Chapter 33 Executive Summary Phys270

Moving charges exert forces on each other that augment (add to) Coulomb's Law

This can be viewed as a two-step process:

1. Moving charges (or equivalently electrical currents) create a magnetic field,

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2},$$

Here $\vec{\mathbf{v}}$ is the velocity of the moving charge.

This is the analog to the expression for the electric field due to a point charge.

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2}$$

Superposition applies if a number of moving charges are present just as in the case of electric fields. As in the case of electric fields, most of your grief will come from trying to apply the principle of superposition.

2. A charge moving through a magnetic field feels a force (called the Lorenz force) given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}.$$

The total electric and magnetic force on a moving charged particle is the sum of the contributions due to electric and magnetic fields,

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}).$$

Just to be confusing this is sometimes called the Lorenz force too.

There are a host of consequences of 1. and 2., which will be explored in this chapter. An example is **parallel currents attract** and **antiparallel currents repel**.

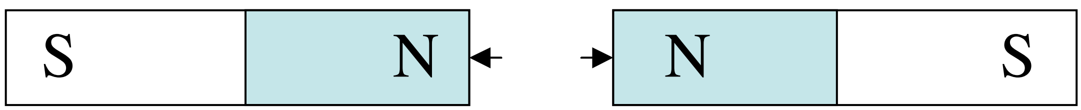
A **big** complication when discussing magnetic fields is the appearance of **vector cross products**. Don't think that you can slide by without learning how to evaluate them.

Some History

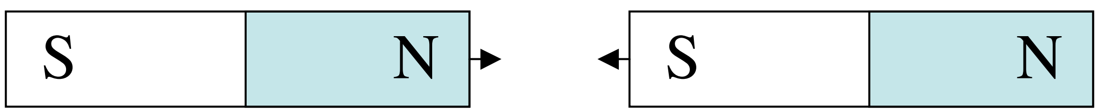
Magnetic materials have been known for over 2000 years. The first compasses (made in China) appeared about 1000 AD. Around the same time Norwegian Vikings navigated to North America without the use of compasses (I just though I'd through that out there.). The connection between electricity and magnetism was discovered by Oersted (a Dane) in 1819.

Magnets have "poles" labeled north and south:

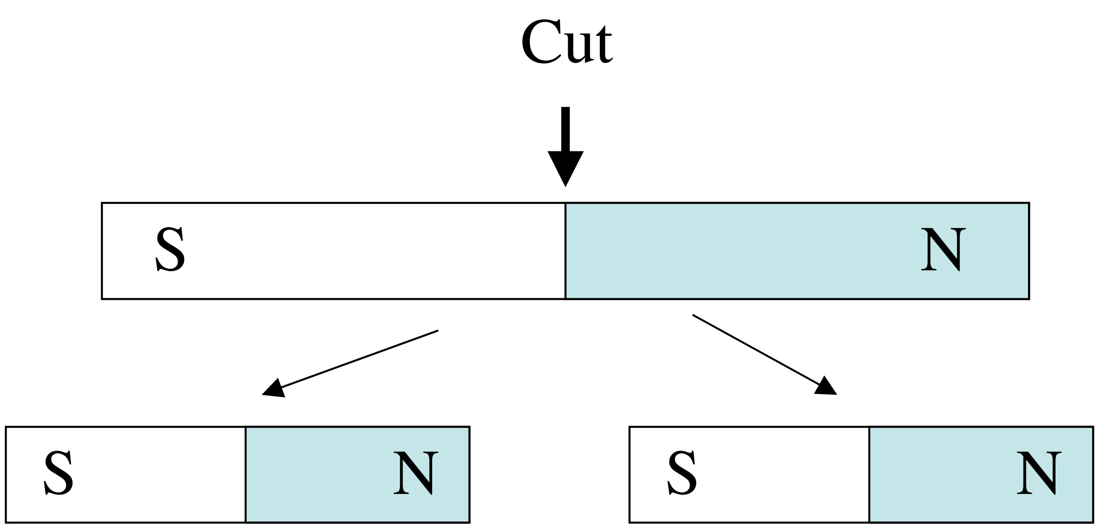
Like poles repel



Unlike poles attract

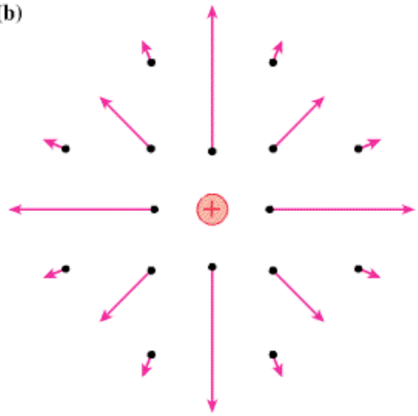


If you cut a magnet in half, both halves will have a north and south pole.

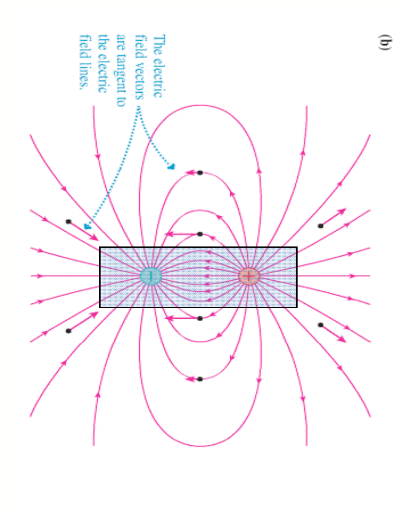


Magnets produce fields, which have a distribution unlike that of a point charge.

