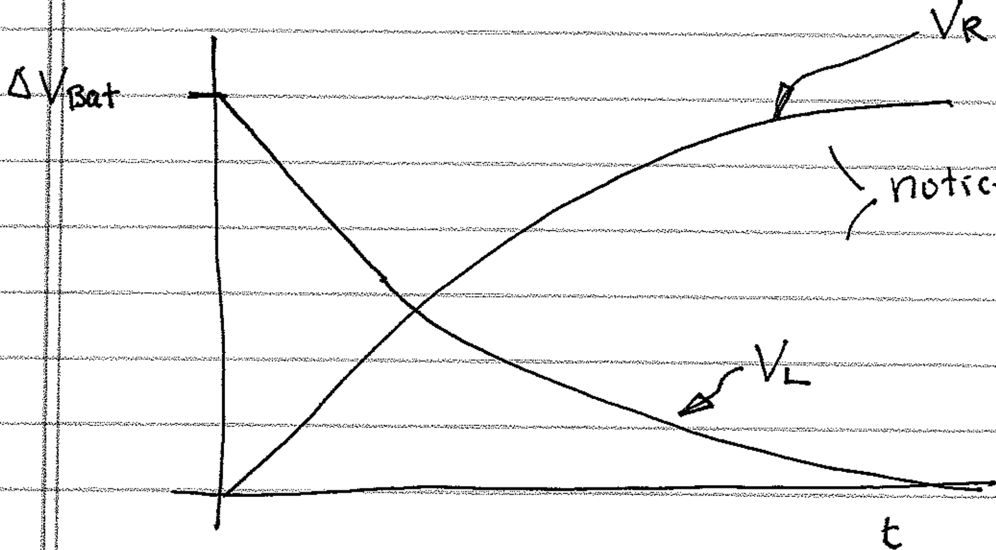
$$V_R = IR = \Delta V_{BAT}\left(1 - e^{-t/\tau}\right)$$

Voltage across inductor

$$V_L = L\frac{dI}{dt} = \frac{L\Delta V_{Bat}}{R}\frac{d}{dt}\left(1 - e^{-t/\tau}\right)$$

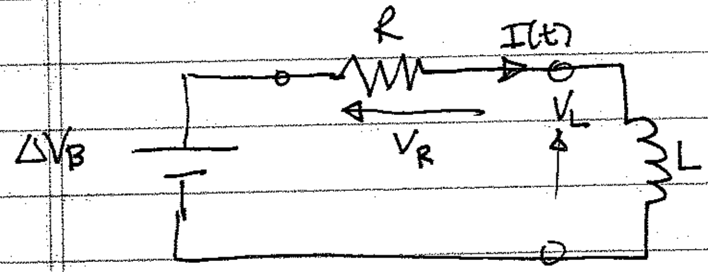
$$= \frac{L\Delta V_{Bat}}{R}(-1)\left(-\frac{1}{\tau}\right)e^{-t/\tau}$$

$$= \frac{L}{R\tau}\Delta V_{Bat}\,e^{-t/\tau} = \Delta V_{Bat}\,e^{-t/\tau}$$



notice $V_R + V_L = \Delta V_{BAT}$ as it shoul

What is going on?



Initially I is small (I can't change rapidly in an inductor)

I is small, so $V_R = IR$ is small.

All the voltage ΔV_{Bat} is applied to the inductor

$$V_L \simeq \Delta V_{Bat}$$

I(t) then begins to increase linearly in time. As I(t) rises so does $V_R = IR$

This means V_L must drop. Since V_L drops rate of change of current

$$\frac{dI}{dt} = \frac{1}{L} V_L$$

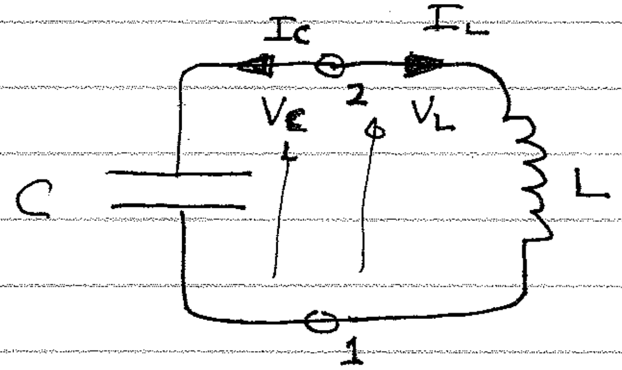
must drop.

