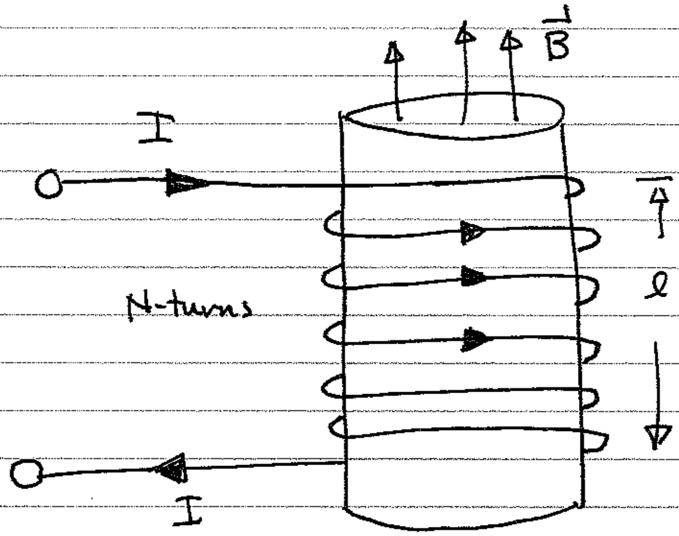
FIND THE INDUCED ELECTRIC FIELD INSIDE A SOLENOID. EXAMPLE 34.9

Consider a solenoid with N turns

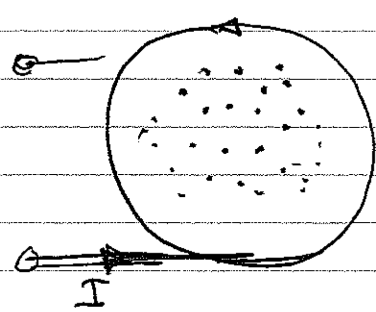


Put your right thumb in direction of I. Fingers give direction of $\vec{B}$ (up inside)

According to our previous calculation using Ampere's Law

$$|\vec{B}| = \frac{\mu_0 I N}{l}$$

VIEW FROM ABOVE



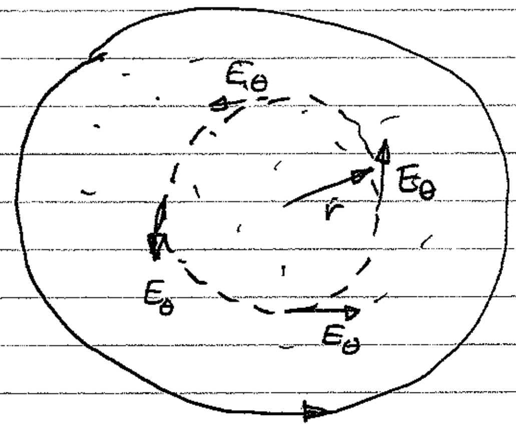
$\vec{B}$ out of page

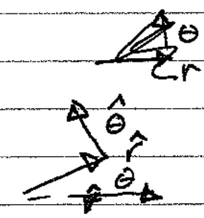
Calculate induced $\vec{E}$-field as a function of r

$\vec{B}$ - out of page

$\vec{B} = B_z \hat{k}$

Consider a loop of radius r





Q: Which direction is $\vec{E}$? $+\hat{\theta}$ or $-\hat{\theta}$

Ans: We don't know. Is B increasing or decreasing?

Call component of $\vec{E}$ in $\hat{\theta}$ direction $E_\theta(r,t)$

Integrate around loop in CCW direction (increasing θ)

$$\oint \vec{E}\cdot d\vec{s} = E_\theta(r,t)\underbrace{(2\pi r)}_{\text{circumference of circle}} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

By RHR $d\vec{A}$ is out of page. Why? put fingers in direction of $d\vec{s}$ i.e. $\hat{\theta}$. Thumb gives direction of $\vec{A}$

$$\int \vec{B}\cdot d\vec{A} = B_z \pi r^2$$

$$2\pi r E_\theta(r,t) = -\pi r^2 \frac{\partial B}{\partial t}$$

$$\boxed{E_\theta = -\frac{r}{2}\frac{\partial B_z}{\partial t}}$$

Now, is Lenz's Law satisfied?

Ans: yes. If B_z increases in time a negative E_θ is induced. If there were a conducting loop at radius r a current in the clockwise direction would be induced. A clockwise current would resist the increase of B_z.

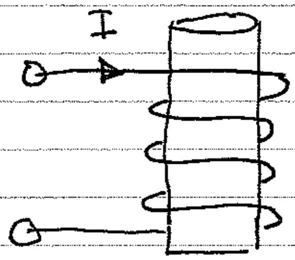
Inductor — Engineering sign conventions

Inductor is basicaly a coil of wire.