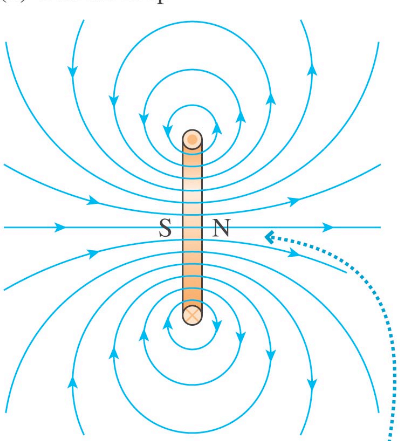
(b)



Rather the field distribution around a magnet is like that of an electric dipole.

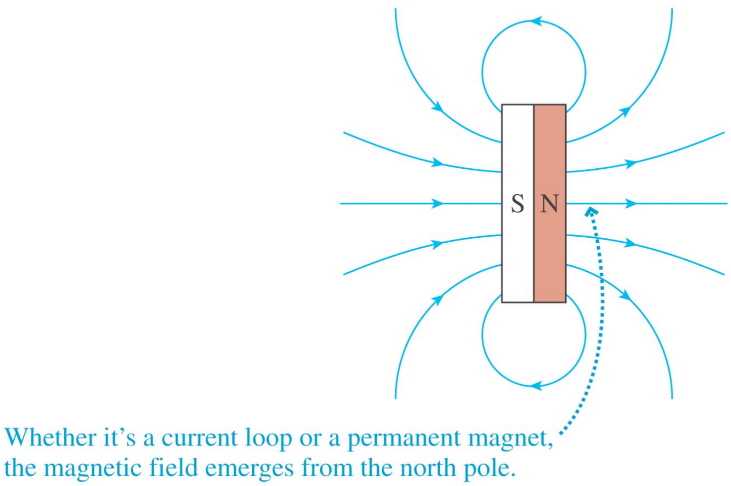


(a) Current loop



Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

(b) Permanent magnet



33.7 Force on a moving Charge

For a stationary charge we have

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}(\vec{\mathbf{r}})\,.$$

For a moving charge there is an additional contribution, known as the Lorenz force,

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})\,.$$

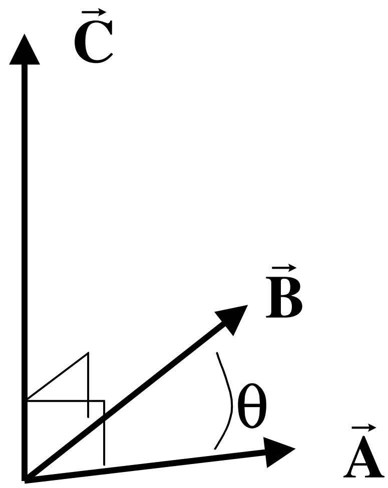
Here, $\vec{\mathbf{v}}$ is the velocity of the moving charge.

Some facts:

1. For a particle at rest, $\vec{\mathbf{v}} = \mathbf{0}$ the magnetic field exerts no force on a charged particle.
2. The Lorenz force is proportional to the charge, the magnetic field strength and the particle's velocity.

3. The vector force is the result of the vector cross product of velocity and magnetic field.

Aside on cross products: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$



Let θ be the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, **always chose this to be less than 180°**.

The magnitude of $\vec{\mathbf{C}}$ is given by $\left|\vec{\mathbf{C}}\right| = \left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|\sin\theta$

The direction of $\vec{\mathbf{C}}$ is perpendicular to both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, and determined by the "right hand rule"

<u>Right hand rule:</u> put the fingers of your **right hand** in the direction of $\vec{\mathbf{A}}$, then rotate them through the angle θ (less than 180°) to the direction of $\vec{\mathbf{B}}$. Your thumb now indicates the direction of $\vec{\mathbf{C}}$.

The order of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is important: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$

If $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel, then *(θ=0)* $\vec{\mathbf{C}} = \mathbf{0}$.

Use of unit vectors and components

While you may be comfortable with the preceding description of the vector cross product, there will be instances where it is too difficult to use. For example if you are given two arbitrary vectors in component form, then how do you find the direction that is mutually perpendicular to both of them? In these instances it is much easier to evaluate the cross product using basis vectors and components. Let's say we are given $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ in component form in Cartesian coordinates.

$$\begin{aligned}\vec{\mathbf{A}} &= A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \\ \vec{\mathbf{B}} &= B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\end{aligned}.$$

How do we find the components of $\vec{\mathbf{C}}$?

$$\vec{\mathbf{C}} = C_x \hat{\mathbf{i}} + C_y \hat{\mathbf{j}} + C_z \hat{\mathbf{k}} .$$

Answer:

$$\begin{aligned} C_x &= A_y B_z - A_z B_y \\ C_y &= A_z B_x - A_x B_z \qquad \text{(my favorite)} \\ C_z &= A_x B_y - A_y B_x \end{aligned}$$

which is the same as taking the pretend determinant

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} .$$

I prefer the component form above. It's easy to remember, the first three components of each line are in x-y-z order, provided that you remember that x comes again after z, viz. z-x-y and y-z-x.