V_L drops to zero in several $\tau = L/R$ times. Once V_L is nearly zero.

I(t) stops changing.

In steady state $I(\infty) = \dfrac{\Delta V_{Bat}}{R}$

34.9 The LC- circuit



If we label the two terminals the same so that there is a common VOLTAGE

$$V_L = V_c = V(t)$$

Then we must draw the current arrows on the inductor and capacitor pointing in opposite directions

Kirchoff's current Law tells us

$$I_c + I_L = 0$$

OR $\quad I_c = -I_L$

SO:

$$V(t) = L\frac{dI_L}{dt} \qquad \text{Inductor Law}$$

$$I_c = C\frac{dV(t)}{dt} \qquad \text{Capacitor Law}$$

differentiate capacitor law with respect to time

$$\frac{dI_c}{dt} = C\frac{d^2V(t)}{dt^2} = -\frac{dI_L}{dt} = -\frac{V(t)}{L}$$

SO

$$\frac{d^2V(t)}{dt^2} = -\frac{1}{LC}V(t)$$

This is a second order, linear differential equation with constant coefficients

It requires two initial conditions
These correspond to the initial charge on the capacitor and the initial current in the inductor.

$$Q(0) = CV_c(0) = CV(0)$$

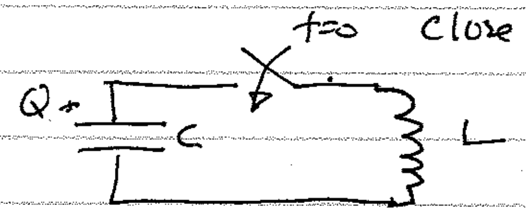
$$I_L(0) = -I_c(0) = -C\left.\frac{dV}{dt}\right|_0$$