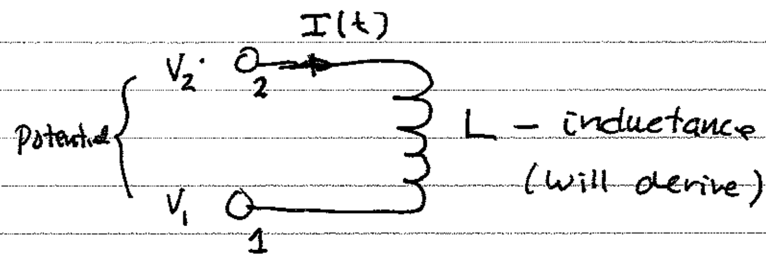
(However, any conductor has some inductance.

It's just usually negligible)

COIL



Electrical Symbul



It is a 2-terminal device. Let's label the terminals 1 & 2 (Soon we'll stop doing this.)

$$\boxed{V_2 - V_1 = V_L = L\, dI/dt}$$

Let's label the current flowing into terminal 2 as I(t). This is just a label as I could be positive or negative

Let's call V_L the potential difference between terminals 2 and 1

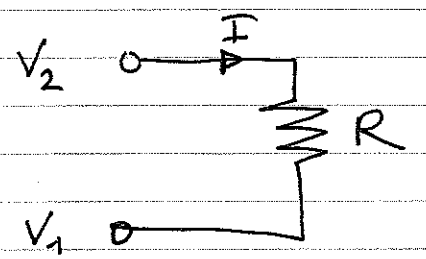
$$V_L = V(2) - V(1) = L\, dI/dt$$

Engineering Convention

If you label the current flowing in one terminal of a device as $I(t)$, then the voltage across the device is the potential at the terminal where the current arrow goes in minus the potential where the current arrow would come out.

If done this way there are no spurious minus signs.

For example consider a resistor

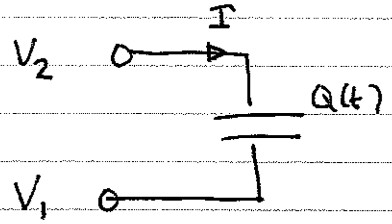


$$V_R = (V_2 - V_1) = IR$$

OHM'S LAW

$$V_R = IR$$

Example #2 a capacitor



$$Q(t) = C(V_2 - V_1)$$
$$= C V_c$$

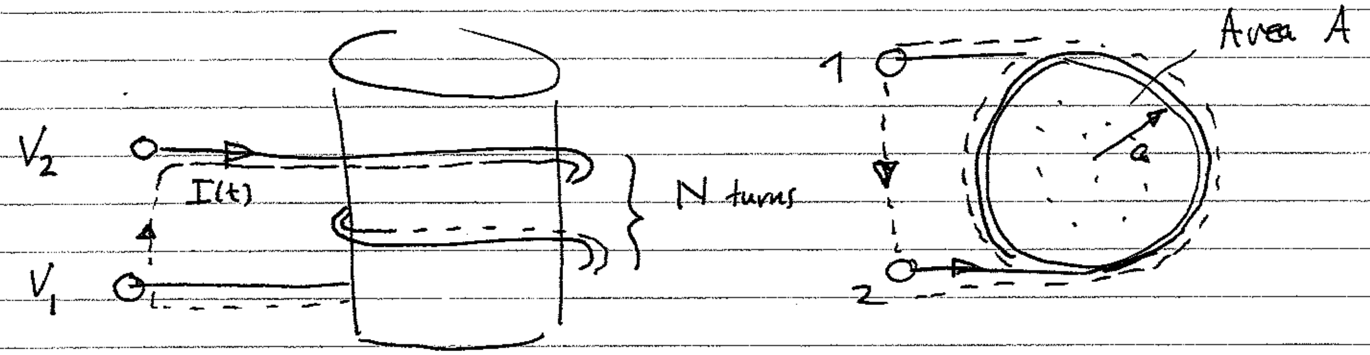
$$\frac{dQ(t)}{dt} = I(t) = C\frac{dV_c}{dt}$$

$$\boxed{I(t) = C\frac{dV_c}{dt}}$$

FINALLY INDUCTOR (will show)

$$\boxed{(V_2 - V_1) = V_L = L\frac{dI}{dt}}$$

Voltage across an Inductor



Side view Top view

Evaluate $\oint \vec{E} \cdot d\vec{s}$ around the dotted closed loop

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} N \Phi$$

$$\Phi = \underbrace{\pi a^2}_{A} B_z \quad \text{flux through } A$$

and $B_z = \dfrac{\mu_0 N I}{\ell}$ – solenoid B

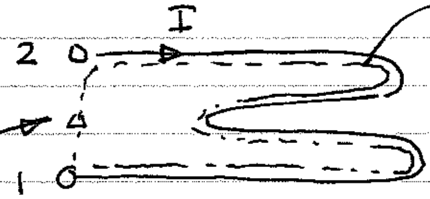
So

$$\oint \vec{E} \cdot ds = -\frac{\pi a^2 N^2 \mu_0}{\ell} \frac{dI}{dt}$$

Call $\dfrac{\pi a^2 N^2 \mu_0}{\ell} = L$ inducance

units Henry

But wait



most of dotted loop is parallel to a good conductor. $\vec{E}\cdot d\vec{s} = 0$ on that portion of the dotted Loop!