Example #1
Example #2

Motion of a charged particle in a magnetic field

Newton's law: $m\vec{\mathbf{a}} = \vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

$$m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

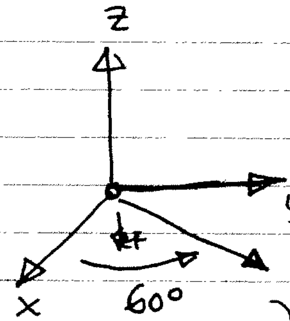
Note: Lorentz force is always perpendicular to velocity. There fore the magnitude of velocity will not change. This implies the kinetic energy of the particle is constant,

$$\frac{d}{dt}KE = \vec{\mathbf{v}} \cdot \vec{\mathbf{F}} = \mathbf{0} , \qquad KE = \frac{m}{2}|\vec{\mathbf{v}}|^2$$

Note also, if velocity is parallel to magnetic field the force is zero and vector velocity is constant.

Tesla

Example #



$\vec{B} = 1T$ in the y-direct

$\vec{V} = 10^6$ m/sec in x-y plan making an angle 60° wrt x

Q: What is magnitude and direction of $\vec{F}$

$$\vec{F} = q\,\vec{V}\times\vec{B}$$

electron $q = -1.6\times10^{-19}$ C

$\vec{V}\times\vec{B}$ will be in the z direction

$|\vec{V}\times\vec{B}| = |\vec{V}||\vec{B}|\sin\theta$ what is θ?

$\theta = 30^\circ$ $\sin\theta = 1/2$

$$\vec{F} = \underbrace{-1.6\times10^{-19}}_{q} * 10^6\,\frac{m}{s} * 1T * \frac{1}{2}\,\hat{k} = -0.8*10^{-13}\,N\,\vec{k}$$

~~$\vec{F}$ zer~~d

Example # 2

$$\vec{V} = 3\hat{i} + \cancel{4}5\hat{j} + \cancel{8}7\hat{k} \quad m/s$$

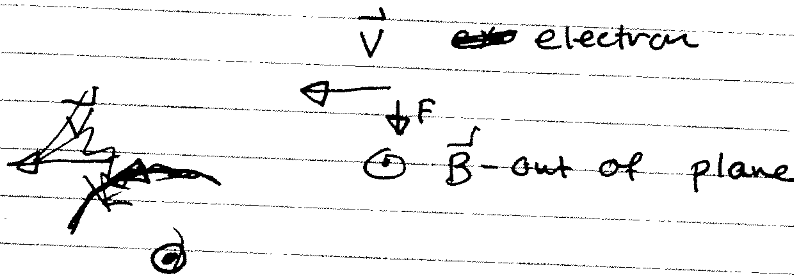
$$\vec{B} = 1\hat{i} + 2\hat{j} + 4\hat{k} \quad T$$

what is $\vec{V}\times\vec{B}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 2 & 4 \end{vmatrix} = \hat{i}(5\cdot4-2\cdot7) + \hat{j}(\cancel{2}7\cdot1-3\cdot4) + \hat{k}(3\cdot2-5\cdot1)$$

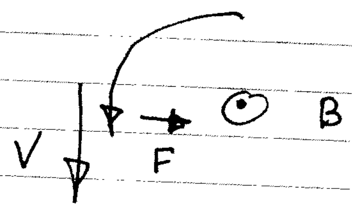
$$= 6\hat{i} \cancel{4} -5\hat{j} + \hat{k}$$

Motion in a uniform magnetic field



what direction is $\vec{V}\times\vec{B}$ (ans up)
what direction is $\vec{F}$ (ans down)

later



what direction is $\vec{V}\times\vec{B}$ (ans left)
" " " F (right)

Electron will execute circular ~~Part~~ motion in ccw direction

Q: What if proton? A: circular motion in CW direction

Q: What if $\vec{B}$ points into plane A: $\begin{cases} \text{ccw} \to \text{cw} \\ \text{cw} \to \text{ccw} \end{cases}$

since of rotation changes

What is the rotation rate?

~~From~~ For circular motion $|\vec{a}| = \frac{V^2}{R}$ ← radius of circle

$$|\vec{F}| = q|\vec{V}\vec{B}| = m|\vec{a}| = m\frac{V^2}{R} \qquad \Omega = \frac{V}{R}$$

Rotation Rate $|\Omega| = \left|\frac{qB}{m}\right|$ cyclotron frequency (independent of $|\vec{V}|$)

Suppose $B = 6\,T$

what is Ω for an electron

$m_e = 9.11 \times 10^{-31}$ kg $\qquad |q| = 1.6 \times 10^{-19}$ C

$$\Omega = \frac{qB}{m} = \frac{\cancel{1.6} \; 1.6 \times 10^{-19} \times 6}{9.11 \times 10^{-31}} = 1.05 \times 10^{12} \text{ Rad/sec}$$

$$f = \frac{\Omega}{2\pi} = 167 \; \overset{\times 10^{9}}{\cancel{GHz}} \; Hz = 167 \text{ GHz}$$

$$\lambda = c/f = 1.8 \times 10^{-3} \text{ m.}$$

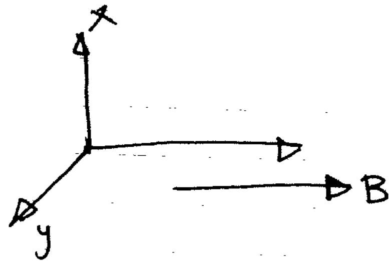
- frequency of microwave oven $\quad f = 2.45$ GHz