THUS: $V(0) = \frac{Q(0)}{C}$

$$\frac{dV(0)}{dt} = -\frac{I_L(0)}{C}$$

Let's suppose ~~the~~ for simplicity $I_L(0) = 0$

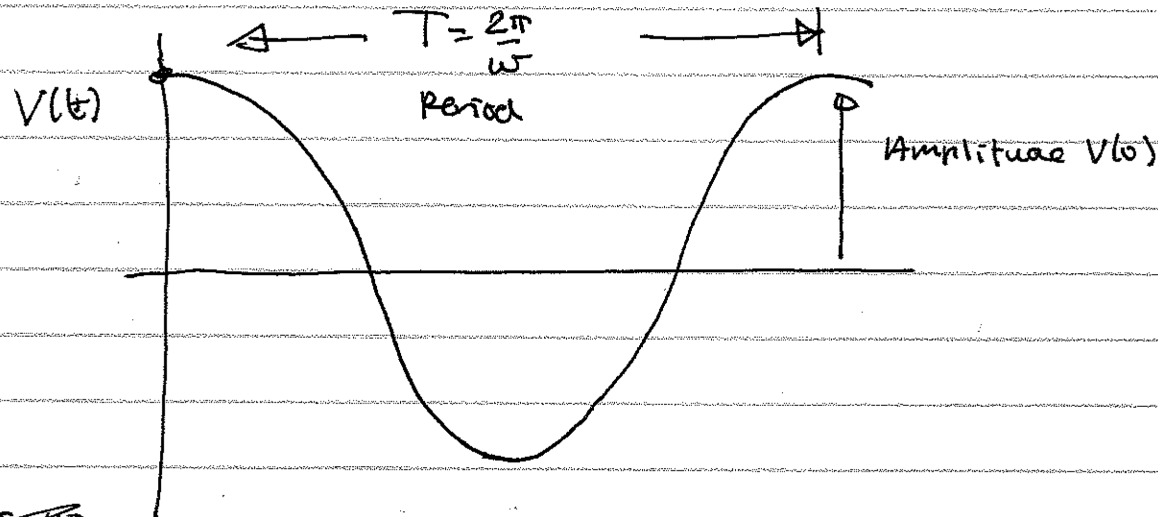
Charge capacitor



Solution for $V(t)$

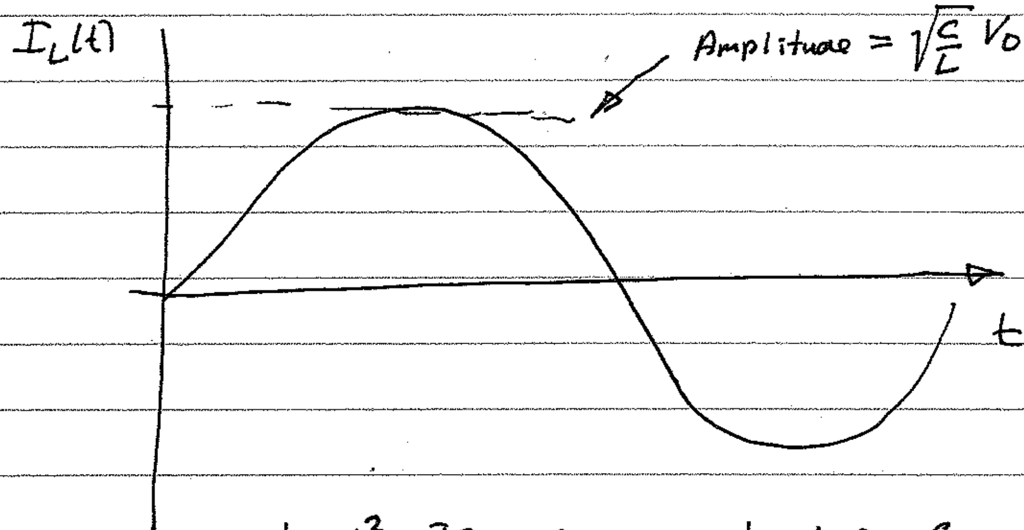
$$V(t) = V(0)\cos\omega t \quad \longleftarrow \text{Verify}$$

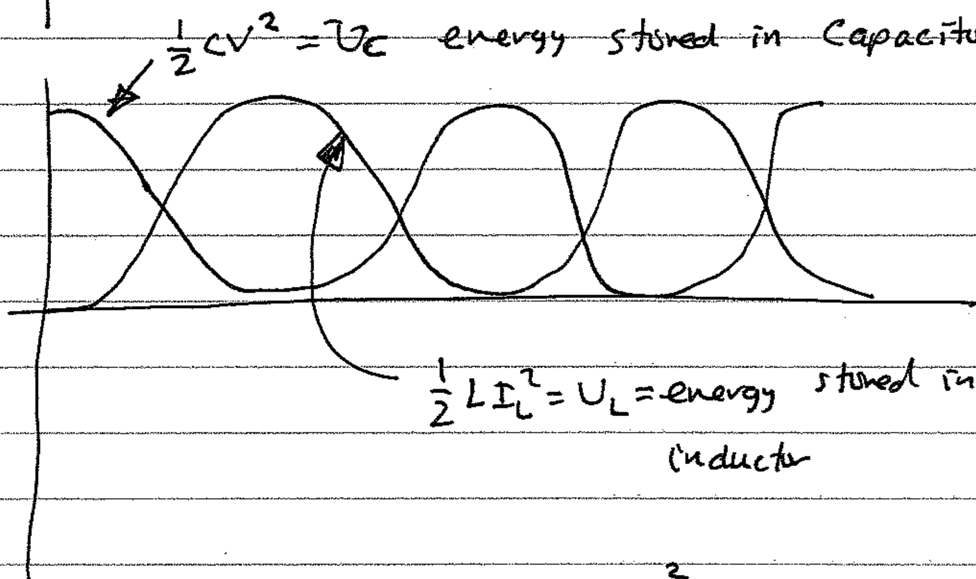
where $\omega = 1/\sqrt{LC}$



~~$I_L(t)$~~

$$I_L(t) = -C\frac{dV}{dt} = C\omega V_0 \sin\omega t = \sqrt{\frac{C}{L}}\, V_0 \sin\omega t$$





$$U_C + U_L = \text{const} = \frac{1}{2}CV(0)^2$$