1 all the contribution to $\int \vec{E} ds$ comes from this portion

So: $$\int_1^2 \vec{E}\cdot d\vec{s} = -(V(2)-V(1)) \quad \text{definition of potential}$$

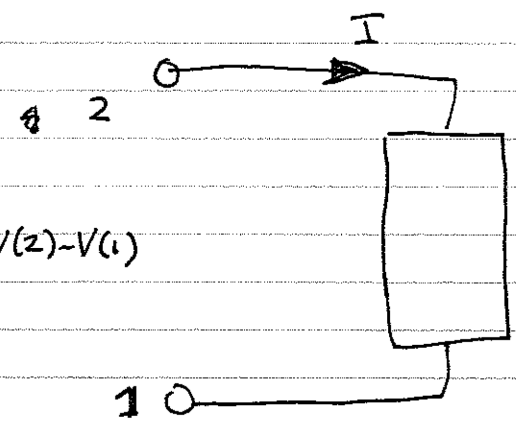
$$= -L\frac{dI}{dt}$$

THEREFORE

$$V_L = V(2)-V(1) = L\frac{dI}{dt}$$

where I is current into terminal #2

POWER and ENERGY IN INDUCTORS



TWO TERMINAL DEVICE

$V = V(2) - V(1)$

Using the electrical engineering convention, the power delivered to a 2-terminal device is

$$P = (V(2) - V(1))I = VI$$

IF THE DEVICE IS A RESISTOR THE POWER DELIVERED TO THE RESISTOR IS

$$P_R = VI \qquad \text{use} \qquad V = IR$$

$$P_R = I^2 R > 0$$

power can only be delivered to a resistor

POWER DELIVERED TO AN INDUCTOR

$$P_L = VI \qquad \text{use} \qquad V = L\,dI/dt$$

$$P_L = L I \frac{dI}{dt} = L \frac{d}{dt} \frac{I^2}{2}$$ ← check

Power may be positive or negative depending on whether I^2 is increasing or decreasing

The energy stored in the inductor is the time integral of the power.

$$U_L = \int_{t_1}^{t_2} dt\, P_L(t) = \int_{t_1}^{t_2} dt\, \frac{L}{2} \frac{d}{dt} I^2(t) = \frac{L}{2} I^2(t) \Big|_{t_1}^{t_2}$$

energy delivered to the inductor during the time interval $t_1 < t < t_2$

Suppose we pick t_1 such that $I(t_1)=0$

Then $U_L = \frac{L}{2} I^2(t_2)$

THIS IS THE ENERGY STORED in the inductor

This Energy is said to be stored in the magnetic field.

Return to our solenoid example

$$L = \frac{(\pi a^2) N^2 \mu_0}{\ell}$$

$$U_L = \frac{\pi a^2 N^2 \mu_0}{2\ell} I^2$$

NOW WRITE U_L in terms of $B = \frac{\mu_0 N I}{\ell}$

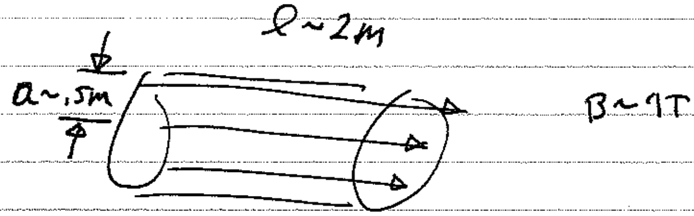
$$U_L = \frac{\pi a^2 \ell}{2} \frac{B^2}{\mu_0} = \frac{1}{2} \text{Volume} \frac{B^2}{\mu_0}$$

This suggests that there is an energy density associated with the magnetic field

energy densith

$$U_L = \text{Volume}\, u_B \qquad \text{where} \quad u_B = \frac{B^2}{2\mu_0}$$

What is the energy stored in the magnetic field of an MRI machine



Volume $= \pi a^2 l = \pi (.5)^2 \cdot 2 = 1.57\ m^3$

$\mu_0 = 4\pi \times 10^{-7}$

$U = \frac{1.57}{2} \frac{1^2}{(4\pi \times 10^{-7})} = 6.25 \times 10^5\ J$

$=$ 1 hair dryer * 625 sec

$=$ 1 hairdryer * 10 minutes