~~R~~ Radius of orbit $\quad R = \dfrac{V}{\Omega}$

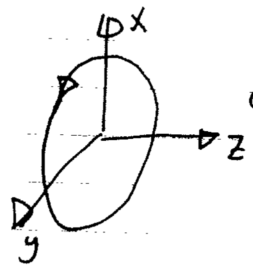
Suppose $\quad V = 0.2 * c \qquad c = 3 \times 10^{8}$ m/sec
speed of light

$$R = \frac{0.2 * 3 \times 10^{8} \text{ m/s}}{1.05 \times 10^{12} \text{ Rad/sec}} = 5.71 \times 10^{-5} \text{ m}$$

What is motion when $V_z \neq 0$



when $V_z = 0$ $V_x, V_y \neq 0$



Circular motion

freq $= \Omega = qB/m$

Linear motion

When $V_z \neq 0$ $V_x, V_y = 0$

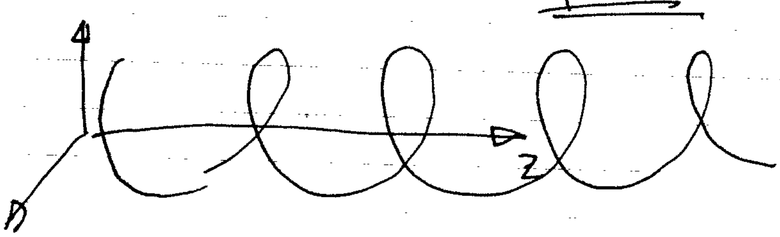
$$\frac{d\vec{V}}{dt} = q\,\vec{V} \times \vec{B}$$ is a linear equation (one power of $\vec{v}$)

if $\vec{V}_1(t)$ is a solution corresponding to Circular motion

and $\vec{V}_2(t)$ is a solution correspondin to linear motin

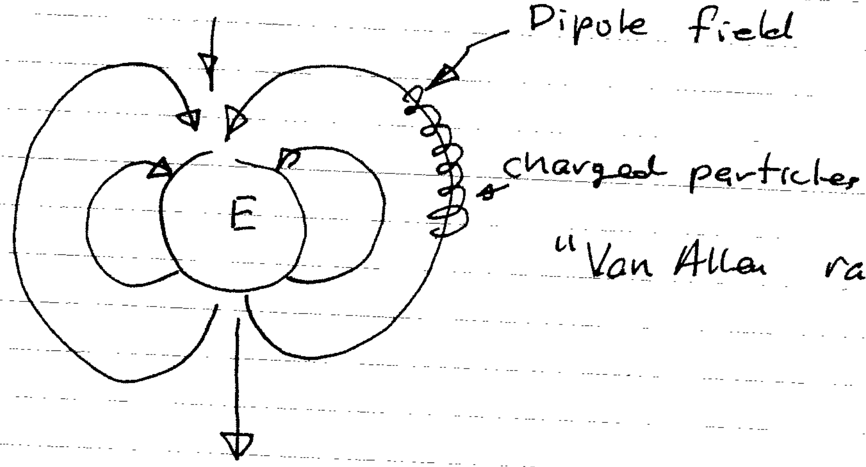
Then $\vec{V}_3(t) = \vec{V}_1(t) + \vec{V}_2(t)$ is also a solution

<u>Spiral</u>



5

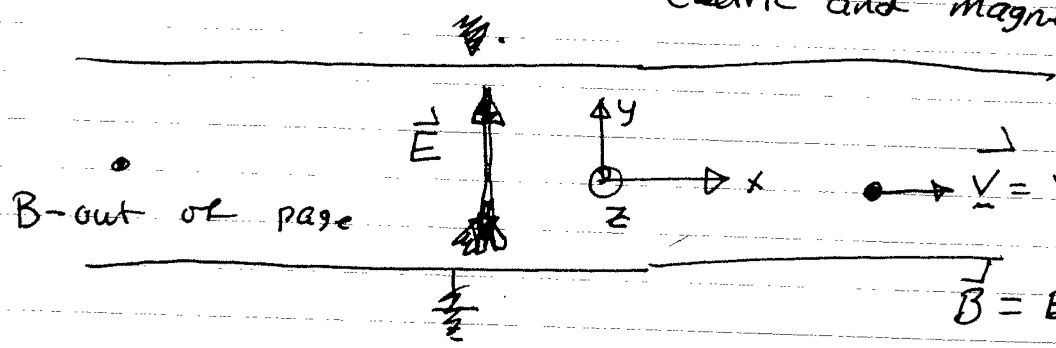
Example Earth is surrounded by plasma (magneto sphere)



"Van Allen radiation belt"

Sometimes ~~part~~ charged particles strike the poles "Aurora"

Motion of a charged particle in crossed electric and magnetic fields



Suppose

$$\left.\begin{aligned} \underline{V} &= V_x \hat{\underline{i}} \\ \vec{B} &= B_0 \hat{\underline{k}} \\ \vec{E} &= E_y \hat{j} \end{aligned}\right\} \text{try these}$$

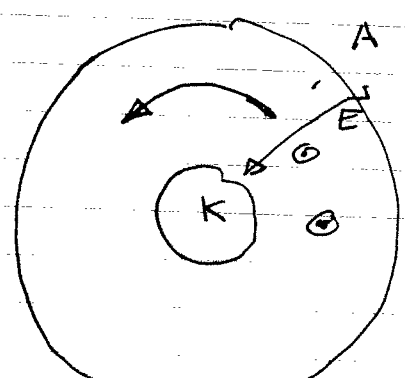
$$m\frac{d\vec{V}}{dt} = q\left[E_y\hat{j} + V_x B_0\, \hat{i}\times\hat{k}\right] = \hat{j}\, q(E_y - V_x B_0)$$

$$\hat{i}\times\hat{k} = -\hat{j}$$

if $V_x = E_y/B_0$ then total force is zero

Particle will move with constant velocity in a direction perpendicular to both $\vec{E}$ & $\vec{B}$.

Cylindrical Version



* $\vec{B}$ out of plane what is direction of motion

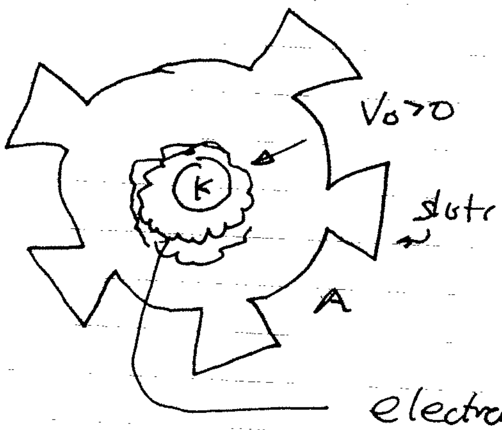
* have we left anything out?

yes radial acceleration

* makes a good HW Problem

Q: Where would you encounter such a thing?

A: On your kitchen counter — magnetron powers a microwave oven



electrons spiral out from cathode, potential energy is converted to radiation.

$f \doteq 2.45$ GHz (determined by slots)

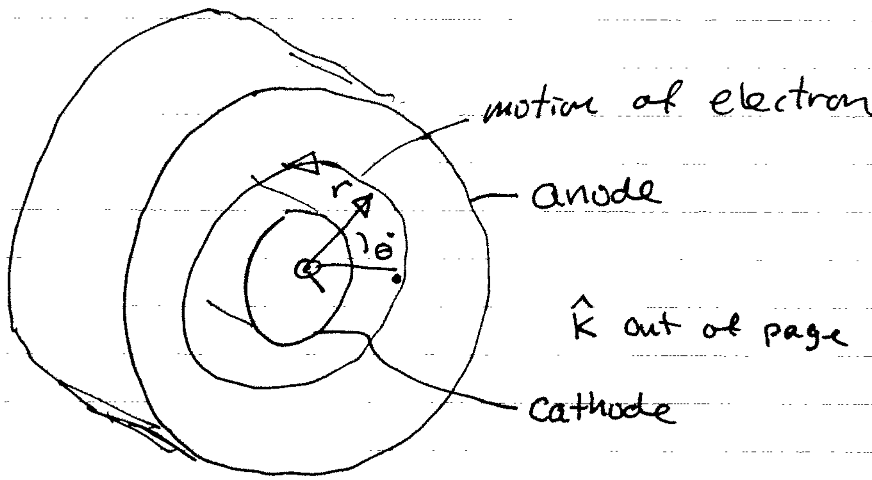
~ 70% efficient

\$12 / unit

Power ~ 1 kW

Never put your cat in a microwave oven.

Motion in crossed electric and magnetic fields



$\vec{B} = B_0 \hat{k}$ $\qquad$ $\vec{E} = \hat{r}\left(\frac{\lambda}{2\pi\epsilon_0 r}\right)$ field due to a line charge

Suppose $\vec{V} = V_\theta \hat{\theta}$ circular motion

$$m\frac{d\vec{V}}{dt} = q\left[\hat{r}E_r + V_\theta B_0 \underbrace{(\hat{\theta}\times\hat{k})}_{\hat{r}}\right]$$

centripetal accelleration

$$\frac{d\vec{V}}{dt} = -\frac{V_\theta^2}{r}\hat{r}$$ find V_θ

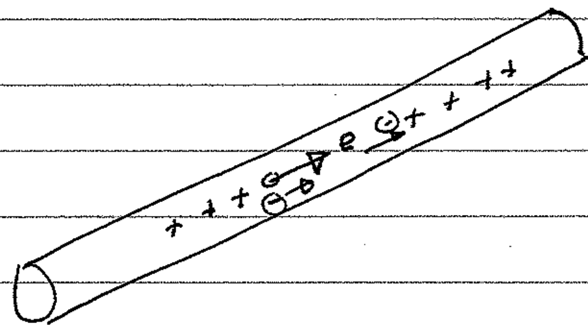
$$-\frac{mV_\theta^2}{r} = q\left(\frac{\lambda}{2\pi\epsilon_0}\frac{1}{r} + V_\theta B_0\right)$$

Could solve for rotation rate

~~SOLVE FOR $V_\theta = \Omega r$~~

Now let $V_\theta = \Omega r$

33.8 Force on a current carrying wire

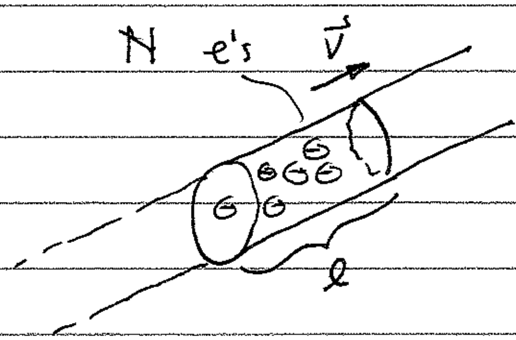


inside the wire we have stationary charges ~~(+/-)~~ BOTH POSITIVE & Negative

moving charges - usually electrons (why?) Called free electrons

* In any length of ~~whire~~ wire the number of positives and negative charges is essentially same (unless wire is charged)

* Can still be a net current even if electrons are moving slowly because there are so many free electrons



consider a segment of ~~eg~~ length ℓ

Suppose there are N free electrons (moving)

$\vec{V}$ is mean velocity of electrons

~~in that~~

~~\#~~ <u>To calculate current</u>

How long will it take all N to leave the volume? (Of course they will be replaced by new e's)

$$\Delta t = \ell / v$$

so during time interval Δt a net charge $Q = \cancel{eN} - eN$

flows through any cross section of the wire

Thus the current in the wire is

$$I = \frac{|Q|}{\Delta t} = \frac{eNv}{\ell}$$

Since ~~we a~~ electrons are flowing the direction of the current is opposite to $\vec{v}$

What is the force on those N electrons?

Force on a single electron

$$\vec{F} = -e\vec{V}\times\vec{B}$$

~~Let~~

Force on the N electrons

$$\vec{F}_{wire} = -eN\vec{V}\times\vec{B}$$

NOW FROM ABOVE

$$eN|\vec{V}| = I\ell$$

Let's make ℓ a vector

$-eN\vec{V} = I\vec{\ell}$ where $\vec{\ell}$ points ~~in~~ ~~direction of current~~ parallel to wire in direction of current (Not ~~a~~ electron velocity)

$$\vec{F}_{wire} = I\vec{\ell}\times\vec{B}$$

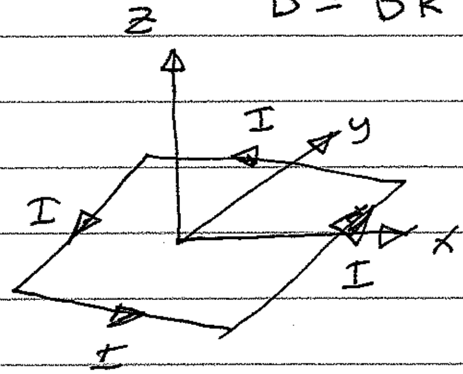
Now the fact that current is carried by negative or positive charges is not relevant

FORCE is $\perp$ to both $\vec{B}$ and $\vec{\ell}$

33.9

~~Force~~ Torque on a current loop

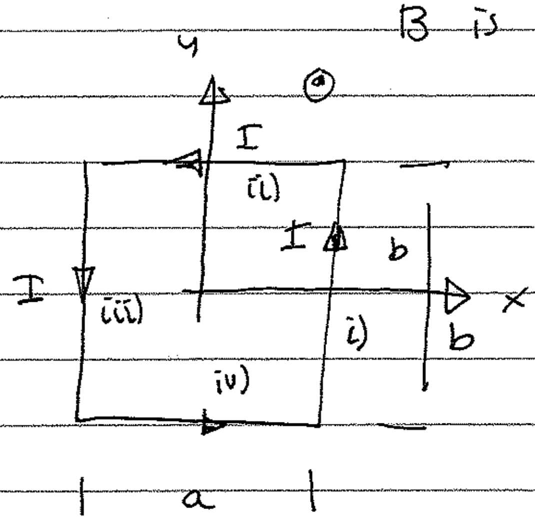
$\vec{B} = B\hat{k}$



current loop is a rectangle in x-y plane $a \times b$

TOP VIEW

$\vec{B}$ is out of page



what is force on each segment?

i) To the right $\hat{i}\ IbB$

ii) up $\hat{j}\ IaB$

iii) To the left $-\hat{i}\ IbB$

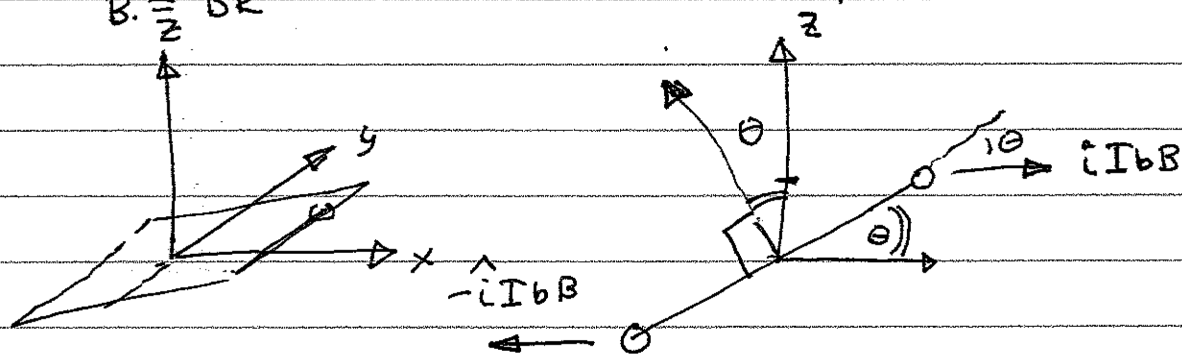
iv) down $-\hat{j}\ IaB$

Net force is zero $\vec{F} = \hat{i}\,(IbB - IbB) + \hat{j}\,(IaB - IaB) = 0$

(Net force on any loop in a uniform field is ZERO. Non uniform $\vec{B}$ can have a net Force

What is torque?

$\vec{B} = B\hat{k}$



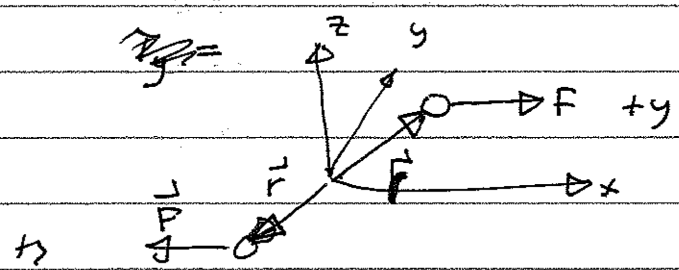
Plane of loop is tilted by angle θ with respect to direction of $\vec{B}$ ($\hat{k}$)

The loop wants to move so that axis and $\vec{B}$ are aligned

~~Torque around y-axis~~

$\cancel{\tau_y =}$ Remember $\vec{\tau} = \vec{r} \times \vec{F}$

$\cancel{\tau_y =}$



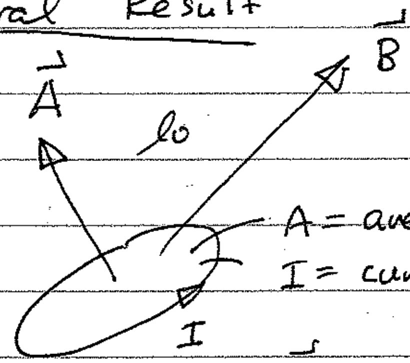
$\vec{r} \times \vec{F}$ is in +y direction

$$\tau_y = 2\left(\frac{a}{2}\right) IbB \sin\theta = I(ab)B\sin\theta$$

area of loop $ab = A$

call $\mu = IA$ call d

General Result



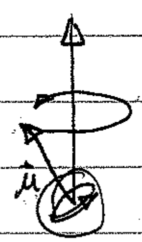
A = area of loop
I = current

$\vec{A}$ = vector in direction determined by Right hand rule

$$\vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

DIRECTION OF torque is to align $\vec{\mu}$ & $\vec{B}$



loop precesses like a top

* MAGNETIC FIELD DUE TO A Current

We have been holding off on this topic now is the time to bite the bullet
~~General~~ ~~results~~

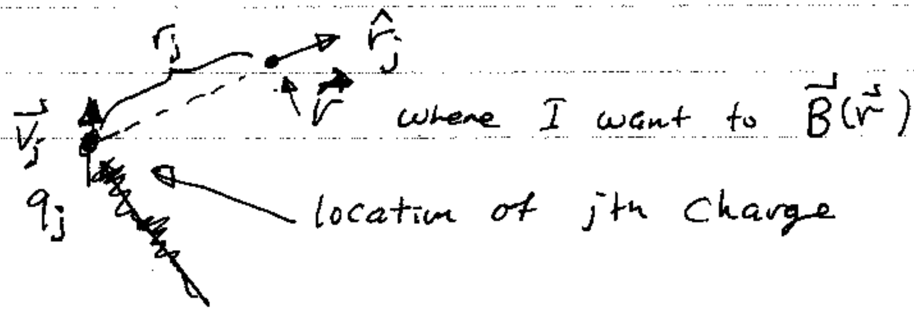
* Magnetic field due to a single charge moving at velocity $\vec{v}$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{(q\vec{v}) \times \hat{r}}{r^2}$$

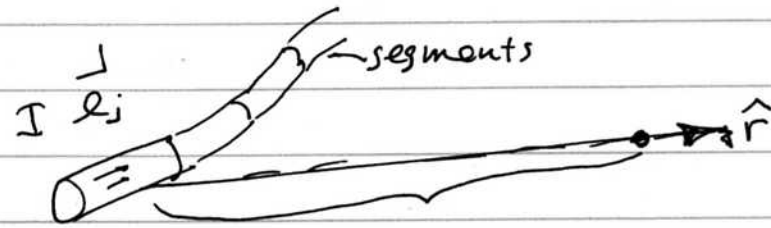
IF many charges apply principle of super position

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \sum_{q_j} \frac{q_j \vec{V}_j \times \hat{r}_j}{r_j^2}$$

$\hat{r}_j =$ unit vector from charge q_j pointing to location $\vec{r}$ where I want to know $\vec{B}(\vec{r})$



When currents in wires are involved we sum over charges in small segments of the wire



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \sum_{\substack{\text{line} \\ \text{segments},\, j}} \frac{I\,\vec{l}_j \times \hat{r}_j}{r_j^2}$$

Each segment will have a differnt $\vec{l}_j$

r_j and $\hat{r}_j$ will be different ~~too~~ for each segment too.

~~let $\vec{r}$ be where you want to know $\vec{B}$~~

~~$\vec{r}_j'$ by location of source segment~~

~~$r_j = |\vec{r} - \vec{r}'|$~~

When you take the limit of many small segments, the sum becomes an integral

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \sum_{\substack{\text{line} \\ \text{segments}}} \frac{I\, \vec{\ell}_j \times \hat{r}_j}{r_j^2} \Rightarrow \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

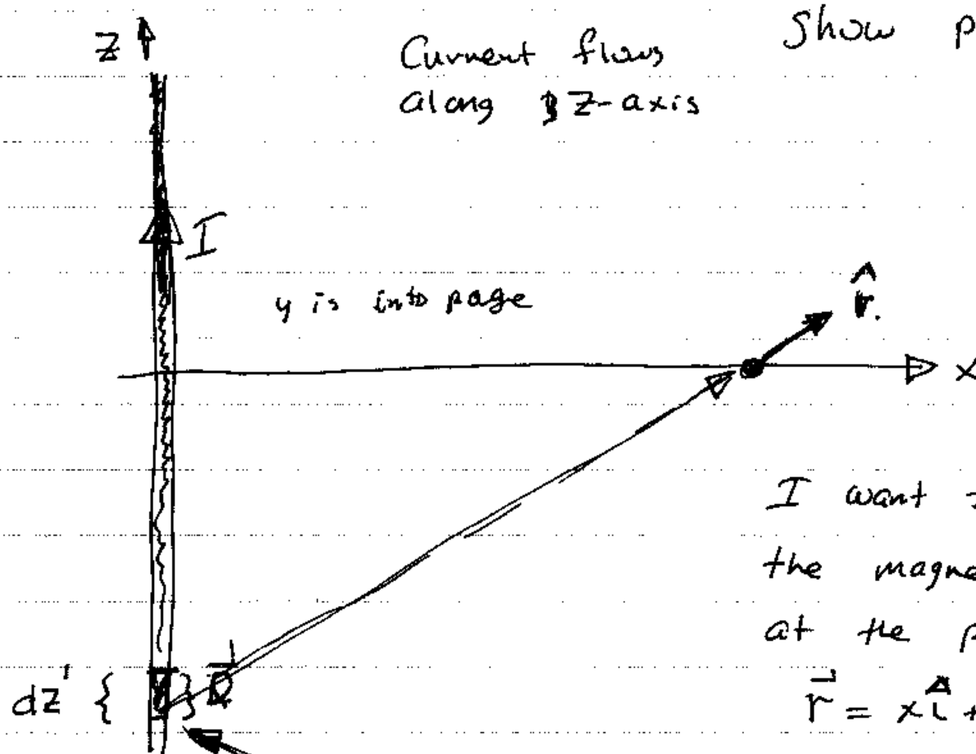
This expression looks simple enough but you have to keep track of two position vectors:

1) $\vec{r}$ which is where you want to know $\vec{B}$. call this the observation point

2) $\vec{r}\,'$ which is the location of the current segments the contribute to $\vec{B}$. This is the variable that you integrate over

Example magnetic field due to an infinitely long wire

Show picture



I want to find the magnetic field at the point $(x,0,0)$

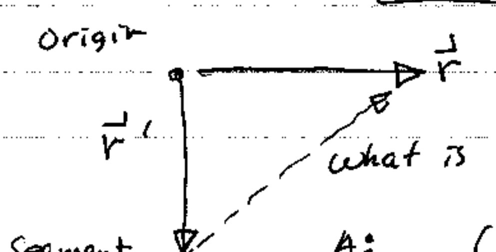
$\vec{r} = x\hat{\imath} + 0\hat{y} + 0\hat{k}$

Segment located at point $\vec{r}\,' = 0\hat{\imath} + 0\hat{y} + z'k$

prime denotes location of current

unprimed denotes location where I want to ~~know~~ know $\vec{B}(\vec{r})$

$\boxed{\text{Summing over segments} \Rightarrow \text{integral over } z'.}$



what is this vector?

A: $(\vec{r} - \vec{r}\,') = (x-0)\hat{\imath} + (0-0)\hat{\jmath} + (0-z')\hat{k}$

Things we will need for the integral:

$$d\vec{l} = dz' \hat{k}$$

$$r^2 = |\vec{r} - \vec{r}'|^2 = \sqrt{(x-0)^2 + (0-0)^2 + (0-z')^2}$$

$$= \sqrt{x^2 + z'^2}$$

$$\hat{r} = \frac{\vec{r} - \vec{r}'}{r} = \frac{x\hat{i} \cancel{+0} - z'\hat{k}}{\sqrt{x^2 + z'^2}}$$

So, putting things together

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int_{-\infty}^{\infty} dz' \, \frac{\hat{k} \times (x\hat{i} - z'\hat{k})}{(x^2 + z'^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} dz' \, \frac{x(\hat{k} \times \hat{i}) - z'(\hat{k} \times \hat{k})}{(x^2 + z'^2)^{3/2}}$$

$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{k} \times \hat{k} = 0$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\,\hat{J}\int_{-\infty}^{\infty}\frac{x\,dz'}{(x^2+z'^2)^{3/2}}$$

In the lecture notes I show

$$\int_{-\infty}^{\infty}\frac{x\,dz'}{(x^2+z'^2)^{3/2}} = \frac{2}{x}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi x}\,\hat{J}$$

Evaluation of

$$I_{nt} = \int_{-\infty}^{\infty} \frac{dz' \; x}{(x^2+z'^2)^{3/2}}$$

Evaluate by trigonometric substitution

Range

Let $\quad z' = x\tan\theta \quad \Rightarrow \quad z' = \pm\infty \;,\; \theta = \pm\pi/2$

$$dz' = \frac{x\,d\theta}{\cos^2\theta}$$

$$(x^2+z'^2)^{3/2} = \left(x^2(1+\tan^2\theta)\right)^{3/2} = x^3\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)^{3/2} = \frac{x^3}{\cos^3\theta}$$

$$I_{nt} = \int_{-\pi/2}^{\pi/2} \frac{x\,d\theta}{\cos^2\theta}\,\frac{x}{x^3/\cos^3\theta} = \frac{1}{x}\int_{-\pi/2}^{\pi/2} d\theta\cos\theta = \frac{2}{x}$$